ILLUSTRATIONS

OF

SCIENCE.

BY

PROFESSORS OF KING'S COLLEGE.
LONDON.

VOL. I.

LONDON:
LONGMAN, ORME, BROWN, GREEN, & LONGMANS,
PATERNOSTER-ROW.
1839.

ILLUSTRATIONS

OF

MECHANICS.

LONDON:
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ILLUSTRATIONS

OF

MECHANICS.

BY THE

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KING'S COLLEGE, LONDON.

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INTRODUCTION.

THIS work is the first of a series, entitled ILLUSTRATIONS OF SCIENCE, by Professors of King's College, London, to be published at intervals of three months, and continued until the circle of the Physical Sciences, and the Sciences of Observation, is embraced in it. The author has proposed to himself the development of that system of experimental facts and theoretical principles on which the whole superstructure of mechanical art may be considered to rest, and its introduction, under an available form, to the great business of practical education. To effect this object, and to reconcile, as far as it may be possible, the strictly scientific with the popular and elementary character of the undertaking, a new method has been sought, the nature of which is sufficiently indicated by its title — Illustrations of Mechanics. The work consists, in fact, of a series of illustrations of the science of mechanics, arranged in

the order in which the parts of that science succeed each other, and connected by such explanations only, as may serve to carry the mind on from one principle to another, and enable it to embrace and combine the whole—a plan which leaves to the author the selection of such elements only of his science as are capable of popular illustration, and as come within the limits of practical instruction; and which enables him to exclude from his work all abstract reasoning, and mathematical deduction.

Throughout, an attempt is made to give to the various illustrations an entirely elementary and practical character; and each illustration forming a short distinct article, the subject of which is enunciated at the commencement of it, the work has assumed a broken form, adapted peculiarly, it is conceived, to the purposes of scholastic instruction.

It is an idea which presents itself to the mind of every man who has children to educate and provide for, which is a constant subject of comment and discussion, and which prevails through all classes of society, that a portion of the school life of a boy ought to be devoted to the acquisition of those general principles of practical knowledge of which the whole business of his

subsequent life is to form a special application; that there ought, in fact, to be commenced by him at school a common apprenticeship to those great elements of knowledge, on which hang all the questions of interest which are to surround him in nature, and which are destined, under the form of practical science, to take an active share in the profession, trade, manufacture, or art, whatever it may be, which is hereafter to become the occupation of his life.

It is the object of this work, and of the Series of which it forms part, to promote this great business of PRACTICAL EDUCATION, by supplying to the instructors of youth a system of elementary science, adapted to the ordinary forms of instruction. No one can doubt that the same capabilities in the scholar, united to the same zeal in the master, which now suffice to carry the elements of a classical education to the very refinements of philological criticism, would be equal to the task of instruction in the nomenclature of the physical sciences, their fundamental experiments, their elementary reasonings, and their chief practical results; nor can it be questioned that the ordinary intelligence of youth, and common diligence on the part of their teachers, would enable them to master the secrets of

the more important of the arts, and the chief processes of the manufactures; and would place within their reach the elements of natural history, the general classification of the animal and vegetable kingdoms of nature, and their various ministries to the uses of man.

These are elements of a knowledge which is of inestimable value in the affairs of life; and the interests of this great commercial and manufacturing community claim that they should no longer be left to find their way to the young mind (if, indeed, they reach it at all) rather as a relaxation of the graver business of education than as a part of it.

That instruction which does not unite with all other knowledge the knowledge of those great truths of religion on which rests, as its foundation, the fabric of human happiness, can at best be considered but as a questionable gift. As a work of education, therefore, any treatise which, having for its object the development of principles of natural knowledge, did not point to the great Author of nature, would be an imperfect work; and, more than this, such a work, considered in a *scientific* point of view, would assuredly bear on its face a blemish; for, were it not an impiety to discuss the infinite mani-

festation of wisdom and goodness in creation otherwise than with sentiments of reverence to the Creator, and deep humility before him, it could at best be considered but as an affectation and a folly. It is under the influence of this conviction that, in the following work, the laws of the natural world have been taught—where the opportunity has been presented — with a direct reference to the power, the wisdom, and the goodness of God.

The illustrations of the mechanical properties of matter and the laws of force are drawn *promiscuously* and almost equally from ART and NATURE.

It is not by *design* that examples taken from these distinct sources thus intermingle, but simply because they suggest themselves as readily from the one source as the other — from nature as abundantly as from art.

An important truth is shadowed forth in this fact.

There is a RELATION between ART and NATURE — a relation amounting to *more* than a resemblance; —a relation by which the eye of the practical man may be guided to that God who works with him in every operation of his skill, and mechanical art elevated from a position

which is sometimes unjustly assigned to it among the elements of knowledge. It cannot be misplaced in this commencement of a work, which has for its object to develop the great principles of natural science, and which bears upon its title the arms and the motto of an institution formed to unite instruction in the precepts of religious knowledge with the elements of human learning, to point out this relation. The following illustration will serve the purpose, and will assimilate with the general method of the work: —

"I take up a work of *art*, I examine it, I see on it stamped the evidence of the power and skill, the judgment and knowledge, of the maker: there is the evidence of *design* in it, there is proof of the economy of *labour* — its material is suited for its *use*, and as little of it as possible is *used*, and its form is controlled by a perception, however imperfect, of the *beauty* and regularity of *form*. These are things, the evidence of which I perceive in the thing itself. It matters not that I saw it not *made*, — that I know not the maker — that he has never instructed me in the secret of his art: for centuries he may have been dead, and may have left no record of the manner of his working.

This matters not, I see plainly the *design* with

which he wrought. The thoughts of his mind rise up before mine as though I were present to them — *stamped* upon it are the traces of intelligence, power, and skill, which have operated in its formation — *invisible* things — no hand any longer works in it — no skill has any longer its visible exercise in it — no name is inscribed upon it — no legend records for me the fact that *there* wisdom, knowledge, and power, were exercised — yet is the existence of these things, and their exercise in that work of art, among the most certain elements of my knowledge: — my reason claims for me the admission of these among the most certain of the things that I may know, deduced by no new or unaccustomed operation of my mind, but by processes of thought which I am daily in the habit of verifying.

Now let me take up a work of nature, and place it beside that thing of art. Evidence such as that which I have found in the artificial thing is to be sought only *in* the thing itself, and essentially *belongs* to it. I may seek it then in this work of nature, as in that of art, and it may, or it may not, be found *here*, as it was found there. — By every mark and sign that I judged of that work of art I judge of this of

nature — every rule, which I applied to the one, I apply to the other; and the conclusion which I draw from the one, with a certainty that never, as I know by experience, fails me, I draw with equal certainty from the other.

Is there in the work of art the evidence of means to an end? I behold the very same evidence in the work of nature. Is there an adaptation of the *material*, in the one case? there is the like in the other. Is the artificial thing collected and arranged as to all its elements for a specific object, to which each element is made subordinate? so is the natural thing. Is the contrivance of the one complicated, involving many subsidiary contrivances, all having their direction towards an ultimate result? so is that of the other. Does the work of art manifest an economy of material and of labour in its construction? there is the like economy apparent in the work of nature. Subject to the adaptation of the form of the artificial thing to its use, and to the economy of its material, and the labour bestowed upon it, is the disposition of its parts governed by a certain perception of beauty and of grace — who shall describe the beauty of nature?

The only difference is, indeed, this, that in

the work of nature all these qualities exist in their infinite *perfection* — in the work of art, in their infinite *imperfection*. The evidence is perfectly alike in kind, although it is the evidence of things infinitely remote in degree.

With whatever certainty, then, I reason of the finite wisdom and power of the artificer from that work of his art, with the same certainty do I reason of the infinite wisdom and power of the eternal God from the works of his hand; and on this evidence I declare with St. Paul, that "the invisible things of him from the beginning of the world are manifest, being plainly seen by the things which are made, even his eternal power and Godhead." (Rom. i. 20.)"

Every work of human art or skill is a thing done by a *creature* of God; a creature MADE IN HIS OWN IMAGE, and operating upon matter governed by the same laws, which HE, in the beginning, infixed in it, and to which he subjected the first operations of his own hands — a creature in whom is implanted *reason* of a like nature with that excellent wisdom by which the heavens were stretched forth — living power as that of a worm, and as a vapour that passeth away, but an emanation of Omnipotence — a perception of beauty and adaptation akin to

that whence flowed the magnificence of the universe — and to control these, a volition, whose freedom has its remote analogy and its source in that of the first self-existent and independent Cause.

It is from this relation between the Author of nature and the being in whom the works of art have their origin that arise those relations, infinitely remote, but distinct, between the things themselves, of which the evidence is every where around us. These are *necessary* relations: it is not that the works of art are made by any purpose or intention in the resemblance of those of nature, or that there is any unseen influence of nature itself upon art — the primary relation is in the causes whence these severally proceed.

Thus it is possible, that in the infinities of nature, every thing in art may find its type; this is not, however, *necessarily* the case, since the causes are infinitely removed, since, moreover, in their operation, these causes are independent, and since nature operates upon materials which are not within the resources of art.

How full of pride is the thought, that in every exercise of human skill, in each ingeniou adaptation, and in each complicated contrivanc

and combination of art, there is included the exercise of a faculty which is akin to the wisdom manifested in creation!

And how full of humility is the comparison which, placing the most ingenious and the most perfect of the efforts of human skill by the side of one of the simplest of the works of nature, shows us but one or two rude steps of approach to it.

How full, too, is it of profit thus to see God in every thing — to find him working *with* us, and *in* us, in the daily occupations of our hands, wherein we do but reproduce, under different and inferior forms, his own wisdom and creative power.

A man may thus hold converse with God as intelligibly in art as in nature, and live with him in the workshop, as he may go forth with him in the fields and upon the hills. And whilst he *feels* himself in those faculties of thought and action, the exercise of which constitute his physical being, to be in very deed a creature made in the image of God, he will not fail to be reminded that the resemblance once embraced with these the qualities of his moral being.

If we conceive space spreading out its dimen-

sions infinitely, still through all its interminable fields does science show it to us peopled with matter—stars upon stars innumerable—a vista in which suns and systems *crowd* themselves, and to which imagination affixes no limit. If, in like manner, we conceive space to be infinitely divided—as its dimensions grow before the eye of the mind yet less and less—still does it appear a region peopled with the infinite divisions of matter.

On either side is an abyss—an interminable *expanse*, through which the creative power of God manifests itself, and an unfathomable *minuteness*.

It is in this last mentioned region of the inaccessible minuteness of matter that the principles of the science treated of in the following pages have their origin. Matter is composed of elements, which are inappreciably and infinitely minute; and yet it is within the infinitely minute spaces which separate these elements that the greater number of the forces known to us have their only sensible action. These, including compressibility, extensibility, elasticity, strength, capillary attraction and adhesion, receive their illustration in the first three chapters of the following work. The

fourth takes up the Science of Equilibrium, or Statics; applies in numerous examples the fundamental principles of that science, the parallelogram of forces, and the equality of moments; then passes to the question of *stability*, and to the conditions of the resistance of a surface; traces the operation of each of the mechanical powers under the influence of friction; and embraces the question of the stability of edifices, piers, walls, arches, and domes.

The fifth chapter enters upon the Science of Dynamics. Numerous familiar illustrations establish the permanence of the force which accompanies motion — show how it may be measured — where in a moving body it may be supposed to be collected — exhibit the important mechanical properties of the centres of spontaneous rotation, percussion, and gyration — the nature of centrifugal force, and the properties of the principal axes of a body's rotation — the accumulation and destruction of motion in a moving body, and the laws of gravitation.

The last chapter of the work opens with a series of illustrations, the object of which is to make intelligible, under its most general form, the principle of virtual velocities, and to protect practical men against the errors into which, in

the application of this universal principle of mechanics, they are peculiarly liable to fall: it terminates with various illustrations of those general principles which govern the reception, transmission, and application of power by machinery, the measure of dynamical action, and the numerical efficiencies of different agents — principles which receive their final application in an estimate of the dynamical action on the moving and working points of a steam engine.

The Appendix to the work contains a detailed account of the experiments of Messrs. Hodgkinson and Fairbairn upon the mechanical properties of hot and cold blast iron: and an extensive series of tables referred to in the body of the work, and including, 1. Tables of the strength of materials; 2. Tables of the weights of cubic feet of different kinds of materials; 3. Tables of the thrusts of semi-circular arches under various circumstances of loading, and of the positions of their points of rupture; 4. Tables of co-efficients of friction, and of limiting angles of resistance, compiled and calculated from the recent experiments of M. Morin. The results of these admirable experiments, made at the expense of the French

government, are here, for the first time, published in this country.

The author has also to acknowledge his obligations to the "Physique" of M. Pouillet, for several valuable illustrations and drawings.

The articles marked with an asterisk, and the whole of the sixth chapter, are recommended to be omitted on the first reading.

ERRATA.

Page 27. last line but one, *for* "is proportional" *read* "is inversely proportional."

67. line 1. *for* "the lower flanch by the depth" *read* "the lower flanch, in inches, by the depth."

129. line 11. from the bottom, *for* "*nine*" *read* "*eight*," and on the same page, in line 4. from the bottom, *for* "9" *read* "8," and in the last line, *for* "9" *read* "8."

190. line 2. *for* "the ratio rose to" *read* "the ratio sank to."

line 4. *for* "it sank again" *read* "it rose again;" line 7. *for* "diminished" *read* "increased."

269. line 15. *for* "five times" *read* "ten times."

CONTENTS

OF

THE ILLUSTRATIONS OF MECHANICS.

CHAP. I.

THE INFINITE MINUTENESS OF THE ELEMENTS OF MATTER.—THE POROSITY OF MATTER—ITS COMPRESSIBILITY—ITS ELASTICITY.

THE POROSITY OF MATTER.

COMPRESSIBILITY.

ELASTICITY.

CHAP. II.

THE STRENGTH OF MATERIALS.

THE FORCES PRODUCING EXTENSION OR COMPRESSION. — THE LIMITS OF ELASTICITY. — RUPTURE. — THE STRONGEST FORMS OF CAST-IRON BEAMS AND COLUMNS. — WOOD AS A MATERIAL IN THE ARCHITECTURE OF NATURE. — THE MECHANICAL PROPERTIES OF METALLIC SUBSTANCES AS AFFECTED BY THEIR INTERNAL STRUCTURE.

CHAP. III.

CAPILLARY ATTRACTION, AND ADHESION.

CHAP. IV.

STATICS.

CHAP. V.

DYNAMICS.

THE FORCE OF MOTION. — ITS PERMANENCE. — THE MEASURE OF IT. — THE POINT WHERE IT MAY BE SUPPOSED TO BE COLLECTED. — MOTIONS OF TRANSLATION AND ROTATION, INDEPENDENT. — THE CENTRE OF GYRATION. — THE CENTRE OF SPONTANEOUS ROTATION. — THE CENTRE OF PERCUSSION. — THE PRINCIPAL AXES OF ROTATION. — THE FORCE OF A BODY'S MOTION IS NEVER GENERATED OR DESTROYED INSTANTANEOUSLY. — ACCELERATING FORCE. — GRAVITATION. — CAVENDISH'S EXPERIMENTS. — DESCENT OF A BODY FREELY BY GRAVITY. — ATWOOD'S MACHINE. — DESCENT OF A BODY UPON AN INCLINED PLANE AND UPON A CURVE. — THE CYCLOIDAL PENDULUM. — THE SIMPLE PENDULUM. — THE CENTRE OF OSCILLATION. — KATER'S PENDULUM. — THE COMPENSATION PENDULUM.

CHAP. VI.

THE RETARDATION OF MOTION. — THE PRINCIPLE OF VIRTUAL VELOCITIES. — THE MEASURE OF THE DYNAMICAL EFFECT OR THE ACTION OF AN AGENT. — THE DYNAMICAL EFFECTS OF DIFFERENT AGENTS. — THE MOVING AND WORKING POWERS IN A MACHINE. — THE MOVING AND WORKING POWERS IN ANY MACHINE ARE EQUAL, ABSTRACTION BEING MADE OF THE RESISTANCES WHICH OPPOSE THEMSELVES TO THE MOTIONS OF THE PARTS OF THE MACHINE UPON ONE ANOTHER. — THE MOVING POWER IN A STEAM-ENGINE. — THE WORKING POWER IN A STEAM-ENGINE.

APPENDIX.

THE

ILLUSTRATIONS OF SCIENCE,

(BY PROFESSORS OF KING'S COLLEGE, LONDON,)

Will appear in the following order:

ILLUSTRATIONS OF ZOOLOGY (in 2 Vols.). By THOMAS BELL, F.R.S. Professor of Zoology in King's College. Vol. I.—*In June.*

ILLUSTRATIONS OF EXPERIMENTAL PHILOSOPHY (in 2 Vols.). By C. WHEATSTONE, F.R.S. Professor of Experimental Philosophy in King's College. Vol. I.—*In September.*

ILLUSTRATIONS OF ZOOLOGY (in 2 Vols.). By THOMAS BELL, F.R.S. Professor of Zoology in King's College. Vol. II.—*In December.*

The following to be published early in the year 1840:

ILLUSTRATIONS OF NATURAL PRODUCTS USEFUL IN THE ARTS AND MANUFACTURES. By J. F. ROYLE, M.D. Vice-President of the Royal Sooiety; Professor of Materia Medica and Therapeutics in King's College.

ILLUSTRATIONS OF ANIMAL PHYSIOLOGY. By R. B. TODD, M.D. F.R.S. Professor of Physiology and General Anatomy in King's College.

ILLUSTRATIONS OF EXPERIMENTAL PHILOSOPHY. By C. WHEATSTONE, F.R.S. Professor of Experimental Philosophy in King's College. Vol. II.

ILLUSTRATIONS OF HYDROSTATICS AND HYDRODYNAMICS. By the Rev. H. MOSELEY, M.A. F.R.S. Professor of Natural Philosophy and Astronomy in King's College.

ILLUSTRATIONS OF BOTANY AND VEGETABLE PHYSIOLOGY, ASTRONOMY, PHYSICAL GEOGRAPHY, &c. will follow in succession.

ILLUSTRATIONS

OF

MECHANICS.

CHAPTER I.

THE INFINITE MINUTENESS OF THE ELEMENTS OF MATTER. THE POROSITY OF MATTER — ITS COMPRESSIBILITY — ITS ELASTICITY.

THE limits of *observation* are soon passed, and imagination almost as soon wearied, when we seek for the *ultimate divisions of matter* and its *atoms.*

Among the many illustrations which offer themselves of the extreme minuteness of its elements are the following: —

1. GLOBULES OF BLOOD.

Blood, when examined under the microscope, is seen to be composed of a transparent colourless liquid, called *serum*, and certain minute *globules* which float in it and give it its colour. These contain, as there is every reason to believe, the principle of the *nourishment* of the body, of which the *serum* is the vehicle. This composition of the blood, which has been found whenever search has been made for

it, we may fairly conclude to be *an essential part* of the animal economy, and to extend through every form of living organisation, even to the lowest.

Although called globules, they are not of a spherical form, but either cylindrical or lens-shaped, and not unfrequently they are to be seen floating in the serum, packed together thus  in groups. In all mammiferous animals they are circular. In birds and fishes their form is elongated. In *man* each globule has a diameter varying from the two thousandth to the four thousandth of an inch. Now there are creatures so small, yet *visible* by the aid of microscopes, that their whole living organisation might be included in the bulk of one globule of human blood; limbs for motion, for defence, and to provide themselves with food; organs of sense and of deglutition; sinews, muscles, nerves: nay, a circulating medium—*blood* composed of serum, and having its own globules of blood. In the milky juice —which is the blood — of certain *plants* also, as the Euphorbia and the Ficus, may be seen globules, like those of the blood of animals, but greatly less; they are probably as essential a part of the vegetable as of the animal economy, extending throughout it, and to its minutest forms.

If the imagination be not yet wearied, it may conceive each of these globules *divided* in respect to the atoms which compose it. Still the minuteness of the elements of matter will never be reached; for the gelatinous consistency of the globule shows that these its component atoms, are *infinite* in number.

2. Crystals of the Cells of Plants.

In certain portions of the cellular texture of many plants, as in the cells of the flower stem of the hyacinth, the bulb of the lilly, and of the squil, &c., may be seen by the aid of the microscope, crystals regularly and perfectly formed, composed, it is said, of oxalate of lime. The very fact of their crystallisation proves to us (by every analogy) that each one of these crystals has an infinity of component atoms. Now, in the cuticle of the Scilla maritima, are to be found such crystals, one five thousandth of an inch in length, and one eighth thousandth of an inch in their greatest thickness.

3. Musk.

It is said that a grain of musk is capable of perfuming for several years a chamber twelve feet square without sustaining any sensible diminution of its volume or its weight. But such a chamber contains 2,985,984 cubic inches, and each cubic inch contains 1000 cubic tenths of inches, making, in all, nearly three billions of cubic tenths of an inch. Now it is probable, indeed almost certain, that each such cubic tenth of an inch of the air of the room contains one or more of the particles of the musk, and that this air has been changed many thousands of times. Imagination recoils before a computation of the number of the particles thus diffused and expended. Yet have they altogether *no appreciable weight or magnitude.*

4. Dust of the Lycoperdon.

The lycoperdon, or puff-ball, is a fungus growing in the form of a tubercle, which, being pressed, bursts, emitting a dust so fine and so light that it floats

through the air with the appearance of smoke. Examined under the microscope, this dust, which is the seed of the plant, appears under the form of globules of an orange colour, perfectly rounded, and in diameter, it is said, about the fiftieth part of a hair; so that, if this calculation be correct, and a globule were taken having the diameter of a hair, it would be one hundred and twenty-five thousand times as great as the seed of the lycoperdon.

5. Metallic Solutions.

Let one grain of copper be dissolved in nitric acid. A liquid will be obtained of a blue colour; and if this solution be mingled with three pints of water, the whole will be sensibly coloured.

Now three pints contain 104 cubical inches, and each linear inch contains at least one hundred equal parts distinguishable by the eye; each cubical inch contains, then, at least one million of such parts, and the 104 cubical inches of this solution 104 millions of such parts: also each of these minute parts of the solution is coloured, otherwise it would not be distinguishable from the rest; each such part contains then a portion of the nitrate of copper, — the colouring substance. Now from each particle of this nitrate, the copper may be precipitated in the state of a metallic powder — every particle of which is therefore less than the 104 millionth of a grain in weight.

6. Colours produced by the Attenuation of Transparent Bodies.

The extreme *attenuation* which may be given to certain forms of matter is a proof of the extreme

minuteness of their elementary particles. In the case of transparent bodies, there is a method of measuring the degree of this attenuation, founded on this principle of optics, "that all transparent bodies become *coloured* when they are formed into plates, attenuated beyond certain limits, and moreover, that the particular colours, which under these circumstances they show, are dependant upon the *degree* of their attenuation;" thus serving as a delicate test and measure of it, so that, knowing the colour, which by being attenuated, a transparent body is made to show, we may know how thin it is.

7. The Thickness of a Soap Bubble.

It is thus that Newton has determined the *top,* which is the thinnest part, of a soap bubble, to be when colours are first seen in it, the ·000,003,937th part, or about the twenty-five-thousandth part of an inch in thickness, and before it bursts to reach an attenuation of at least the four-millionth part of an inch.

8. Attenuation of the Wings of Insects.

By the same means we know that the transparent wings of certain insects, are not more than the hundred-thousandth of an inch in thickness, and that as great an attenuation as this may be given to glass, by blowing it in bubbles, until it bursts like the bubbles of soap.

The property of matter, by which it may be made to receive an extreme degree of attenuation, is of extensive application in the arts.

9. The Attenuation of Gold Leaf.

An ounce of gold is equal in bulk to a cube, each of whose edges is five-twelfths of an inch, or nearly half an inch, in length, so that placed upon a table it would cover nearly one quarter of a square inch of its surface, standing nearly half an inch in height. This cube of gold the gold-beater extends until it covers 146 square feet; and it may readily be calculated, that to be thus extended from a surface of $\frac{5}{12}$ths of an inch square to one of 146 square feet, its thickness must have been reduced from half an inch to the 290,636th part of an inch. Fifteen hundred such leaves of gold placed upon one another, would not equal the thickness of the paper on which this is printed.

10. The ordinary Process of Gilding.

Gilding, according to the process usually adopted in the arts, presents a remarkable example of the minute division, and the attenuation of which gold is capable.

The following is that process. Gold is dissolved in mercury in the proportion of one part to five or six, by placing the two metals in these proportions in an iron ladle and bringing them to a boiling heat. A half a pound troy of gold, in minute portions, may thus be dissolved in six times its weight of mercury in twenty or twenty-five minutes. This solution of gold in mercury is called an amalgam.

It may be thickened in its consistency by straining, by means of pressure through a piece of chamois leather through whose pores the mercury, not in actual union with the gold, escapes; or it may be diluted by heating again with more mercury. With this amalgam, the surface to be gilded, which is usually of copper or brass, is to be covered by means of a brush or otherwise; but that an intimate cohesion or union of the two may take place, it is found to be necessary first to wash over the surface with a liquid, technically called quick-water, which is made by dissolving about a table-spoonful of mercury into a quart of nitric acid. The effect of washing the surface with this liquid is, to cover it with an exceedingly thin amalgam of the metal which forms the surface. Although the amalgam of gold will not unite itself *directly* with the surface to be gilded, yet it will unite itself with this amalgam of the surface, and thus by the adherence to the surface of its own amalgam, and of the gold amalgam to that, both become fixed upon it.

If now the mercury could be removed, the particles of gold only would remain upon the surface, and the gilding would be complete. The property of mercury by which it is converted into a vapour like water at the temperature at which it boils, makes this an easy process.

The various articles thus covered with amalgam of gold have only to be subjected to a powerful heat in a kind of oven of iron specially contrived for that purpose, and the mercury is evaporated, nothing but the gold remaining, and the surfaces being gilded.

A polish is usually given to surfaces thus gilded by rubbing them with a polished mineral known to chemists as black hæmatite, which is a natural steel. This process is called burnishing. They are usually, moreover, subjected to a chemical process called colouring.

A *perfect* and *continuous* surface of gold is thus placed upon the gilded article, not the minutest aperture or uncovered space is perceivable in it with the most powerful magnifying glass or microscope. Nitric acid*, if it be washed with it, will find no aperture by which it may reach and attack the substratum of copper or brass. But what is the thickness of this coating of gold? It may be spread by the process above described more thinly upon brass than copper; surfaces of brass, when gilded, are said to be similored, and upon these a grain of gold is commonly made to cover about 40 square inches: this being the case, it may readily be calculated that the thickness of this coating of gold is about the $\frac{1}{233000}$th of an inch.

11. The Gilding of Thread for Embroidery.

This process is thus described by Reaumur as practised in his time. A cylinder of silver, 360 ounces in weight †, is cased with a cylinder of gold at most 6 ounces in weight. ‡ This cylindrical

* Nitric acid will not attack gold.

† The weights and measures spoken of in this article are French.

‡ A French inch equals $\frac{17}{16}$ths of an English inch, and a French ounce $\frac{98}{100}$ths of an English ounce.

mass of 366 ounces of metal is then drawn by a powerful force through a series of circular holes in a plate of steel continually diminishing in diameter, until it attains the state of a wire so thin that 202 feet in length weigh but the sixteenth of an ounce: the whole length of the wire into which it is now drawn being 1,182,912 feet, or about 98½ leagues. This wire is then passed between rollers which in the act of flattening it elongate it one-seventh, and its total length thus becomes 112½ leagues. The width of the flattened thread is now $\frac{1}{8}$th of a line, or $\frac{1}{96}$th of an inch; and supposing, with Reaumur, that a cubical foot of gold weighs 21,220 ounces, and a cubical foot of silver 11,523 ounces, it may readily be calculated that the thickness of this gilded thread is very nearly the $\frac{1}{3108}$th part of an inch. Now what is the thickness of the plate of gold which envelopes it? Calculating on the same principles as before, we readily arrive at the conclusion, that the thickness of this plate of gold is $\frac{1}{713136}$th of an inch. Now gilded threads are made by a process similar to this, in which only $\frac{1}{3}$d the proportion of gold is used. There is spread over these, therefore, a continuous plate of gold less than the two-millionth part of an inch in thickness. The silver may be taken out of its gold case by plunging the thread in nitric acid, by which the silver will be attacked through the extremities of the gold case and dissolved, whilst the gold will remain untouched by it. This being done, and the hollow gold case being examined, it is found to be a perfectly continuous plate, and to possess in this

state of extreme attenuation all the sensible and all the chemical properties which belong to the metal.

Another but less striking evidence of the minuteness of the elements of matter is found in the extreme tenuity of certain natural and artificial fibres and threads.

12. Tenuity of Fibres of Silk.

The thread of the silk-worm is a perfectly smooth cylinder, whose diameter is from the one thousand seven hundredth to the two thousandth part of an inch.

13. The Tenuity of Fibres of Wool.

Each hair of wool is a cylinder of from the seven hundredth to the two thousandth part of an inch in thickness, covered with what appear to be overlapping scales, which are laid in the direction in which the hair grows, and the roughness of which we feel when we draw our fingers along it in the opposite direction.

14. The Fibre of Cotton.

Under the microscope, each fibre of cotton-wool appears to be composed of two tubular cylinders, at a slight distance from one another, but joined together by a membrane. Its section is somewhat in the form of the figure 8. It is about the thousandth part of an inch in diameter.

15. The Fibre of Flax.

Each fibre of flax is a fasciculus of other fibres, which appear under the microscope jointed and ir-

regular. Some of these have been ascertained to be the two thousand five hundredth part of an inch in diameter. The appearance of the fibre of flax under the microscope is very different from that of cotton. It is by this difference that the fine cerecloths of the mummies have been determined, by Mr. Bauer, not to be of cotton fabric, but of linen.

16. Fibres of the Pine Apple Plant.

Some of these have been measured, and are ascertained to be from the five thousandth to the seven thousandth of an inch in diameter. They are perfectly cylindrical, and when twisted into threads and woven, are said to form cloths of a very beautiful texture, and to offer a useful substitute for silk.

17. Tenuity of the Fibres of a Spider's Thread.

The most remarkable example of the tenuity of a natural fibre is, however, to be sought in the spider's thread, of which two drachms by weight would, it is said, reach from London to Edinburgh.

It appears from the observations of Reaumur, that the thread of the spider results from the expulsion of a peculiar viscid matter through six teats under the animal's belly, each of which, being pierced by certain minute apertures, not less, probably, than one thousand in number, yields by each of these a separate fibre, which fibres, uniting with one another from each teat and adhering — and sometimes the com-

pound threads from different teats thus uniting — form those threads which we see composing the web. Now the head of each teat is so small as scarcely to be visible. What then must be the tenuity of the component fibres of the spider's thread, of which more than 1000 spring from the head of each teat?

It is by reason of the exceeding fineness of many natural threads, that they are made to minister so greatly to the luxury of life under those forms of woven tissues, for which the weavers of India were formerly, and our own manufacturers have been of late, so celebrated.

18. Tenuity of Cotton Yarn.

There is a specimen of Dacca muslin in the museum of the India House, of which the yarn, spun by the hand, was ascertained by Sir J. Banks to be so fine, that a weight of it equal to one pound avoirdupois would extend 115 miles, 2 furlongs, 60 yards. When the muslin made from this Dacca yarn is laid on the grass, and the dew falls upon it, it is said to *be no longer visible*. The natives, in their metaphorical language, call it *woven air*.

Cotton yarn has been spun by machinery in England, of which one pound would extend 167 miles; but this has never been woven.

There are various methods of drawing artificial threads and wires to an extreme tenuity.

19. Threads of Glass.

Glass is artificially drawn out almost to the fineness of the fibre of silk. A rod of glass is melted in the middle, in the flame of a blowpipe. One portion is then fastened to a small wheel, which being turned rapidly round, the melted glass is drawn out from the other part of the rod which is still held in the flame. Glass *tube* has been thus drawn out to the fineness of silk, *and liquids have afterwards been made to pass through it.* It is a remarkable fact, that whatever was the form of the bore of the original tube, the same form is retained in the *drawn* tube, however great may be its tenuity.

20. Platinum Wire.

Wires are used in the arts as fine almost as hairs. There is, however, a mechanical limit fixed to the thinness to which a wire can be *drawn*, by the force necessary to draw it; which force, when the wire becomes thin, breaks it. This limit has, however, been greatly passed by a method of art, of which the following is an illustration. Wishing to obtain a wire of extreme tenuity to be used in a micrometer, Dr. Wollaston placed a platinum wire one hundredth of an inch in diameter, in the axis of a cylindrical mould one-fifth of an inch in diameter, and cast round it a cylinder of silver. This cylinder he then drew out by the common method, until it became a wire so thin that it would no longer sustain the force necessary to draw it. This wire of silver, along the axis of which ran a wire of platinum, he then immersed in boiling nitric acid, by which the silver was dissolved, and a platinum

wire was separated, the *three millionth* of an inch in diameter; being an artificial thread of which 140 must be placed together to equal in thickness a fibre of the finest silk.

THE POROSITY OF MATTER.

All bodies have between their elementary particles or atoms, interstices through which *heat* penetrates into them, and into some of them, *air*, *water*, and other *fluids*. These last are said to be POROUS.

21. THE POROSITY OF WOOD.

Wood is but a fascicle of tubes permeated when it is growing by the sap. It is a common experiment with the air pump, to make mercury pass through these pores of wood. The mercury being placed in a cup, the bottom of which is a piece of wood cut transversely to the fibre, and this cup being hermetically fixed upon an aperture in the receiver of an air pump; when the air is extracted from beneath it in the receiver, the pressure of the external air on the surface of the mercury, no longer balanced by the elasticity of the air within the receiver, presses it with such force as to drive it through the pores of the wood. At the extremity of each pore a minute globule is seen, and these globules, at length, descend in a minute shower of silver. When wood is carbonised, its pores are very easily traced by means of the microscope. Dr. Hook found them extending through the whole length of the wood, and counted in the eighteenth of an inch 150 of them; so that in a piece of charcoal one

inch in diameter, there are more than five millions and a half of them.

22. Wood ceases to be buoyant when its Pores are filled with Water.

If a piece of wood be subjected to a great pressure of water in a hydraulic press, or by sinking it deep in the sea, the water will be driven into its pores, expelling from them the air, and remaining fixed in them by capillary attraction, the wood thus becomes too heavy to float. Being placed in the water, it will sink like lead. Boats used in the whale fishery have been dragged to great depths in the sea by the entanglement of the rope attached to the harpoon with which the whale has been transfixed. These, when brought to the surface again, have been found useless, by reason of the water which has been incorporated with them.

23. The Porosity of Rocks.

That many rocks are thus porous, the infiltration into caverns and the formation of stalactites sufficiently proves; and it is thus in winter when, in the act of freezing, the water they have imbibed *expands*, that their surfaces exfoliate, and they crumble away.

24. The Porosity of Hydrophane.

Among silicious stones is one called *hydrophane*, a kind of agate, whose porosity causes it to present a very remarkable phenomenon. In its ordinary state it is only semi-transparent, but after being plunged in water it takes up about $\frac{1}{6}$th of its bulk of it, and becomes nearly as transparent as glass.

25. Porosity of Metals.

That metals are porous was proved in 1661 by the academicians of Florence, who submitted a hollow ball of gold filled with water to a great pressure, by which the water was made to weep through the pores in the surface of the gold. This experiment has often been repeated.

That all bodies are more or less permeable to heat or porous to fluids, sufficiently accounts for the fact that all bodies are more or less compressible.

COMPRESSIBILITY.

In many bodies their compressibility is a property familiar to us.

A Sponge, for instance, by compression, gives out the water that it imbibes, and may thus be reduced to one third of its bulk.

26. Compressibility of Wood.

Wood is compressed by passing it between iron rollers to form the pins or bolts used in ship-building; it is thus commonly reduced to one half its bulk.

A cork immersed 200 feet in the sea, will be so compressed that, instead of rising when left to itself, it will sink. And a bottle of fresh water corked up and sunk a great depth in the sea, will return with the cork still in it as when it descended, but the water will be found to taste of salt. The cork has in fact *compressed* so as to allow the salt water to mingle with the fresh. Having at the same time,

become heavy, it has sunk in the bottle, and, as the bottle rose again to the surface, it has expanded to its original dimensions, rising and re-occupying its place in the neck of the bottle.

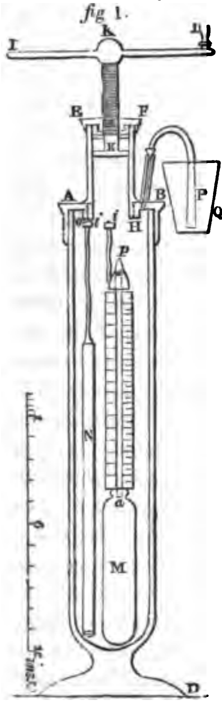
27. Compressibility of Aëriform Bodies.

Of all the different forms of matter, the aëriform is that under which it is most compressible. In some recent experiments, a large body of air has been mechanically compressed by Œrsted, a Danish philosopher, into the one hundred and tenth part of its original bulk. He used for this purpose powerful forcing-pumps originally constructed for compressing air into the receivers of certain air-guns belonging to the king of Denmark. It is not only common air that is thus compressible, but all aëri-from bodies. Thus, the gas used in our streets is so compressible that a sufficient quantity may be forced into an iron bottle of comparatively small dimensions, to supply a number of lights for a considerable time. The stand which supports a light, being cast hollow, has thus been made the reservoir, whence gas was supplied to it, sufficient to feed the flame for several evenings. A company was a few years ago established for the purpose of selling gas under this compressed form as portable gas. It was commonly sold thus compressed under a pressure of 450lb. on the square inch, into $\frac{1}{30}$th part of its ordinary bulk.

* 28. Compressibility of Water.

The compressibility of water was long disputed. The question has lately, however, been set com-

pletely at rest by the experiments of Œrsted.* The apparatus used by him was that represented in the accompanying figure; ABCD is a strong glass



cylindrical vessel, having firmly affixed to it at the top a cylinder of smaller dimensions of metal, AEFB, in which is moveable, by means of a screw, an air-tight piston K. M is a glass bottle, into the neck of which is fixed, by grinding, one extremity of a capillary tube *aa*, which is open at both ends. The bore of this tube must be extremely fine, and the precise fraction of the contents of the whole bottle, which each inch in length of its bore will hold, must be ascertained with great accuracy. This is done by weighing the quantity of mercury which the bottle will hold, and the quantity which an inch of the bore of the tube will hold. Whatever fraction the one weight is of the other, the same is evidently the contents of one inch of the tube of the content of the bottle. In some of the tubes used by Œrsted, each inch in length was found to hold 80 *millionths* of the contents of the bottle. Let us suppose these to have been the tubes

* Transactions of the Royal Society of Sciences at Copenhagen, 1818—1822.

with which his experiments were made. Let now the bottle and tube be conceived to be filled with water. Any pressure exerted upon this water which will have caused its surface in the tube to descend one inch will have compressed it by 80 millionths of its bulk. Divisions were, however, marked upon a scale annexed to the tube $\frac{1}{40}$th of an inch apart. A depression of the water in the tube through any one of these divisions would therefore indicate a compression of two millionths. But how is this compression to be produced? The bottle, and its apparatus, are to be introduced into the glass vessel ABCD, the part AEFB having been screwed off to admit them. This vessel is then to be filled with water, and the cylinder AEFB is to be replaced, its piston K having been first screwed down to H. This piece being firmly fixed, and the piston then screwed back towards its position K, water will be drawn into the vessel by the syphon BP, which communicates with a vessel of water Q. When it is full a cock closes the communication of the syphon with the vessel, and the piston K being screwed back again, or downwards, the pressure begins.

From the piston and the water in the vessel the pressure is propagated through the tube *aa* to the water in the bottle; and the pressure thus produced within and without the bottle is precisely the same.*

But how is this pressure to be measured? By this simple contrivance:—N is a glass tube, closed

* By the law of the equal distribution of fluid pressure. See *Mechanics applied to the Arts*, Art. 243.

at the top and open at the bottom, and equal divisions are marked along it. This tube, being loaded by a rim of lead at the bottom, is immersed in the water of the vessel, in its inverted position, at the same time that the bottle and tube are introduced. And when the pressure is applied, the air which it contains is compressed continually into a less and less bulk, the diminution of its bulk being precisely proportional to the pressure.* Thus, by observing the degree to which the air is compressed in this tube, or the height to which the water is raised in it, the pressure which the screw is exerting and the water in the bottle sustaining, is always known.

Since the whole vessel as well as the tube and bottle are filled with water, a question arises how is the descent of the surface of the water in the tube *a a* to be distinguished? Some separation must evidently be made between the surface of the water in the tube *a a*, and the water in the vessel which presses upon it. To produce this separation, when the tube is sunk, care is taken that it shall not be completely full of water; and to keep in the air which thus occupies the top of the tube, and cause it to make a permanent separation between the water within and that without the bottle, a funnel-shaped glass vessel *p*, open at the bottom and loaded round its lower edge, is inverted over it. This vessel is thus, when the instrument is sunk, nearly filled with air, which by the pressure is made continually to occupy a less and less space, and driven into the tube, so that when the water of the vessel at length

* By Marriotte's law, afterwards to be explained.

reaches the top of the tube and enters it, there intervenes between it and the water already within the tube, a column of compressed air, forming a separation of the two, which may easily be seen without. *i i* are cork floats, attached by strings for the convenience of removing the apparatus when the experiments are completed.

The experiments of Œrsted, made with this apparatus, not only establish the fact of the compressibility of water, the water sinking in the tube about half an inch for each additional pressure of an atmosphere; but they ascertain its amount by the methods explained above, to be $46\frac{1}{10}$ millionths of its bulk for each such additional pressure of one atmosphere or of about 15 pounds the square inch.* Thus for each additional equal pressure the water is compressed by the same fraction of its bulk. This is a remarkable law, which is found to govern the compression of all other bodies.

The same method applied by Œrsted to the compression of water, manifestly enabled him to compress and measure the compression of any other liquid. For that purpose, he had only to cause that liquid to replace the water in the bottle and tube. Table I., in the Appendix, presents the results thus

* The pressure of an atmosphere on any surface is a pressure equal to that which is exerted upon it by the weight of the air: the pressure of two atmospheres is twice the pressure of the air, and so on. This pressure of the air upon any surface is equivalent to the weight of a column of mercury having a base equal in size to that surface, and a height equal to the height at which the barometer stands. Its mean value is 15 pounds to the square inch.

obtained. Beneath them are results similarly obtained by Messieurs Colladon and Sturm.

Out of the great and unexplained difference between these results of Colladon and Sturm and those of Œrsted, has arisen an interesting discussion as to the correction which should be made for the compression of the substance of the tube and bottle by reason of the pressure which they sustain within and without. M. Poisson (Mém. Ac. Sci., 1827, 1828,) has arrived at the theoretical conclusion, that by this compression the capacity of the bottle is *diminished;* and he has given a very simple rule for the correction. Œrsted denies, however, the accuracy of this correction. He states, indeed, the fact altogether inconsistent with it, that the recession of water in the capillary tube is invariably about 1½ millionths greater when bottles of lead and tin were used instead of bottles of glass.

* 29. The Compression of Solids by Œrsted's Apparatus.

The method of Œrsted lends itself to direct experiments on the compression of solid bodies. To determine the compression of a solid under any given pressure it is placed in the bottle M, the tube being taken out to admit it. The bottle is then filled with water, the tube replaced, and the whole subjected to the pressure of the screw, under the same circumstances as before. The descent of the column of water in the tube shows the joint amount of the compression of the water and the solid. Also, the amount of the compression of the water

is known by the preceding experiments; that of the solid, then, is easily found.

* 30. The Adaptation of Œrsted's Apparatus to High Pressures.

Wishing to try the effect of higher pressure in the compression of air than could with safety be applied to the glass vessel, Œrsted replaced it by one of metal.

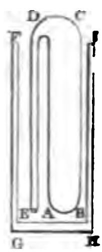
Now, however, the vessel being no longer transparent, it became necessary to contrive some method by which the pressure produced by the screw, and the degree of compression of the air, might register themselves *permanently*, so that they might be read off, when the apparatus was taken out of the vessel.

This permanent registration of the pressure was effected by using, instead of the large tube N, a smaller tube expanded at its closed extremity into a bulb, and having a short column of mercury suspended in it, on whose surface floated an index, to which was affixed a hair spring, pressing it against the side of the tube, so that the index would stick at the extreme point to which the mercury might have raised it, when the latter should again recede.*

By the position of this index, when the apparatus was taken out, the extreme pressure which the screw should have produced would evidently be known. To *subject* the air, on which the experiment was to be made, to this pressure, and to measure the amount of its compression, the inge-

* This contrivance is the same with that in Six's self-registering thermometer.

fig. 2.



nious and simple apparatus shown in the accompanying figure was used. F G H I is an open vessel containing mercury; A B C D E a glass vessel drawn out into a slender tube E D, which is turned downwards, as shown in the figure. This vessel, whose only opening is at E, is made to contain the air on which the experiment is to be made, and is then sunk in mercury in the position shown in the figure; and in this state the whole is plunged in the receiver, A C D B, of the compressing apparatus. (*Fig.* 1. *p.* 18.) The pressure being then applied, its effect is to drive the mercury up the tube E D, and into the vessel C B, compressing the air above it, and falling to the bottom of that vessel. When the pressure is withdrawn, only that portion of the mercury which is contained in the tube E D will return, and the *volume* of that contained in the vessel D C B A being added to the volume of this which was contained in the tube, will equal the volume by which the air was diminished during the experiment, as shown by the maximum pressure of the index. Œrsted thus compressed air into $\frac{1}{65}$th of its original bulk, and measured the pressure, which he found to be just 65 times the ordinary pressure of the atmosphere. And in a number of other similar experiments, he found, that by however many times he wished to diminish its bulk, by exactly so many times was it always necessary to increase the pressure upon it, or in other words, that the compression was always proportional to pressure applied; twice the ordinary pressure upon the air producing twice the

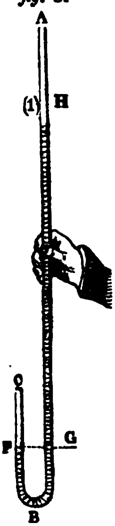
compression; three times, thrice the compression; this relation of the compression to the pressure is called that of perfect elasticity. It is not peculiar to the air, but is common, within certain limits of pressure, to all aërial and solid bodies, and it appears, from the preceding experiments of Œrsted, to all liquid bodies.

ELASTICITY.

* 31. Marriotte's Experiment.

The perfect elasticity of air was first proved by Marriotte. The following is (with a slight variation) his experiment. A B C (*fig.* 3.) is a curved cylindrical tube, graduated in equal parts, closed at C and open at A.

fig. 3.



Let mercury be poured into this tube, so as to occupy a portion, H B F, of it, towards the open end A, whilst the rest, F C, contains air.

Let this tube now be laid flat on a perfectly horizontal table, and let the division which separates the mercury and air be observed. Place it then in an upright position, and again observe the division at which the mercury and air are separated; and moreover, the whole height of the column of mercury *above the level* of that division.* When the tube was laid

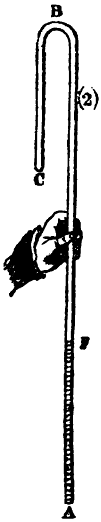
* Thus, if the division of the air and mercury stand at any point F, in the shorter branch, it is the height of the column H G, which is *above the level* of F, that is to be measured.

flat, the mercury was supported entirely upon its sides, and did not press at all upon the air, so that the space occupied by the air was that which it would occupy *out of the tube*, or in its natural state, that is, under a pressure equal to that of the barometric column *; but when it is placed in an upright position, the weight of the whole column of mercury, above the level of the common surface of the air and mercury, presses upon the air. The air is therefore pressed more than in its natural state by the weight of this column; and it is *compressed*, and the amount of the compression is easily measured by a comparison of the length of the tube which the air now occupies, with that which it occupied when it was laid flat. Now, suppose that the height of the column of mercury above the level of its division with the air, to equal the height of the barometric column at that moment. The *natural* pressure upon the air equalling the weight of this column, and an *artificial* pressure of the same amount being added to it, the whole pressure upon the air in the tube will be *double* what it was before. Now it will be found, that under these circumstances, the space occupied by the air will be *halved;* and if, in like manner, the column of

* By the pressure of the barometric column is here meant the weight of the column of mercury as it would stand at the time of the experiment in a barometer whose tube had the same diameter with that used in the experiment. The column in the barometer being supported by nothing but the air, is greater as that pressure is greater, and less as it is less; its weight is exactly equal to the pressure of the external air on a surface equal to the base of the column.

mercury in the tube had been made equal to twice the barometric column, so as to *triple* the whole pressure upon the included air, then the space occupied by it would be reduced to one third; and generally, it will be found that if the whole pressure upon the air be by these means increased in any proportion, the space occupied by it will be diminished in a like proportion.

fig. 4



Moreover, by inverting the position of the tube, as in *fig.* 4. we may *diminish* the pressure upon the included air, instead of increasing it, and this diminution of pressure will then just equal the weight of the column of mercury, F A, which is suspended beneath the level of the surface F, which separates it from the air. So that if this column equal in height one half of the barometric column, then the pressure upon the air will be diminished one half; if it equal two thirds the barometric column, then the natural pressure will be reduced to one third, &c. Now in these cases it will be found that the space occupied by the air will be doubled, tripled, &c. So that in general, the pressure upon air, whether it be more or less condensed than in its natural state, is proportional to the space it occupies. This is called the law of Marriotte.* In all cases,

* It is a necessary precaution to the accuracy of this experiment, that the air should be perfectly freed from moisture; the presence of water materially affecting the conditions of its elasticity. To dry it perfectly the tube should be heated, and

the air when released from the pressure applied to it, instantly recovers its original bulk; the force with which it tends to recover that bulk, being, in fact, that which must be overcome to compress it.

This property is not peculiar to aëriform bodies. Œrsted has proved it of water and other liquids, enumerated in the table I. in the Appendix. It appears, indeed, that aëriform bodies are but liquids under a diminished state of pressure; so that by increasing the pressures upon them very greatly, they may be all made to assume a liquid form.

32. The Elasticity of the Metals.

With the elasticity of metallic bodies every one is conversant. It is a property which, as it belongs to steel, iron, and brass, contributes eminently to the resources of art, and ministers largely to the uses of society. Were it, indeed, not for this property, it would be *in vain* that the metals should be dug out of the earth and elaborated into various utensils. Infinitely more brittle than glass, they would immediately be dashed to pieces by the slight shocks to which every thing is more or less subject; a shower of hail, or even of rain, would be sufficient to *indent* * their surfaces, and the im-

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then for several days made to communicate with a vessel containing muriate of lime, or some other substance which extracts from the air its moisture.

* It will be shown in a subsequent part of this work, that the force which accompanies the *impact* of a body, is in its nature infinitely greater than any force of that kind which we call *pressure*. Now of the class of forces of pressure, are those

pact of the minute particles of dust blown against them by the wind would be sufficient permanently to destroy their polish.

33. The Law of the Elasticity of Metals.

The force with which metals, when extended or compressed, tend *to recover their form*, that is, the force necessary to keep them extended or compressed, is proportional to the *amount* of the extension or compression they have received.

Thus double the extension or compression of the same body requires double the force; triple, triple the force; quadruple, quadruple the force. Similarly, one half the compression or extension, or one third of it, or one fourth, requires one half, one third, or one fourth, the compressing or extending force.

This is the law which constitutes *perfect* elasticity, and which has been shown to belong to liquids and gases. It was first accurately proved in respect to metal *wires* by S. Gravesande.

34. Experiments of S. Gravesande on the Elasticity of Wires.

The apparatus used by S. Gravesande is represented in the accompanying figure. The wire, whose elasticity was to be determined, was extended

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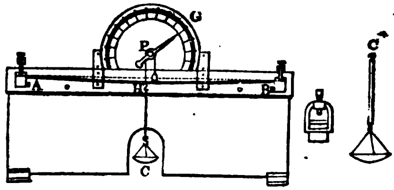
cohesions which hold together the particles of solid bodies. These therefore of necessity yield to any force of impact; and were it not for the force of elasticity by which the displaced particles recover their positions, any such force of impact would produce a permanent indentation.

fig. 5.



between the two fixed points A and B. A light scale-pan, C, suspended from a silken thread C H, was hung upon its middle point H; and to balance this scale-pan, a continuation of the silken thread, which suspended it, passed over a pulley P, and supported *a counterpoise*. The pulley P carried an index P G, pointing to equal divisions on a dial plate. Exceedingly small weights were placed in succession, and very gently, in the scale-pan, and the deflexions of the wire produced by these were observed by the motion of the index P G. This deflexion of the wire being thus known, and also the distance A B, in a straight line, between its extremities, its length A H B corresponding to each such deflexion, became known by easy rules of geometry. The difference between this length and its original length was its *elongation*. It will be observed, that the weight in the scale-pan is not exerted in the direction of the *length* of the wire; nevertheless it does produce a *certain* strain or tension in that direction; now the amount of this strain, exerted in the direction of the length of the wire, can be determined by a very simple rule of mechanics, to be explained hereafter (see Parallelogram of Forces); and it is this strain or tension which was to be compared with the elongation of

the wire. It resulted from the experiments of Gravesande, that *this strain was exactly proportional to the corresponding elongation.*

35. Elasticity of Ivory.

The elasticity of ivory is sufficiently shown by the impact of billiard-balls. The following experiment presents it, however, in a yet more striking form. Let an ivory ball be let fall perpendicularly upon a smooth and hard plane — of stone or metal for instance — which has been first rubbed over with oil. It will be seen to rebound *very nearly* to the height from which it has fallen; the cause which has interfered with its re-ascent exactly to that height being the resistance of the air. Now let the spot where it has struck the plane be examined; the traces of the impact will be seen in the oil, not at a point only, as would have been the case had the surface of the ball not yielded at the instant of impact, but over a considerable surface, which is greater as the ball is allowed to fall from the greater height. Thus whilst by the rebounding of the ball its elasticity is shown, by this mark on the oil, its compression, or the flattening of its surface at the instant of impact, is proved. Balls of wood, stone, glass, and metal, present the same phenomena as those of ivory.

36. Elasticity of Torsion.

If a wire be twisted, it will tend to recover its natural state, with a certain force which is called its elasticity of torsion. The law of this force is this, that it is always proportional to the angle

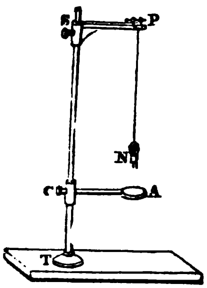
through which the body has been twisted. This property may be shown to result from that other *universal* property of bodies by which any portion of them being displaced within certain limits, tends to return to its position with a force proportional to the displacement, and indeed it proves that property. — (See *Moseley's Mechanics applied to the Arts*, Art. 199., &c.) It exists not only in bodies such as steel, brass, wood, &c., with whose elastic properties we are conversant but more or less in all bodies.

37. Coulomb's Torsion Balance.

From the facility with which metal wires and threads of various substances may be twisted, and the perfect regularity and precision with which they tend to return to their former positions with forces proportional to the angles of torsion; they have come to be used for measuring certain forces too minute to be estimated by the ordinary methods. Coulomb was the first to make this application of the elasticity of torsion: by means of it, he succeeded in determining, by direct experiment, the laws which govern the variation of magnetic and electric forces; and it was by means of the torsion balance that Cavendish afterwards detected and measured the almost evanescent attraction of gravitation in balls of lead

The torsion balance consists of a stand T, supporting a hollow vertical rod S T, which, in the balance of Coulomb, was of *pewter*, that all magnetic and electric influence might be avoided. On

fig. 6.



this rod there are two sliding pieces C A and S P; the lower of which carries a plate A, with a circle divided like a dial-plate upon it; and the upper, a piece P, to which the torsion wire or thread is to be fixed. N is a small bar-piece, with a screw which clips the extremity of the wire whose torsion is to be experimented on, to which a weight, or an index, or both, may be attached.

The following were the principal results obtained by Coulomb: —

1. *The wire, being loaded with different weights, did not rest in the same position of the index.* That is, by adding to the weight borne by the wire, or taking away from it, the index was always made to rest in a different position.

2. *The oscillations of the index were isochronous.* That is, when the index, being deflected from its position, was then left to itself, it always returned to that position again in the same time, whether the deflection was great or small; in the one case moving faster, and in the other slower, precisely in the proportion necessary to preserve this equality of time or isochronism.

It appears from the theory of dynamics, that this one observed fact is sufficient to establish the principle, that *the force with which the wire tends to return is proportional to the angle of torsion,* so that, observing the angles through which the index is

twisted by the action of different forces, we can *compare* the forces: this is the *use* of the balance.

The isochronism of the oscillations only obtains, however, within certain limits of torsion; thus, if an iron wire, so slender that six feet in length weigh but five grains, and nine inches in length, be deflected, its oscillations will be isochronous so long as they do not exceed half a circumference. But if it be deflected through three circumferences, so as to oscillate at first through six, then the oscillations will be slower by about $\frac{1}{30}$th than before.*

38. The Elasticity of Lead and Pipe-clay.

Experiments similar to the above, made with wires of *lead* and thin cylinders of *pipe-clay*, show that these and many other substances, *apparently* yielding and inelastic, possess, in reality, elastic properties as perfect as those of steel. A wire of lead, for example, one fifteenth of an inch thick and ten feet long, suspended as in the experiments of Coulomb, and twisted, being let go, oscillated *isochronously*, showing that the *force* was proportional to the *angle* of torsion, and, therefore, that the elasticity of the substance was perfect. A similar experiment with a thin cylinder of pipe-clay gave the same result.

* 3. *The wire being loaded with different weights, the times of isochronous oscillation were as the square roots of these weights.*

4. *The lengths of wire being different, the times were as the square roots of these lengths.*

5. *The diameters of the wires being different, the times were as the square roots of these diameters.*

In all these cases the oscillations are supposed to be small enough to be isochronous.

39. The Torsion of Bars of Iron.

Whilst a wire of small diameter, a few inches or even a few feet long, may be in a degree homogeneous, this quality is not to be expected in a bar. Thus the conditions of torsion, which in a wire are so simple and uniform, become in a bar complicated and anomalous. In the Appendix to this work will be found tables containing the results of experiments on the torsion of bars, made by Mr. Banks, Mr. Dunlop, of Glasgow, and M. Duleau.

40. Elasticity a common Property of Aëriform Bodies, Liquids, and Solids.

That aëriform bodies, liquids, and solids, should possess, in common, the property of elasticity, may appear to us the less singular, if we consider that these are but different *forms* under which the same body may exist subject to different conditions of heat.

Steam, for instance, an *aëriform* * vapour, condenses into *liquid* water, a certain abstraction being made of its heat; and this water, by another reduction of temperature, becomes *solid* ice. And, to take an example of this process of transition in the

* Steam, in an entirely uncondensed state, appears strictly under the form of an air: it is perfectly clear, colourless, and transparent, and may be seen in this colourless transparent state in the bubbles of steam which ascend in a vessel of boiling water from the bottom, where they are generated. It is when, coming in contact with the air, the steam begins to *condense*, that it assumes that cloudy appearance which we usually associate with our idea of steam.

opposite direction, a *solid* metal becomes a *liquid* by a certain addition of heat; and a yet greater intensity of heat *volatalises* it. When the partial abstraction of heat is aided by a powerful *pressure*, a *permanent* aëriform body or *gas* may be converted into a liquid.

41. The Liquefaction of the Gases.

This interesting experiment was first made by Faraday. Two chemical substances, from which, when brought together, the gas to be liquefied would be liberated (concentrated sulphuric acid and carbonate of ammonia, for instance, when carbonic acid was to be liquefied), were made to occupy opposite extremities of a bent glass tube, which was then hermetically sealed.* By inclining this tube, the two substances were then brought together, and the gas evolved with immense force; and, being held compressed within the narrow chamber of the tube, was seen to assume a liquid form in the opposite leg of the tube to that in which the two substances mingled.

In some experiments, the gas to be operated upon was made to occupy a portion of the tube separated from the rest by a drop of coloured fluid. In the other portion of the tube, carbonate of ammonia and sulphuric acid were placed in small ex-

* Care was taken to introduce the acid by means of a long capillary funnel, so as not to wet with it that portion of the tube into which the neutral salt, or other substance from which the gas was to be liberated, was placed. When this precaution was not taken, the disengagement of gas prevented the tube from being effectually sealed.

panded chambers apart. The tube was then sealed, —the acid and salt were brought together by inclining the tube — the liberated carbonic acid gas drove the drop of fluid before it, compressing the gas included at the opposite extremity of the tube until it liquefied it. The condensation was assisted by artificially cooling that extremity of the tube where it was to take place.*

Table II., in the Appendix, states the pressure in atmospheres, and the temperature at which the liquefaction of the gases enumerated in it took place.

The liquefaction of carbonic acid gas is now produced by means of powerful forcing-pumps.†

When the pressure is removed, the liquid reassumes its gaseous form ; and the gas being allowed to escape, the jet, in the act of expanding itself, so depresses its temperature as to congeal at a temperature lower than any other known to exist.

* The specific gravity was measured by introducing, before the tube was sealed, minute bulbs of glass, whose specific gravity had been before determined by observing in what fluids of known specific gravity they would float. The degree of pressure was measured by a contrivance similar to that used in Œrsted's experiments (page 20.), the tube being here, of course, exceedingly minute, drawn over the blow-pipe.

† Sir H. Davy found that by a given accession of temperature the expansive power of gas in a liquid state was much *more increased* than by an equal addition of heat to gas in a gaseous state. He found, for instance, that the expansive force of liquid carbonic acid at 12° F. was increased by an accession of 20° of temperature from 20 atmospheres to 36. He conceived the idea, that by reason of this property the expansion of the liquefied gases might with advantage be used as a moving power in machinery.

CHAP. II.

THE STRENGTH OF MATERIALS.

THE FORCES PRODUCING EXTENSION OR COMPRESSION. — THE LIMITS OF ELASTICITY. — RUPTURE. — THE STRONGEST FORMS OF CAST-IRON BEAMS AND COLUMNS. — WOOD AS A MATERIAL IN THE ARCHITECTURE OF NATURE. — THE MECHANICAL PROPERTIES OF METALLIC SUBSTANCES AS AFFECTED BY THEIR INTERNAL STRUCTURE.

THERE is no form under which the property of the elasticity of matter offers itself to our notice fraught with more interest or importance than as it affects the strength of the materials of construction.

All these are necessarily subjected, in the uses to which they are applied, to various degrees of pressure; and it becomes a matter of great importance to know, in the *first place,* how far they will lengthen themselves under a given *strain,* or compress themselves under a given thrust *; in the *second place,* how far this strain, or thrust, may be carried *without rupture.*

With regard to the *amount* of the *extension* of materials under given *strains,* it is to be regretted

* A bar or a timber is said to suffer a *strain* when the forces which act upon it tend to *lengthen* it, and a *thrust* when they tend to *compress* it.

that few *direct* experiments have been made; and in respect to the amount of their *compression* under given *thrusts* (it is believed) none.

42. THE EXTENSIBILITY OF IRON AND WOOD.

It appears by the experiments of the engineers of the Pont des Invalids, made with every precaution upon the *direct* strain of bars of the best wrought iron, that they increase their length by about 82 *millionths* under a load of one ton upon the *square inch*.

M. Vicat, from experiments made with a view to the use of iron in the construction of suspension bridges, found that, when formed into bundles firmly bound together, or cables, as they are called, iron *wire* was much more extensible than *bar* iron, and that it was the more extensible as it was thinner. Its elongation varied from 85 to 91 millionths for a load of one ton per square inch.

That a fascicle or bundle of wires, having together a section of one square inch, should be more extensible than an iron bar of the same section, and that such a fascicle should be more extensible as the wires which compose it are *thinner*, are exceedingly interesting facts, inasmuch as it will hereafter be shown that, under the same circumstances in which iron is thus more extensible, it is *stronger*. So that, on the whole, we arrive at the conclusion that iron acquires in a remarkable degree that quality which we understand by *toughness*, by being thus drawn out into wire.

The elongation of oak is about 14 times greater than that of bar iron under the same load of one

ton per square inch. According to Tredgold, bar iron will bear to be elongated by the $\frac{1}{1400}$th part, or by 714 millionths of itself, without permanent alteration of structure, or injury. Cast iron and brass admit of an extension slightly greater; but the woods ash, elm, mahogany, fir, oak, and pine, may with safety be extended more than *three* times as much, according to the experiments of Barlow. Of all the woods, larch and beech appear to admit of the least extension without injury. Tables will be found in the Appendix, containing the results of the experiments from which these conclusions have been drawn. (See Table III.)

43. The Extensibility of Bar Iron when approaching a State of Rupture.

MM. Minard and Desormes suspended weights to bars of iron varying in section from ·12 to 1·63 parts of a square inch, until they broke. All the bars were 7·874 English inches in length, and the mean of 25 experiments gave $\frac{1}{400}$th part as the elongation due to a load of 15 tons the square inch, $\frac{1}{100}$th to 18 tons, $\frac{1}{30}$th to 20 tons, and $\frac{1}{20}$th to 23 tons; 25 tons per square inch produced rupture. Thus, whilst approaching a state of rupture, each additional ton weight per square inch produced a much greater elongation of the bar than in the commencement of the extension. *Then* it produced an elongation of but 714 millionths; but when the load, as in these last experiments, is augmented to 15 tons per square inch, each additional ton, up to 18 tons the square inch, produces an elongation of 2,500 millionths; from that load to 20 tons the square

inch, of 5,000 millionths; and from that load again to 23 tons, of 10,000 millionths.*

Now it cannot be doubted that, before the elongation of the bar, all the parts of it were perfectly elastic. How, then, is this subsequent deviation from the law of perfect elasticity to be explained? By the fact, that all the parts of the bar, by reason of their different densities, and the different circumstances of crystallisation to be found even in wrought iron, are not equally *extensible*, and that the material of the iron has been internally ruptured, and its cohesive power, in many concealed parts, destroyed long before it attains a state of *actual rupture.*

According to the experiments of M. Lagerhjelm, made in Sweden in the year 1826, the most ductile Swedish bar iron elongates the $\frac{27}{100}$th part, or nearly $\frac{1}{4}$th of itself, before it breaks.

44. The Volume or Bulk of an Iron Bar, and of a Copper Wire, are increased in the Act of Extension.

M. Lagerhjelm found that, before it broke, the iron of a bar subjected to extension had diminished its section to the 0·722th part, and its specific gravity by $\frac{1}{105}$th part, and therefore increased its bulk by $\frac{1}{99}$th part.

M. Cagnard de la Tour enclosed a copper wire in a long tube filled with water, and then subjected

* It is remarkable that the elongation thus produced by each additional ton per square inch, in the state approaching to rupture, varied in these experiments in geometrical progression, each being double of the preceding.

it to extension. Having allowed time for the effect of the heat given out by the extension of the wire to pass away, he found that more water was displaced by it after extension than before; showing that its volume had increased in the act of extension.

45. The Theoretical Variation in the Diameter of a Solid Metallic Cylinder subjected to Extension.

M. Poisson has shown theoretically, that if a cylinder an unit in length be uniformly elongated, its diameter will be diminished by one fourth the amount of its elongation; whence it may be calculated that the increase of its volume will equal one half the volume of the elongated part.

46. The Limits of Elasticity.

It has been shown that, when displaced, the particles of a body tend to return to the position they before occupied in it, with a force proportional to the amount of the displacement. That this may be the case, the displacement must, however, be confined within certain infinitely minute limits. If those limits of displacement be passed, the displaced particle may be wholly separated from the rest of the body in the direction from which it has been moved, and thus a partial rupture may take place; or, other particles of the body occupying the space which it has left, and through which it has moved, it may take up its position under a new arrangement of particles exactly as it did under the preceding, and enter into precisely the same relation with them as before; so that, in every respect, the qualities of

the body shall remain unaltered under this new arrangement of its particles. In this last case it is said to have taken a *set*, and the phenomenon described under this name includes all that we understand by ductility and malleability, which terms but imply different ways in which this same property of taking a set is called into operation.

47. The Elasticity of a Body is not injured when a Set is given to it.

Thus, in S. Gravesande's experiments, wire, after it had permanently lengthened, was tried, and found as perfectly elastic as ever. In Coulomb's experiments on torsion, wire, which had been twisted so far that it would not return to its former position, was found to retain its elasticity of torsion as perfectly as before. Now, the making of a wire from a bar of metal, or, as it is called, the drawing of it out, is but the gradual producing of a *set* among its particles; and, since the wire retains the elastic properties of the bar, we may conclude that these are not affected by the *sets* which the particles of the bar are successively made to take.

When *beams* of iron are so loaded in the middle as to cause them to take a permanent deflexion, or a *set*, their elasticity is found to remain unimpaired by it; so that, when again loaded, they tend to recover themselves with forces which are, as before, proportional to the deflexion. Whilst some portions of the substance of a metallic body are made to take a *set*, others may, however, be ruptured. Its elasticity may then remain, but its extensibility will be greater, and its strength impaired.

48. Malleability.

The surface of a body always *yields* to an impact, however slight. If a metallic surface thus yield beyond the limits of elasticity, it takes a *set*. This property, by which a *set* is given to metals by impact, is called *malleability;* and is that property of matter which, perhaps, more than any other ministers to the uses of society. It gives shape to the tools by which all other substances are moulded, by which the earth is broken up and cultivated, and by which ships are made, and a communication established between regions separated by the ocean.

There is no case in which the property of malleability exhibits itself more remarkably than in the art of the copper-smith. From a flat plate of copper he beats out a hollow vessel without seam or joint, and of a given shape, contriving, by the skilful use of his hammer, so to move about the particles of the metal, that, although, to give to this flat piece of copper its hollow form, he must of necessity in some places contract its surface, and in others expand it, he causes it yet to retain the same thickness throughout. All this is effected by giving to its parts minute *sets*, of which, although the result of each is perhaps imperceptible, the aggregate is a displacement which he can carry to any finite extent. Operating thus *minutely* and by degrees, the substance of the metal becomes *soft* under his hands, and he may mould it as though it were clay. There are certain metals, and certain states of the same metal, in which this property of malleability exists in a greater degree than in others. Thus, for in-

stance, cast iron is not perceptibly malleable (except in a slight degree when *annealed*): it flies to pieces under the hammer; but when converted into wrought iron it becomes perfectly malleable.

49. The Stamping of Metallic Surfaces.

It is by a property analogous to their malleability that metallic surfaces are *stamped*. Thus, for instance, in the embossed metallic plates which form the surfaces of plated goods, the pattern is moulded from a steel die, or a block of steel, in which, when it is soft, the pattern is sunk by means of punches, and which is then hardened. Over this die a heavy weight is suspended, which can be made to descend between two upright pieces which guide its descent like the pile driver; the string which suspends this weight passes over a pulley, by means of which it can be raised again. On the under surface of the descending weight is fixed a thick plate of lead, and upon the die beneath it is laid the metallic plate to be embossed. The effect of the impact of the weight upon the die is to force the soft substance of the lead, and with it the intervening thin plate of metal, into the cavities of the die, where both take at the first impact a partial set; and, the impact being repeated, eventually the surfaces are made to adapt themselves perfectly to one another, and a complete copy is obtained.

50. Coining.

It is by a property analogous to that of malleability that metals are made to take the impression of moulds into which they are stamped. It is thus

that the precious metals are coined. The die, in which is sunk the impression which the metal is to receive, is fixed at the extremity of a powerful screw, which is driven impulsively by an effort of the workman applied to a horizontal arm fixed across the axis of the screw, and carrying, at its extremities, two heavy weights. The metal, thus driven with great force into the cavities of the die, takes there a *set*, and retains the impressions.

51. The Rolling of Metals.

The ductility of metals is most effectually called into operation by rolling them. It is thus that iron and copper plates and bars are made; and the iron rails used on railroads receive their form by being passed between rollers, in which are cut channels of a corresponding form. According to the experiments of M. Lagerhjelm, rolled bars are nearly of an *uniform* density, whilst the density of forged bars is extremely *variable*. Within the limits of elasticity, the forces producing given extensions are the same in the two kinds of bars, but a *set* is given to the rolled bar sooner than to the forged one. The resistance to rupture appears to be the same in the two cases.

52. Engraved Steel Plates.

By a like process, duplicates of engraved steel plates are obtained in any number. The steel of the engraved plate of which the duplicate is sought having been hardened, a cylindrical piece of *soft* steel is *rolled* over it under an exceedingly heavy pressure. This pressure causes the soft metal of

the roller to be pressed into all the lines of the engraving, of which it thus receives on its surface a perfect impression in relief. The soft steel of this roller is then hardened, and, thus hardened, it is made, under the same heavy pressure as before, to roll over the surface of the plate of soft steel to which the engraving is to be transferred; and, in the act of thus rolling over it, it indents it with all the lines which it had itself received from the original plate. There is scarcely any limit to the number of duplicates which can thus be obtained of the same engraved steel plate; or, therefore, to the number of prints which may be taken from the same engraving. This method is now largely used, and with great advantage, in calico printing. The same pattern being here to be repeated over a large surface, a small but complete portion of this pattern is engraved on a block of steel, and thence transferred, by the method described above, to a roller. This roller is then made to traverse, under a heavy pressure, the surface to be engraved, until it has *repeated* the pattern over as wide a surface as is required.

53. RUPTURE.

When the parts of a body are, by any external cause, separated beyond the limits of ductility, the separation becomes *permanent;* and, if it extend far enough, this separation constitutes a *rupture* of the mass.

The rupture of a bar of wood or metal may take place either by a strain or tension in the direction of its length, to which is opposed its

TENACITY; or by a *thrust* or compressing force in the direction of its length, to which is opposed its power of resistance to the CRUSHING OF ITS MATERIAL; or, *each* of these powers of resistance may oppose themselves to its rupture, the one being called into operation on one side of it, and the other on the other side, as in the case of a TRANSVERSE STRAIN. Or, lastly, the bar may be ruptured by TORSION.

54. TENACITY.

In the Appendix will be found a table of the tenacities of different materials, or the resistances they offer to forces tending to tear them asunder, as these have been determined by the best authorities, and by the mean results of numerous experiments. From this table it will be seen, that of all the materials experimented on, that which has the *greatest* tenacity, or which requires the *greatest* strain per square inch to tear it asunder, is thin iron wire — a number of pieces of it being placed side by side, and bound together, so as to form what is called a cable of wire. Moreover, that cables of wire thus formed are stronger, as the wires which compose them are thinner.

The first of the experiments enumerated in this table was made by M. Lamé, at St. Petersburg, on wire of the best Russian iron, $\frac{1}{30}$th of an inch in diameter. The result is extraordinary. A tenacity of 91 tons on the square inch must be considered as an extreme, and, perhaps, an *anomalous*, power of resistance.

Nevertheless, it results from the experiments of

Lamé and of others, that cables of fine iron wire, of from $\frac{1}{25}$th to $\frac{1}{30}$th of an inch in diameter, may be safely assumed to have the enormous tenacity of 60 tons per square inch.

The experiments of Telford give 40 tons per square inch, for the tenacity of wire $\frac{1}{10}$th of an inch in diameter.

It is by reason of this marvellous strength of wire cables, that they have come to be extensively used on the Continent, in the construction of suspension bridges. There is a bridge suspended by cables of iron wire at Fribourg, which is 700 feet in span between the abutments, or 100 feet wider in span than the great catenary of the Menai Bridge.*

Next in tenacity to cables of fine iron wire, is cast steel, in bars, well tilted or forged.† By the experiments of Rennie (Phil. Trans., 1813), its tenacity appears to be nearly 60 tons on the square inch. Sheer steel, reduced by the hammer, has, on the same authority, a tenacity of 57 tons. (Phil. Trans., 1818.)

Steel, by reason of its great tenacity, has — in Germany, where it is manufactured at a compa-

* The bridge of Fribourg is said however of late to have become unsafe. If this be the case, it is probably owing not to a want of tenacity in its material to resist the *ordinary* strain upon it, but to the impulses of *vibratory* motion to which, from its lightness, it is liable, in high winds, or from the rapid motion of vehicles.

† It is a remarkable fact that cast steel has its tenacity nearly doubled by being *tilted*.

ratively small expense — been used instead of iron, in the construction of suspension bridges.*

Russia bar iron (which is perhaps the best) appears, by the experiments of Lamé, made at St. Petersburg, with an hydraulic press, in 1826, to have a mean tenacity of about 27 tons on the square inch. Common English, and other bar or wrought irons of an average quality, may be considered to have the mean tenacity of 25½ tons on the square inch.†

Platinum in wire appears, by the experiments of Morveau (Ann. de Chimie, 25–8.), to have a tenacity a little less than bar iron.

Silver wire, *gun-metal*, and *forged copper*, follow next in the order of tenacity, having respectively tenacities of 17, 16¼, and 16 tons, on the square inch.

Gold wire has (by the experiments of Sickingen, Ann. de Chimie, 25–9.) only *one half* the tenacity of wrought iron.

The *best grey cast-iron* may be taken to have a mean tenacity equal to *one third* that of *Russia bar iron;* that is, equal to nearly 9 tons on the square inch; whilst the *ordinary* cast-iron has one third the

* Steel bridges, in common with wire bridges, are, however, by reason of that very lightness which is the great element of their strength, peculiarly liable to those vibrations which are calculated more than any thing else to try it.

† Of the experiments recorded of wrought iron, one by Muschenbroek gives to it the tenacity of 41 tons on the square inch. This is the highest recorded, and it is a problematical result. The iron used was German bar iron, mark B R. The *best* Swedish and Russian bar irons have, however, for the most part, exhibited a tenacity of upwards of 30 tons.

tenacity of *common* bar iron, or about $8\frac{1}{2}$ tons on the square inch.

Of *woods*, *box* has the tenacity of the *best* cast-iron, and *ash* that of *common* cast-iron; that is, one-third the tenacity of *wrought iron*.

Deal, *oak*, and *beech*, have about $\frac{1}{5}$th the tenacity of *wrought iron*, and *mahogany* $\frac{1}{7}$th. Thus, 7 rods of mahogany, taken together; 5 of deal, oak, or beech; 3 of box, or of cast-iron; 2 of gold; $1\frac{1}{2}$ of silver, or copper, have respectively the same tenacity as 1 rod of the same section of *wrought iron;* or as a rod of $\frac{5}{12}$ths that section of steel or fine wire cable.

55. Resistance to Rupture, by Compression.

The results of experiments on this subject are to be found in a parallel column of the same table as the last.

A cube, whose edges are each $\frac{1}{4}$ of an inch, of the kind of cast-iron known by the name of gun-metal, requires, according to an experiment of Mr. Reynolds', 10 tons to crush it, or a compressing force of 160 tons on the square inch. No other material on which experiments have been made, exhibits a power of resistance approaching to this.

From experiments made by Mr. G. Rennie (Phil. Trans., 1818), it appears that *horizontal* castings of iron, from which cubes were taken of the same dimensions, offered a resistance equivalent to from 62 to 76 tons on the square inch; whilst similar cubes, from *vertical* castings, resisted crushing with a force

of from 70 to 90 tons. The more recent experiments of Mr. Hodgkinson, which have been made with remarkable care, give to the Coedtalon iron, No. 2., a resistance to compression of only 36 tons on the square inch; to the Buffery iron, No. 1., 41 tons; to the Carron, No. 3., 51 tons. Brass offered very nearly the same resistance as horizontal castings of iron.

Bar iron, according to Rondelet, crushes, with $31\frac{1}{2}$ tons on the square inch, with less than *one half* the pressure which Mr. Rennie found cast-iron to bear; *Aberdeen granite*, with *one sixth; Italian marble*, with *one seventh; Portland stone*, with *one tenth; brick-work*, with from $\frac{3}{4}$ of a ton to $1\frac{3}{4}$ tons.

But the most remarkable feature presented by this column of the table, is the small resistance which *wood* offers to a crushing force, acting in the direction of the length of its fibre. Experiments on this subject are somewhat uncertain* and variable in the results they give; they nevertheless fully establish the fact of the small comparative power of wood to resist a force tending to compress it in the direction of its fibre.

In every other substance enumerated in the table, it will be seen that the resistance to rupture by compression is *greater* than to rupture by extension; in wood it is *less*. A fact on which, as will hereafter be shown, there depend important principles in the theory of construction.

* This uncertainty appears to depend upon some unknown condition of the *adhesion* of the fibres of the wood to one another.

56. Influence of the Height of a Prism upon the Resistance to the Crushing of its Material.

The experiments on which the conclusions stated in the preceding article were founded, were made with *cubes* of the material. When the cube was converted into a prism of a different height from its width, the results became greatly modified, the strength diminishing as the height increased. Thus, when a cube from a horizontal casting of iron was replaced in succession by prisms having the same base of $\frac{1}{4}$ of an inch square, but each higher than the preceding by $\frac{1}{8}$ of an inch, until the last was 1 inch, their power of resistance to compression passed from 72 tons per square inch gradually to 45 tons. This fact probably accounts for the difference of the results stated in the last article. In all cases when a certain height is passed, rupture takes place by the sliding of one portion of the prism in an oblique section upon the other; and the angle of this oblique section is, in all cases, the same for the same metal. Extensive and accurate experiments have recently been made, on the much neglected subject of compression, by Mr. Hodgkinson of Manchester.

* 57. Rule, by Rondelet, for the Strength of Columns of Wrought Iron, and of Oak and Deal.

From a great number of experiments on columns of wrought iron, varying from half an inch to an inch square, and from an inch and a half to twenty

feet in length, Rondelet has derived the rule, that the load necessary to compress a cube of wrought iron being assumed to be 512 lbs. on the square line (or the $\frac{1}{144}$th of a square inch), the loads necessary to bend and break columns of any given square section, which are in length successively 27, 54, 81, 108, 135, 162, 189, 216, 243 times the side of the square of their section, are respectively 256 lbs., 128 lbs., 64 lbs., 32 lbs., 16 lbs., 8 lbs., 4lbs., 2 lbs., 1 lb., upon each square line of section. It will be perceived, that the first numbers are as the arithmetic progression, 1, 2, 3, 4, 5, 6, 7, 8, 9; and the last as the geometric progression, 2^8, 2^7, 2^6, 2^5, 2^4, 2^3, 2^2, 2, 1.

From similar experiments, made with columns of oak and deal, the same author deduced the rule, that assuming 44 lbs. per square line to be the load necessary to crush a cube of oak, and 52 lbs. one of deal, the loads necessary to bend and break columns of any given square section, which are in length successively 12, 24, 36, 48, 60, 72 times the side of the square section, are respectively $\frac{5}{6}$th, $\frac{1}{2}$, ——, $\frac{1}{6}$th, $\frac{1}{12}$th, $\frac{1}{24}$th of the force necessary to compress a cubical piece of the column.

Rondelet found that a square column of oak or deal *began* to yield by bending when its height was 10 times the side of its section. The weights and measures used by Rondelet, and mentioned in this article, were of the old French system, in which one pound weighs 7,561 English grains troy: and one foot 12·78933 English inches.

58. A COLUMN OF CAST-IRON, WHOSE EXTREMITIES ARE ROUNDED, WILL SUPPORT BUT ONE THIRD THE WEIGHT OF A SIMILAR COLUMN WHOSE EXTREMITIES ARE FLAT.

This remarkable fact is one among a great number which have been developed by the recent experiments of Mr. Hodgkinson of Manchester.

Having caused a series of cylindrical columns of cast-iron, of different diameters, to be accurately turned, with their extremities *rounded*, so as to support an insistent weight by the apex of the rounded end, — that is, by a single point in the extremity of the axis; — and having caused another series of columns to be turned, exactly similar and equal to the last, but cut off flat at their extremities, he broke the two series of cylinders by the compression of a powerful lever, made to act vertically in the direction of their length, by the intervention of a cylindrical hardened steel bar, acting like a solid piston through a hollow cylinder, which served it as a guide.

In all these experiments he found *the cylinders with the rounded ends to break with a pressure which was scarcely one third that of the cylinders with the flat ends.*

When *one* end of the cylinder was rounded, and the other flat, the breaking pressure was about *two* thirds that which broke the cylinder when both ends were flat; so that, in the three cases, the strengths of columns, equal in every other respect, were as the numbers 1, 2, 3.

59. The Strongest Form of a Cast-iron Column.

In all Mr. Hodgkinson's experiments, before described, the cylinder was observed to break in its *middle point*, indicating that to be the weakest. He commenced, therefore, a series of experiments on columns in which the middle section was increased at the expense of the extreme sections, with a view to ascertain that form of the column in which, when breaking in the middle, it should be about to break at every other point; this being manifestly the strongest form. From these it resulted, that the strength of a column of cast-iron, containing a given weight of metal, whether it be solid or hollow, is much greater when it is cast in the form of a double cone; that is, with its greatest thickness in the middle of its height, and tapering to its extremities, than when cast in any other form. The precise results of these valuable experiments have not been published: we hope, however, to be able to publish them in the Appendix.

60. The Pressure to which Materials may be subjected with safety in Construction.

In the actual *practice* of construction, materials cannot with safety be subjected to constant strains, or thrusts approaching to those which produce *rupture*. They are liable to various occasional and accidental pressures; and to others of a permanent kind, resulting from settlement, and other causes of which no previous account can be taken, for which allowance must nevertheless be made.

The engineer and the architect will therefore in their practice be in a degree guided by the example of ACTUAL STRUCTURES.

From a comparison of numerous examples of these, Navier has deduced the rule, that stone and wood have, in existing structures, with safety, been subjected, — the former to $\frac{1}{10}$th the thrust, and the latter to $\frac{1}{10}$th the strain, which breaks them; and iron, cast or wrought, to $\frac{1}{4}$th.*

61. ADHESION OF THE FIBRES OF WOOD TO ONE ANOTHER.

Mr. Barlow found the force necessary to separate two parts of a piece of deal, by causing them to slide upon one another *in the direction* of the fibre, to be about 5 cwt. to the square inch; for oak 82½ cwt. to the square inch was required. When the force was applied in a direction *perpendicular* to the direction of the fibre, 20½ cwt. to the square inch was required for oak, 15 cwt. for poplar, and from 8½ cwt. to 15½ cwt. for larch.

* One of the greatest pressures to which the stone of any building is known to have been subjected, is probably that borne by the central column of the Chapter-House at Elgin. It amounts to more than 40,000 lbs. the square foot: nevertheless, this stone would certainly bear ten times that pressure, without crushing. It is, however, dangerous to subject stones to any pressure approaching to that at which they crush: one half that pressure causes them to chip; and the tendency of the overloaded stone to yield increases with the time; —$\frac{1}{8}$th the crushing pressure is generally taken as the limit, which should not be exceeded. Telford gives 50,000 lbs. per square foot as the maximum pressure to which the voussoirs of an arch should be subjected.

62. The Neutral Axis in a Beam.

Let a beam be supposed to be bent by a weight placed in the middle of it: it is clear that the side of the beam nearest to the weight, will, in the act of flexure, be *compressed*, whilst the opposite side will be *extended*.

fig. 7.

The point where the extension *terminates*, and the compression *begins*, sustains manifestly neither extension nor compression. This point is called the *neutral* point: or, rather, there are a series of such points across the thickness of the beam, which all lie in an axis, called the *neutral axis* of the beam.

Since, throughout its neutral axis, the beam is neither extended nor compressed, its strength is not there at all called into play, and is, in point of fact, of no use; so that the beam would bear as great a weight if a *hole* were cut through it along this axis.

63. The Strength of a Beam.

What constitutes the strength of a beam is its resistance to extension on one side of its neutral axis, and its resistance to compression on the other. These act on either side of the neutral axis, like antagonist forces at the two extremities of a lever; *if either of them yield the beam will be broken.*

64. To cut a Beam One Half through, without diminishing its Strength.

It is evident that the resistance of the compressed side of a beam to *compression*, would not be at all affected by cutting it through, provided it were cut only so far as the compression reached, especially if we could cut it with a saw so thin that none, or scarcely any, of the material should be removed.

This experiment has actually been made, first by Du Hamel. He found that the strength of a wooden beam was not at all impaired by cutting it one half through on its compressed side — and scarcely impaired by cutting it $\frac{3}{4}$ths through; and, by filling up the saw-cut with a harder wood, he found that he could actually strengthen the beam by thus cutting it.

Barlow found that the compressed portion of a beam extended to about $\frac{5}{8}$ths of the depth. Through $\frac{5}{8}$ths of the depth it might then be cut, without in the least affecting its strength.

65. The Relation of the Forces necessary to tear Materials asunder, and to crush them.

If a beam yield either on its compressed side or its extended side, it will be broken. But on which of these is it likely *first* to yield? Does the material of which the beam is made yield first to compression or to extension? And in what proportion does it yield differently to these causes of rupture?

In parallel columns of a Table in the Appen-

dix, will be found the forces, reduced to the square inch, which are necessary to tear asunder the materials enumerated, and to crush them. From a comparison of these columns, there will appear the remarkable fact, that whilst the *metals* require a much *greater* force to crush them than to tear them asunder, the *woods* require a much less.

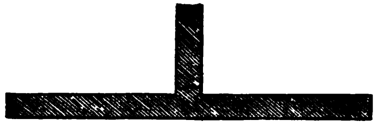
Experiments on the compression of *wood* are peculiarly uncertain; and the numerical results stated in the Table are probably to be received only as distant approximations. Still, the fact remains indisputable, that wood crushes with a force less than that with which it tears asunder; whereas the metals require a much greater force to crush them than to tear them asunder. Cast-iron seven or eight times as much.

*66. To make a Beam or Girder of Cast-iron which shall be four times as strong when turned with one Side, as when turned with the other Side, upwards.

A very ingenious experiment was made by Mr. Hodgkinson, of Manchester, to illustrate the fact stated in the preceding article.

He caused two castings of iron to be made from the same mould 5 feet in length. The form of the

fig. 8.

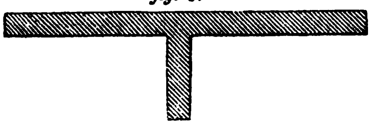


section was that shown in the figure. It may be described as made up of a large flanch 4 inches in

width, along the back of which runs a smaller upright rib 1 inch in height. Mr. Hodgkinson's object was to make one of these castings break, by the *extension* of this rib, and the other by the *compression* of it; and to compare the load necessary in these two cases to produce fracture. He anticipated that a greater force would be required to break that casting which yielded by the *compression* of the rib, than that which yielded by its extension. But how was he to break the one by the extension of this rib, and the other by its compression? Let the reader imagine these castings to be placed between supports 4 ft. 3 in. apart, the one with the rib *upwards*, the other with the rib *downwards*, and both to be loaded in the middle.

Let us take the case in which the rib is *upwards* (as shown in the last cut), and therefore *compressed*, and the flanch *extended*. Were the flanch only of the *same size* as the rib, and did it exert its strength under similar circumstances, it, being the *extended* part, might be expected the first to yield: but it is very greatly larger than the rib; and it was made so greatly larger, that its greater size might make up for its less power of resistance — it actually *did more* than make up for it; for the casting did not yield by the extension of its lower part, but by the compression of its upper, the rib — it broke with a load of 9 cwt. The other casting, placed with the rib *downwards*, of course

fig. 9.



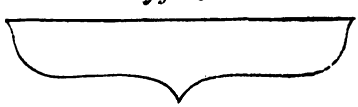
yielded by the *extension* of that rib; the extended part being here not only weaker, but smaller than the compressed part.

This casting broke with 2½ cwt. Thus we find that to break the casting by compressing the rib, required nearly four times as great a load as to break it by extending the rib: a result agreeing with the before observed fact, that cast-iron resists compression with greater force than extension. Here, then, was a form of iron beam, which was nearly four times as strong when turned one way as when turned the other: and here was an indication of the fact, that the strength of such a beam may, with the same *quantity* of material, be prodigiously influenced by the way in which that material is distributed.

*67. A Wedge, driven out by the Compression of the Rib.

In the experiment when the rib was uppermost, and it was broken by compression, there started out from it, when in the act of yielding, a wedge, of which the length was four inches, and depth ·98 of

fig. 10.



an inch, and which was exactly of the same form and dimensions in all other experiments with castings from the same mould. The wedge is accurately shown in the accompanying cut.

*68. The Strongest Form of Section of a Cast-iron Beam.

What, then, is the best way of distributing the material of a beam? This was the problem which Mr. Hodgkinson undertook to solve, by the method of experiment; and of which his solution is one of the most important practical results that have been, in modern times, obtained.

In the first place, let the reader be again reminded of the fact, that a beam, in bearing a load, sustains it by the resistance of its material to compression on one side, and to extension on the other; and that these forces act on opposite sides of its neutral axis, like forces acting at either extremity of a *lever,* the yielding of either destroying the balance, and breaking the beam. Moreover, let his attention be called to the fact, that the farther these forces are placed from the fulcrum, the greater will be their *effect:* so that all the forces resisting compression will produce their greatest effect when collected the farthest possible from the neutral point; and, in like manner, all the forces resisting extension. Thus, all the *material* resisting compression will produce its greatest effect when collected at the top of the beam; and all the *material* resisting *extension,* at the bottom. And thus we are directed to this first *general* principle of the distribution of the material, that it should be collected in two flanches, one at the top, and the other at the bottom of the beam, joined by a comparatively slender rib. This is the *first* step in the distribution: this is not, however, all; it does not give the *strongest* form of beam.

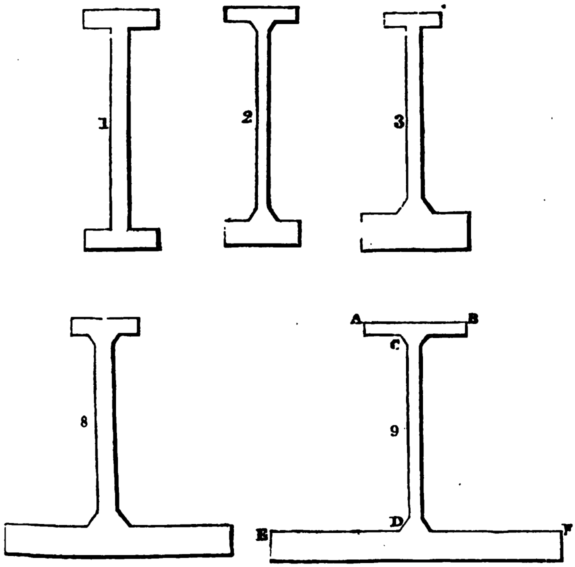
To understand *why*, let the reader's attention be called to this general principle.

That form of beam is the strongest, whose material is so distributed, that at the instant when it is about to break by extension on one side, it is about to break by compression on the other: for if when it is about to break by extension on the one side, it is not about to break by compression on the other, then may some of the material be taken from the compressed side without making that break, and added to the extended side, to prevent that breaking; so that *now* the beam is made to bear the weight which before it would not, or it is strengthened. And this is a *general* principle. So long as the distribution of the material is not such, as that the compressed and extended sides would yield *together*, the strongest form of section is not attained.

Now it seems clear that since cast-iron yields to extension sooner than compression, if the upper and lower flanch were of the *same size*, the lower or extended one would yield *first*. The compressed side cannot yield at the same time as the extended one, unless it be greatly less than it. On the whole, then, the strongest form of beam will evidently have its lower flanch much larger than its upper.

But in what proportion? Mr. Hodgkinson's experiments were directed to the determination of this point. He made a series of castings, gradually increasing the lower flanch, at the expense of the upper—as shown in the accompanying diagrams; and, as he had anticipated, he found the beams, in this state of transition, to grow stronger and stronger.

fig. 11.



No. of Experiment.	Ratio of Surfaces of Compression and Extension.	Area of Section in Inches.	Strength per square Inch of Section in lbs.
1	1 to 1	2·82	2368
2	1 to 2	2·87	2567
3	1 to 4	3·02	2737
4	1 to 4½	3·37	3183
5	1 to 4	4·50	3214
6	1 to 5½	5·0	3346
7	1 to 3·2	4·628	3246
8	1 to 4·3	5·86	3317
9	1 to 6·1	6·4	4075

In the first eight experiments, each beam broke by the *tearing* asunder of the *lower* flanch. The distribution by which both would be about to yield *together* — that is, the *strongest* distribution — was not therefore, up to that period, reached. At length, however, in the last experiment, the beam yielded by the *crushing* of the *upper* flanch, from which a wedge flew out.

In this experiment, then, the *upper* flanch was the weakest. In the one before it, the *lower* was the weakest. For a form between the two, therefore, the flanches were equal in strength to resist the pressures to which they were severally subjected; and this was the *strongest* form.

In this strongest form the lower flanch had *six times* the material of the upper.

In the best forms of girders used before these experiments, there was never attained a strength of more than 2,885 lbs. the square inch of section. There was therefore, by this form, a gain of 1,190 lbs. the square inch of section, or $\frac{2}{5}$ths of the strength of the beam.

The great girders cast in Manchester are now commonly cast on this principle; and there has resulted, it is said, a *practical* economy in the iron of full 25 per cent.

69.* Rule for the Strength of a Beam cast on Mr. Hodgkinson's Principle.

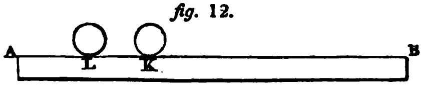
From the comparison of a great number of experiments, Mr. Hodgkinson has deduced the following rule to determine the weight necessary to break beams cast on his construction:—*Multiply*

the area of the section of the lower flanch by the depth of the beam, and divide the product by the distance between the two points on which the beam is supported. This quotient, multiplied by 536 *when the beams are cast erect, and by* 514 *when they are cast horizontally, will give the breaking weight in cwts.*

70.* TO VARY THE SECTION OF A BEAM AT DIFFERENT DISTANCES FROM THE POINTS OF SUPPORT, SO THAT FOR A GIVEN QUANTITY OF MATERIAL ITS FORM MAY BE THE STRONGEST.

The strength of a beam, to bear a load, is different according as it is loaded in the centre of its length, or nearer to either of its extremities. It is, for instance, evident that a beam will bear a load placed upon it very near to one of its points of support, when it would not bear the same load placed over its middle point. It appears from a mathematical inquiry into this subject, that the effect of a given

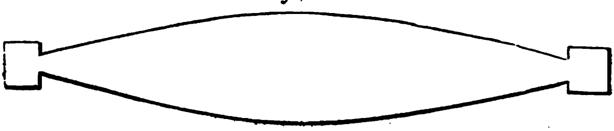
fig. 12.



load to break the beam, varies when it is placed over different points in it, as the products of the distances of these points from the two points of support of the beam. Thus the effect of a weight placed over the point L, is to the effect of the same weight placed over the point K, as the product of AL by LB is to the product of AK by KB; A and B being the two points of support. Since, then, the effect of a weight to break a beam is not so great at points nearer to its extremities, as in the middle, the beam need not be so strong any where as at its

middle point; and, guided by the law stated above, it appears that its strength at different points should vary as the products of the distances of those points from the points of support. Now this difference of strength may be given in two ways; either by varying the *depth* of the beam according to this law, or by preserving its depth every where the same, and varying the dimensions of its upper and lower flanches according to this law. Whether we thus vary the depth of the beam or the dimensions of its flanches, the law in question will give, for the outline, in the one case of the *elevation* of the beam, in the other of the *plan* of the flanch, the geometrical curve called a parabola. The cut represents the flanch, according to this form, adopted by Mr. Hodgkinson; the upper and lower flanch were of the same form; but the dimensions of the latter were six times those of the former.

fig. 13.



71. The Qualities of Wood as a Material of Construction.

Such is the form which, guided by experiment and such other resources of science as we possess, we find ourselves led to give to the substance, iron, which, forming part of the solid materials of the earth, and ministering *there* to some wholly different use, we dig up and apply to our purposes of construction.

Now let us turn to the architecture of trees, and examine Nature's material, and let us consider whether, guided by the light which our efforts to *economize* this *artificial* material of construction may have given us, we may not discover, in the material which has been elaborated wherewith to build up those stately structures, some feeble traces of that mighty and all-perfect wisdom of which ours, feeble as it is, is yet an *emanation*.

And let the principle first of all be stated, *as one observable throughout all nature*, that *creative power, infinite in its development, is infinitely economised in its operation*.

Were wood but as *durable* as iron and stone, it would supersede their use as a material of construction.

If other evidence were wanting, the unparalleled boldness of the structures erected with wood would, for itself, speak to the fact.

What have we to compare with the structures erected in wood? There is no arch of iron or stone, for example, that approaches to the span of the wooden arches which have been erected by Wiebeking in Germany, or to that arch at Philadelphia, which, with one vast span of 340 feet, crosses the Schuylkill.

The superiority of wood to iron or stone, as a material of construction, results from the extraordinary lightness which it unites with its strength. Thus deal has only one fifteenth the weight of cast iron, although it has considerably more than one half the tenacity, and sixteen bars of it would weigh only the same as one bar of the same dimensions of

wrought iron, although they would have together more than the *strength* of three.

Now it is evident that a building erected with a material, however strong, which was in the same proportion *heavy*, might, and probably would, be a *weak* building.

Such a structure, notwithstanding the great strength of its material, might load itself with its own mass to the utmost that it would bear, so that the slightest *additional* pressure would cause it to yield — as it is the last ounce which breaks the camel's back. Many, and memorable, are the instances of this weakness in artificial structures. The case of the Brunswick theatre, whose iron roof fell in by the pressure of its own weight, and that of Mr. Maudeslay's manufactory in London, and of the Conservatory at Brighton, are in every body's recollection.

But wood falls short of other materials in *durability*.*

The food of *living* vegetation is extracted from *decayed* vegetation; decay was thus, for the great purposes of nature, made its inseparable concomitant.

This decay —which was a necessary property then of timber, as a material of nature's architecture — *unfitted* it for that of man; who, reserved for im-

* The recent discovery of Mr. Kyan has, nevertheless, given, it is said, to wood an artificial durability almost equal to that of iron. The great agent in its destruction is a *fungus* whose ravages we are familiar with under the name of *dry rot*, and the experiments of Mr. Kyan, confirmed by those of Dutrochet, appear to show that this fungus will not grow in timber steeped in *corrosive sublimate*.

mortality, and struggling, even here, in an unceasing combat with the fleeting and transitory character of all that surrounds him, would construct for himself an abode whose durability may laugh to scorn the shortness of his tenure, and digs its material from among those mineral substances out of which the mass of the earth itself is builded up, and whose duration is coeval with it.

72.* The Adaptation of Wood as a Material to the Architecture of Trees in respect to its Distribution.

So much for the quality of the material as evidencing the infinite skill of the mighty Architect.

Now for the distribution of it. Can we see, imperfect as are our faculties, any traces of that perfect wisdom which governs the distribution of that material?

The experimental fact (ascertained with certainty), that its power of resisting extension, when subjected to transverse strain, is so nearly balanced by its power of resisting compression, as to bring its neutral point, at the instant of rupture, nearly in the *centre* of the beam (only one eighth of its thickness from it), is manifest evidence of this.

To make this appear, let us imagine that this nicely balanced equilibrium had not existed, as, in the case of iron, it does not. Let us suppose, in short, that iron were the material of trees. To give the most economical distribution to its material, a beam of wood must, then, be of that form which we have discovered to be the best for a beam of iron; that is, one side of it must contain six times the material

that the other side does. But such a beam is only calculated to bear a pressure acting upon *one* side of it, and to bear it in one particular direction. If fixed, for instance, firmly *upright* in the earth, and made to be acted upon powerfully by the *wind*, it might bear it, and would be of the form *best calculated* to bear it when it blew in one direction, but not when it blew in the opposite direction. To make it resist equally a force in *either* direction, the flanch must evidently be of an equal size on either side: but if you make it *thus*, all the *economy* of the distribution of the material is gone. To preserve this *economy*, the relation, of the resistance to compression, to the resistance to extension, must be *changed* in such a way that an *equality* of the flanches may constitute the most *economical* arrangement of the material.

Now in wood precisely this relation appears to obtain. The proximity of the neutral axis to the centre, as determined by Duhamel and others, sufficiently indicates the near equality of the forces resisting extension and compression as they are called into action in the transverse strain of a beam, and renders it extremely probable that in the transverse strain of the *cylindrical* trunk of a tree, whether hollow or solid, this equality becomes absolute.

Supposing then a beam of wood formed like that of iron, of which we have spoken, and conceiving its flanches to have the form of longitudinal slices of a hollow cylinder, and the circumstances of resistance to be similar to those in a beam of wood; if these two flanches be very nearly of the *same size*, when

one is about to yield by extension, the other will be about to yield by compression. This has been before shown to constitute the most economical distribution that can be conceived. To sustain the force of the wind on its opposite sides, a *timber*, of this form, then, with *equal* flanches, would have the most perfect form. Let the reader conceive a number of such equal timbers to be placed so that the ribs which join their flanches may all intersect along the centre of their length, and their flanches be brought side by side like the staves of a barrel, and let him imagine a hoop to be placed round it; and he will have conceived a structure whose material is of perfect *economy* in itself, and whose mass is *distributed* with a perfect economy, so far as these things may be comprehended; he will have embraced a principle which shapes out the bones of every living animal, which distributes the material of the stem of every weed and flower, and of a great family of trees. Surely "God is wise in heart; He is mighty in wisdom; He is wonderful in counsel, and excellent in working."

But it may be asked, If this economy of material be a *principle* of creative wisdom, why is not every tree made thus with a hollow stem, as are the great tribe of grass-like trees, and the bones of animals? Let us speak with reverence when we speak of the designs by which Providence directs her vast operations. Is it, however, an unjustifiable conclusion, that, in building up the trees of the forest, there was taken into that far-reaching view the *uses* to which, in the vast economy of human life, they should

hereafter be placed? How materially would that utility have been impaired had they been but hollow tubes? And how vast an influence would it have exercised on the destinies of our race, if for this reason large buildings had never been framed together, ships never built?

73. Various Circumstances which affect the Strength of Metals, as Materials of Construction.

Of the various circumstances which affect the strength of metallic substances, the most important appear to be those which connect themselves with crystallisation.

The crystallisation of bismuth and sulphur may be taken as examples of what little is known of the circumstances under which crystallisation takes place generally.

74. Crystallisation of Bismuth.

If bismuth, melted in a crucible, be poured into a mould (heated to receive it), and when it has cooled, so that a crust is formed over its surface, if the mould * be inverted, so that the weight of the liquid metal beneath the crust may break through it and run out, the cavity beneath will be found to be surrounded with beautiful crystals of the metal adhering to the crust and to the sides of the vessel. These crystals will be larger as the process of cooling takes place more slowly, and as the melted mass

* The crystals will be very beautifully seen if a test-glass be used for the mould.

is greater. They present to the eye the order and arrangement according to which its parts solidify; which order and arrangement, therefore, characterise their solid state.

A similar experiment may be made with sulphur. M. Mitscherlich, from a melted mass of fifty pounds, slowly cooled for four or five hours, obtained crystals half an inch thick.

75. Saline Crystallisations.

The process of crystallisation is much more easily observed in crystals which form, as do various salts, from aqueous *solutions* than it is in those which form in the act of cooling from a *melted* state. It is nevertheless probable that these two processes of crystallisation have much *in common;* and, in this point of view, the subject has great interest, as connected with the mechanical properties of metals. If the water in which a salt has been dissolved be slowly evaporated from it, the crystals of the salt will, after the evaporation has passed a certain limit, begin to re-appear in it: each minute crystal will be seen to be terminated by plane surfaces, to have a definite form, and certain inclinations of its planes. As its dimensions increase, which they will be seen to do continually so long as it remains unmoved, it will never lose this definite form or this given inclination of its containing planes. The amount of the increase will, however, probably be different on its different faces; a circumstance which, on examination, will be found to depend upon the different quantities of the fluid mass to which its

different faces are presented, and the different degrees of facility with which the saline particles of the solution have access to them; so that by turning the crystal round into different positions in the solution, different faces may be made in succession, to receive the greater increase. The *definite* character of this accumulation of the particles of the crystal is strikingly illustrated by the fact, that if a crystal, whilst the process is going on, be taken out of the solution and broken at its angles, so as to make its surfaces rough and uneven, and to destroy the form under which the accumulation of its particles was taking place, and if this broken crystal be then *replaced* in the solution, the process will recommence upon it by a restoration of the broken part, a filling-up of the roughness of its faces, and a re-formation of the crystal *upon its original model.* A yet further evidence of the *definite* arrangement of the particles of the crystal is found in the circumstances of its *cleavage:* these it partakes of in common with many crystals, which have never been formed by *artificial* means, but which are found in nature composing part of the earth's surface, and have probably resulted from igneous fusion. These, which are, some of them, excessively hard, have almost in every case certain particular directions in which they can be divided or cleft, called planes of cleavage. When they are thus cleft, the divided surfaces appear perfectly smooth and even, like the faces of artificial crystals. Every crystallised substance has *several* of these planes of cleavage, and the *same substance* has always, in every fragment or specimen of it, the same number of them; and these

inclined to one another at the same angles: so that cleaving any such fragment until all its faces are planes of cleavage, the resulting crystal will always, for the same substance, be of the same form. If it be an artificial crystal, this form will be exactly the same as that in which it forms itself from the solution out of which it crystallises.*

76. Crystallisation may take place in a Mass which is in an imperfect State of Fusion.

Wrought iron is obtained by the forging of masses of *cast* iron which are *heated*, but only imperfectly *fused*. This process, and others which it undergoes, separate it from the carbon which is combined with it, and from various other impurities under the form of scoriæ; and, as it thus becomes more pure, it becomes more difficult of absolute fusion, and less perfectly fused: nevertheless, cooling from this *imperfect* state of fusion, it assumes that crystallised structure, which is so apparent in wrought iron. It is, perhaps, in consequence of this crystallised structure of wrought iron, that its strength is so greatly modified by drawing it into wire: we have seen that its tenacity may thus be increased from 25½ tons the square inch to from 60 to 90 tons; that is, it may be tripled. The tenacities of iron, in the three states of cast iron, wrought iron, and fine iron wire, are as the numbers 9. 25. 80.

* Recent experiments have rendered it probable, indeed certain, that electricity in slow currents is the great agent in determining natural crystallisations; and that these are, for the most part, of aqueous origin.

77. The Influence of the various Conditions of Crystallisation on the Cohesive Force of Cast Iron.

This influence is fully shown by an experiment of Mr. G. Rennie. He took a cube of iron, whose edges were each $\frac{1}{8}$th of an inch, from the centre of a large casting, where the crystals being slowly formed were perfect and large, and plainly seen: he found that it crushed with a pressure of 1440lbs. He took a second cube, of the same dimensions, from a *small* casting, where there was *not* the same appearance of crystallisation, but a close compact grain: this crushed with 2116lbs.; that is, it required half as much force more, to crush it.

78. The Influence of Pressure upon the Solidification of Metals.

The *pressure* under which the solidification of metals takes place, has an evident influence on their internal structure. Thus, to the strength of a cannon, whether it be cast in a vertical or a horizontal mould —that is, whether in the act of cooling it sustains a greater or less weight of superincumbent material; and whether the muzzle or the breech be cast upwards,—are things of importance to its strength.

The experiments of Mr. Rennie show, moreover, that bars of metal differ in cohesive power, as they are cast vertically or horizontally.

Thus a prism, cast *horizontally*, he found to crush under a load of 9006lbs.; another prism, cast in the same mould, but *vertically*, required 9328lbs. to

crush it; and, generally, a vertical casting was best adapted to bear a vertical force.

Mr. Hodgkinson found so great a difference between the strength of iron girders, according as they were cast horizontally or vertically, that he has given different rules for calculating them. In illustration of the same fact it may be mentioned, that great bells are found to be of a different quality of metal at the top of the mould in which they are cast, and at the bottom; and that, from the experiments of Sir J. Hall, it appears that carbonate of lime may be made to assume all the forms under which it is presented in nature, from chalk to limestone-rock and marble, by subjecting it, at *high pressures*, to different extreme temperatures.

79. Malleable Platinum.

Platinum cannot be obtained from the ore in any considerable quantities by direct fusion, as other metals are. The voltaic pile and the oxygen blow-pipe will indeed melt it; but these can be made to operate only on small portions at a time: to obtain it in larger quantities, under a malleable form, was long a *desideratum* in science. It is now accomplished as follows:—The ore, being made to pass through a series of chemical solutions, at length yields a *hydrochlorate* of platinum and ammonia; which, under a high temperature, decomposes and leaves pure platinum, under the form of a fusible attenuated mass, called (from the resemblance) spongy platinum.

Under this form it is as far as can be conceived removed from malleability; and here lay the great

difficulty of the process. It is overcome by subjecting the spongy platinum to the action of heat, in iron mortars, under a *high pressure*: it then assumes the state of a semi-fluid, which becomes malleable platinum, under the *forge*.

This is a remarkable example of the influence of pressure upon solidification.

80. Cast Iron.

Of all the causes which affect the mechanical properties of iron, the most remarkable are those which result from the union with it, in different degrees, of that subtle element, which is called by chemists *carbon*, and which is, under a pure form, the substance known to us as *charcoal*. It is this element, which, in the process of smelting, passes from the charcoal or coke, which is mingled in given proportions with the mass of iron ore in the furnace; and uniting itself with the pure iron, gives it the properties of a fluid. Run into moulds, and allowed to cool, this compound of carbon and iron becomes *cast* iron.

In the *melted* state, *fluidity* is that property which the iron (in itself scarcely *fusible*, and when melted scarcely *liquid*) receives in a greater or less degree from the greater or less quantity of carbon combined with it—that is, from the greater or less quantity of coke mingled with it in the furnace, and the better or worse quality of the coke. In the *solid* state, according as it contains *more* carbon, cast iron is softer under the file or chisel, and possesses less strength as a material. As it contains *less* carbon, it is *harder*, and possesses

more strength as a material. This property of hardness, however, which it acquires as the proportion of carbon combined with it is diminished, ultimately passes into *brittleness*, and, beyond a certain limit, it thus loses its strength as a material, for the ordinary purposes of construction. It is only used in its most highly carbonised state, because in that state, by reason of its fluidity when melted, it may be made to run into the finest and most delicate mouldings, so as to present, when cooled, a minute and perfect reproduction of the model. For castings, on which less minute and accurate mouldings are required, iron combined with less carbon is used, because of its greater strength. Irons of these two qualities, of greater or less carbonisation, and suited to these distinct purposes, are known to the founders as the irons, No. 1. and No. 2. A third quality of cast iron, known as No. 3., is, in some places, made for castings of great size and strength, with a yet less admixture of carbon, and possessing less fluidity than No. 2. And there is a fourth quality, called *bright* iron, yet further without carbon, of an extremely imperfect fluidity when melted, and applicable only to the largest castings.

There are two qualities of iron — *mottled* and *white*—which are obtained from the furnace with yet less degrees of carbonisation than *bright* iron. These are, however, so thick when melted, and so brittle when cooled, as to be *wholly* unfit for the purpose of casting. When combined with carbon in a less proportion than in these qualities, iron does not *melt* in the furnace, and cannot be separated there from the ore.

To be obtained in this lower state of combination with carbon, it must first be extracted from the ore in the last-mentioned state of cast iron, or in some of the other states before mentioned, and then have more of its carbon, by an independent process, taken from it. Cast iron, *thus* deprived in a greater or less degree of its carbon (and other foreign ingredients), becomes *wrought iron*. It is remarkable that iron in this state, united with less and less proportions of carbon, *re-acquires* rapidly those properties of softness and malleability in its texture, which before, by the deprivation of its carbon, it *lost*, and soon greatly surpasses them. Whilst its resistance to the file, the chisel, and the hammer, have thus, by de-carbonisation, become less, its *tenacity* has tripled itself; the *brittleness* of cast-iron has wholly disappeared from it, and it has become that material, whose union of weight, strength, durability, and hardness, adapt it, above all others, to the various utensils and tools of art; and whose malleability, when heated to a red heat, and the facility with which it is welded, enable us to mould it into any required form upon the anvil, and to frame and unite any number of distinct portions of it into a continued structure.

It is the difficulty of melting wrought iron, its extreme softness nevertheless, when brought to a red heat, and that property by which it admits of being *welded* *, which, as much as its greater toughness and ductility, distinguish it from *cast* iron.

* The welding, or joining together of different pieces of iron and steel, is performed by bringing the surfaces to be joined to a temperature bordering upon that of fusion; a glossy appearance, like varnish, then appearing upon them, they are spee-

81. The Manufacture of Wrought Iron.

Cast iron is decarbonised, so as to convert it into *wrought* iron, by exposing it to the action of the air, for a considerable length of time, in a melted state: its carbon combines, in this state, with the oxygen of the air, and deserts the fused metal. This process of fusion is twice undergone: the first time it is called *refining* the iron. Mingled with the requisite quantity of fuel, the pig-iron is placed in a trough-like furnace, whose sides are of iron plate, and its bottom of masonry, and round whose sides, externally, a stream of water is made continually to run. The fuel being lighted, a powerful blast of air is impelled upon it, and the metal — having been kept in a state of fusion, with this blast upon it, for not less than two hours, and having lost a large portion of its carbon — is run into a long shallow mould, and cooled: it is then broken into pieces, and carried to a furnace of the kind called a reverberatory furnace, where the powerful flame of a large body of fuel, under combustion, in the grate of the furnace, is made to pass over it, and at length it sinks, in a state of fusion, on what is called the *hearth* of the furnace; a space which is wholly separated from the fuel, and open to a free access of air. Here the melted mass is kept in a state of continual motion, and stirred up from the bottom by the workmen, with long iron rods; a process, which is called *puddling*. In this state the metal

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dily scraped, placed in contact, and hammered together. *Cast* steel, in common with *cast* iron, loses the property of welding.

may be seen to swell and emit its carbon, gradually losing its fluidity, until at length it can, with the workman's rod, be accumulated into semi-fluid balls of 70 lbs. or 80 lbs. in weight; these he removes from the puddling-furnace, and places under the heavy hammer of a forge. After having been well hammered, and received a flattened form from this forge, they are passed between rollers, and converted into bars; five or six of which, being piled upon one another and brought to a welding heat in a reverberatory furnace, and again passed through successive pairs of rollers until they are reduced to bars of the required dimensions, the process is finished.

82. The Manufacture of Steel.

If cast iron, after having been deprived of its carbon, and other foreign ingredients, and thus brought into the state of *wrought iron*, be *re*-carbonised by a process about to be described, it will not return to the state of cast iron, but to a state in which, admitting of fusion, and *re*-acquiring more than the hardness of *cast* iron, it admits of being hammered, forged, and welded like *wrought* iron; and, when tempered, becomes equally pliant and yielding, and far more elastic. In this state it is known as Steel.

The process by which wrought iron is carbonised into steel, is this: In two long troughs or boxes of fire-stone, built up on either side of the fire-grate of a reverberatory furnace, are piled upon one another the bars of iron which are to be subjected to the process of carbonisation: between each layer of

bars is spread a thick layer of *charcoal*-powder; and when the piles are thus completed (usually to the weight of ten or twelve tons), the top of each trough is closely covered over with a bed of sand. The fire of the furnace is then lighted, and the boxes, and their contents, are kept at a red heat for eight or ten days.

In this heated state the iron attracts, and incorporates with itself, carbon from the charcoal which surrounds it. If, at the expiration of the time mentioned, the process is found to have proceeded far enough, by the examination of a bar drawn for that purpose, the furnace is allowed gradually to cool.

The bars, on examination, are now found greatly to have swelled their dimensions, and to have raised their surfaces every where into blisters; for which reason the steel, formed by this process (called *cementation*), is called *blistered* steel. These bars are then heated to redness, and well forged under a powerful forge-hammer, made to strike with great rapidity, commonly by the action of a water-wheel, and called a *tilting hammer:* the hollow, vascular texture of the blistered steel is, by this forging, reduced to a close continuous granular structure, and the metal becomes *tilted* steel. When the bars of blistered steel are first broken, and then welded upon the surfaces of one another, and then tilted, then broken and again welded and tilted, and this operation is several times repeated, they become bars of *German* or *shear* steel.

However great the care with which the process of cementation is carried on, the bars of blistered steel which result from this process are never *uni-*

formly carbonised. To give to steel that *uniform* quality, which, for cutting instruments, is so desirable, Mr. Huntsman, of Sheffield, conceived the idea of *casting* it; and this process is now commonly pursued. The blistered steel bars are broken into small pieces, and put into crucibles of fine clay, whose mouths are covered with vitrifiable sand, to prevent the access of the air, and the consequent decarbonisation: the crucibles are then subjected to an intense heat, which, in four or five hours, *fuses* the included steel. About twenty tons of coals are required thus to *fuse* one ton of steel; a fact sufficient to account for the high price of cast as compared with other steel.

An impure and variable kind of steel, called German, or furnace steel, is obtained by carbonisation directly from the ore, or from cast iron.

It has hitherto been found impossible to convert English bar iron into good steel. The iron used is all Swedish or Russian: it is brought thence, and manufactured into steel at Sheffield, for every market in the world.

83. Casehardening.

This is a process for converting the *surfaces* of wrought iron articles into steel. The manufacture of the articles having been completed, except the polish, they are placed in an iron box, in layers; a layer of animal carbon (horns, hoofs, skins, or leather, first so burned as to admit of their being reduced to powder) being spread over each: the box being then carefully covered and luted with an equal mixture of clay and sand, is kept at a light

red heat for half an hour, and its contents emptied into water. There is thus obtained, over the whole of the article, a surface of hardened steel, the depth of which depends upon the time during which the process of heating has been carried on: in half an hour, it will, it is said, be extended to a depth somewhat less than the thickness of a sixpence. This method is peculiarly applicable to articles which are required to receive a certain degree of *external* hardness and a *polish*. It does not do for cutting instruments.

84. Effect of Heat on the Strength of Cast Iron.

Messrs. Fairbairn and Hodgkinson found, in some recent experiments, that when the temperature of cast-iron bars was raised from the freezing point (by covering them with snow) to a blood-red heat, the strength was varied in the proportion of 950 to 723. The effect of heat on the strength of iron is not, however, limited to the period during which it is subjected to the heat; in some cases it becomes *permanent*.

85. Permanent Diminution of the Tenacity of Iron Wire by Heating.

If iron wire, after it has been first drawn out, be put into the fire, heated red hot, and then allowed to cool gradually, it will be found to have acquired great additional flexibility, and to have lost *nearly one half of its strength;* all the extraordinary tenacity acquired by *drawing* it is thus lost. The same is true of wires of other metals.

86. Annealing of Cast Iron.

It appears, from the experiments of Mr. Watt and Sir J. Hall, that whether a body in the act of cooling from a liquid state shall assume a *crystallised* form or a continuous *glassy* texture, depends upon whether it be *gradually* or *suddenly* cooled. Upon the texture of a metal, as influenced by these circumstances, depend many of its most important mechanical properties; as, for instance, its hardness and brittleness, or its softness and malleability. The former qualities are given by cooling it rapidly; the latter by cooling it slowly.

When cast iron has, by too *rapidly* cooling, acquired the quality of *hardness*, it may, in some degree, be taken from it again by heating it a second time, and cooling it *gradually*. A number of pieces are piled upon one another, and covered with a heap of turf or cinders; this *is* set on fire, and when the iron has acquired a red heat, the fire is allowed to go out of its own accord. Sometimes a gradual cooling is effected by burying the iron, when at red heat, in a heap of dry saw-dust.

The character of cast-iron is not in any other way altered by this process, which is called *annealing*, except that it is rendered more malleable.

87. The different Mechanical Properties of Hot and Cold Blast Iron.

It has recently been discovered by Mr. Neilson, of Glasgow, that a prodigious saving of fuel in the smelting of iron from the ore may be effected by

propelling into the furnace, instead of the usual blast of *cold* air, a similar blast of air previously *heated*.

The blast is now commonly heated, before it is propelled into the furnaces of the Clyde Iron-works, to 600° of Fahrenheit; and the expense of coals to smelt each ton of metal (including those used for heating the blast), averaged, in 1833, the blast being thus heated, 2 tons 5 cwt. In 1829, when the same furnaces were worked with the *cold* blast, it averaged 8 tons 1¼ cwt. These coals were, moreover, formerly converted, at a considerable expense, into coke before they were used; with the hot blast they are used *un*-coked.

From the first it was observed, that there was a difference in the mechanical properties of irons from the same ore melted by the hot and cold blast, and the former were believed to be of inferior quality. Accurate experiments, recently made by Messrs. Hodgkinson and Fairbairn of Manchester, have however shown that this is far from being the case: a Table, contained in the Appendix, gives the results of their experiments. From this, it appears that the absolute strength of some irons, both as regards their tenacity and their powers of resisting compression, is materially increased by the use of the hot blast; this remark applies indeed to all the irons experimented on, excepting only the Buffery, No. 1. The Carron, No. 3., acquires, by the use of the hot blast, an additional power of resisting compression, amounting to no less than 8 tons on the square inch, and an additional tenacity of 1½ tons on the square inch.

It is to be observed, that hot blast iron possesses

softer qualities under the hammer than cold blast, being of a more yielding and malleable nature. These properties have an analogy to those of annealed cast iron; they, perhaps, connect themselves ultimately with the operation of the same causes.

88. The Tempering of Steel.

Steel is said to be most hardened when it is raised to the highest temperature which it can receive — a white heat — and then suddenly cooled by being plunged in mercury or an acid, or into a mass of lead. If, instead of these substances, water or grease be used to cool it, the temper obtained is not so hard: corresponding to every different degree of heat to which the metal is raised, there is a different hardness; but as *these* are all different degrees of red heat, which it is very difficult to distinguish from one another, the workmen avail themselves of a remarkable property by which the metal can be made to *lose*, to any degree, the hardening which it has acquired, by heating it again to an inferior degree, and allowing it to cool gradually. This is the process to which they have given the name of *tempering*. Communicating, in the first place, to the steel a hardness above that which they require, they then heat it again over charcoal, and cool it gradually, until sufficient of the hardness is taken from it, or until it is tempered to the required degree. This process is facilitated by certain remarkable changes of colour which appear in it as it undergoes this process of a second heating. These colours are, *straw-coloured yellow, purple red, violet blue, blue, clear watery blue.*

The straw-colour indicates the point at which the seond heating should be arrested, to obtain the temper of razors and pen-knives: the purple, that for table-knives; the blue, for watch-springs; and the commencement of the red, for coach-springs.

Steel, when it has received the highest degree of hardness, is more brittle than glass; and thus the *dies* used in coining, which are of the hardest steel, not unfrequently break by mere atmospheric changes of temperature.

Cutting instruments, if highly hardened throughout, would be exceedingly liable to break; it is therefore customary not to harden the parts near the handle, or to temper them more than the rest, that the yielding of these may prevent the parts about the point from snapping.

89. The Tempering of the Alloy of Copper called Tam-Tam.

It is remarkable that copper possesses properties, in respect to its hardening and tempering, which are the opposite of those of cast iron and steel; when cooled *slowly* it becomes hard and brittle, but when cooled *rapidly* soft and malleable. In a yet more remarkable degree is this anomalous property possessed by an alloy, composed of four parts of copper and one of tin called tam-tam, used in the construction of gongs, and other musical instruments. The circumstances under which it becomes malleable have only of late years become known in Europe; and gongs are now made here nearly as perfect as those of the Chinese.

90. The Annealing of Glass.

Glass admits of being hardened to a very high degree; and, like steel, and by the same process, it may be made to lose, in any degree, its hardness.

In the act of cooling, under the hands of the workman, from the state of fusion in which it is blown, every article of glass becomes irregularly hardened; and, if taken in that state into use, its brittleness would be so great, that the slightest shock or the slightest change of temperature would be sufficient to break it: it is therefore transferred from the hands of a blower into a large furnace or oven, called the *leer;* where it is for some time subjected to an uniform heat, and then allowed *gradually* to cool. It thus becomes tempered. Whatever care may be taken in this process, certain phenomena of the polarisation of light nevertheless show that the same temper cannot be diffused *uniformly* through any piece of glass, however small. Glass has, however, been so effectually tempered by Mr. Dent, as to admit of being formed into the balance-springs of chronometers.

91. Prince Rupert's Drops.

This name is given to pieces of glass, which, being let fall into water when in a state of fusion, acquire a long oval form, tapering to a point; which point being afterwards broken off with the fingers, the whole of the drop is thereby made to burst into minute parts, with a loud explosion.

These drops present a remarkable instance of the

irregular tempering of glass. The outside of the drop is suddenly contracted, hardened, and rendered *brittle*, whilst the interior, cooling slowly, retains its *elasticity*: an equilibrium appears, in the process of cooling, to establish itself between the cohesive force of this external sheet and the elasticity of the mass of glass which it compresses; to which equilibrium the *entireness* of the *surface* about the point or tail of the drop is necessary: the explanation of this last remarkable circumstance is unknown. The fact is however unquestionable: the drop may be struck a sharp blow on the *thick* part, and even ground down on a cutler's wheel, without breaking; but if even a *scratch* be made upon it near the *point*, it will burst into a thousand atoms.

Many of these drops burst in the water, in the act of making.

If they be heated, and then gradually cooled or annealed, they lose entirely their property of exploding.

With these facts manifestly connect themselves the influence of heat upon crystallisation.

92. Mitzcherlich's Experiments on Changes in Crystallised Forms of Bodies by the Operation of Heat.

M. Mitzcherlich was first led to observe this fact from certain changes in the *optical* properties of sulphate of lime, at different temperatures. Subsequently he ascertained that sulphate of nikel, when exposed to the rays of the sun in summer, in a closed vessel, without any change in its external form or appearance, changed in a few days its

crystalline structure, from the prismatic form to that of the square octohedron; this fact was determined by breaking it. An exactly similar change took place in seleniate of zinc, when exposed to the action of the sun on a sheet of paper.

Crystals of sulphate of zinc and sulphate of magnesia, when heated in alcohol, by degrees lose their transparency; and when broken, are found to be composed of exceedingly small crystals, differing totally from the original crystalline forms of the salts.

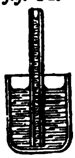
CHAP. III.

CAPILLARY ATTRACTION, AND ADHESION.

93. Ascent of Water in Capillary Tubes.

If one extremity of an open glass tube be plunged

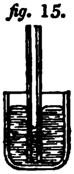
fig. 14.



in water, and the tube be held in an upright position, the surface of the water *within* will be seen to change from a plane to a concave surface, and to rise above the level of that without it.

94. Depression of Mercury in Capillary Tubes.

fig. 15.



If an open glass tube be similarly plunged in *mercury*, the surface of the mercury within it will become convexed, and will sink beneath the surface of that without it.

95. Depression of Water in Capillary Tubes, whose Surfaces cannot be wetted.

If the surface of a capillary tube be such that it cannot be wetted; if, for instance, it be covered with a thin coat of oil, so that moisture will not adhere to it, the phenomena which it will present when plunged into water, will be precisely the same

as those which are presented by a clean glass tube when plunged in mercury. The water will be repelled and depressed all round and within it.

96. The Phenomena of Capillary Attraction and Repulsion are not confined to the internal Surfaces of Tubes, but common to the Surfaces of all Bodies, and only more apparent in these.

Thus, if a fluid be capable of wetting the sides of the vessel which contains it, as well as the tube plunged in it, it will be seen to be raised, and to have become concave all round the sides, and round the outside as well as the inside of the tube; as in the last figure but one: and, in like manner, if the surface of the vessel and the tube be *incapable* of being wetted by the fluid, this will be *depressed* all round it, and have its surface convex; and all round the *outside*, as well as the inside of the tube. This effect is shown in the last figure.

97. The Rise of Water between parallel Plates of Glass.

If two plates of glass be kept slightly asunder, and made perfectly parallel to one another by placing between them two pieces of *wire*, cut from the same length, and if they be then plunged in water, a plate of water will be seen to rise between them. If a tube be taken, whose bore will just admit the wire, or whose diameter equals the distance of the plates, the water will be seen to ascend in this tube precisely to the same height that it does between the plates.

98. The Wick of a Lamp.

The wick of a lamp or of a candle feeds the flame by capillary attraction: it is, in point of fact, a fascicle of threads, the surfaces of which, being very *nearly in contact*, cause the ascent of the oil or melted tallow between them by capillary attraction.

99. An Iron Wick for a Lamp.

If a short capillary tube of iron be placed upright in a reservoir of oil, the oil will ascend in the tube, by capillary attraction, to its top, and may there be lighted.

100. A Syphon Filter, made with Threads of Cotton.

If a fascicle of threads of cotton, such as forms the wick of a lamp, have one of its extremities immersed in a vessel of water, and be then brought over its edge, and be made to hang with its other extremity beneath the level of the surface of the water in the vessel, then, by its capillary attraction, the water will ascend into the space between the threads of cotton; and, on the principle of the syphon, it will *flow* out in drops at the other extremity. If there be any impurities in it, these will be stopped in its progress by the fibres of the cotton.

101. Heavy Bodies made to float by Capillary Repulsion.

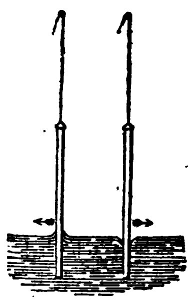
If a small body repelling a fluid, or incapable of being wetted by it, be placed gently upon it — as, for instance, a small ball of wax, or a needle rubbed with oil, upon water — it will float. The repulsion of the body causes a displacement of the fluid beneath and all around it; and since any body, immersed in a fluid, is buoyed up with a force equal to the weight of the water it *displaces*, it follows that this body will float, provided the water it displaces, and by the weight of which it is buoyed up, equals *its own weight*; and it may be made to displace that quantity of water by its repulsion, when otherwise it would not.

102. Insects supported on the Surface of Water by Capillary Repulsion.

It is thus that certain species of insects are supported, and move about freely on the surface of water. Their feet are lubricated with an oily substance, similar perhaps to that which gives a like property to the feathers of aquatic birds. This repulsive substance produces, beneath each foot of the animal, a hollow in the surface of the fluid, which is in fact a boat, supporting it; and its body is thus buoyed up on as many such boats as it has feet.

103. THE ATTRACTIONS OF CAPILLARY RODS WHEN SUSPENDED IN A FLUID.

fig. 16.



If two capillary rods be suspended in a fluid, parallel to one another, then, so long as they are at such a distance that the disturbed surfaces of the fluid immediately about them do not *cross* one another, no attraction or repulsion of the rods upon one another will be perceivable; but when the disturbed surfaces do thus interfere, such attraction or repulsion will immediately become apparent. If *both* surfaces be raised, or *both* depressed—that is, if both rods attract the fluid or both repel it — then the rods will attract one another: but if one surface be raised and the other depressed, as in the preceding figure — that is, if one of the rods be *capable* and the other *incapable* of being wetted by the fluid—then they will repel one another. This experiment was made by Haüy, with two laminæ, — one of which was of talc, and the other of ivory; the former of these substances does not admit of being wetted, that is, it is repulsive of water, the latter substance attracts it — thus, one *depresses* the contiguous surface of the water, and the other *elevates* it; and where the elevation is made to *meet* the depression, a repulsion is immediately apparent.

104. The Attraction and Repulsion of Floating Bodies.

Two small balls of pith, or of wood, each of which *attracts* water, and, therefore, when made to float, elevates it all round the line of contact with its surface, being brought near one another, so that the elevations interfere, will attract one another. In like manner, two small balls of wax, or smoked pith balls, which *repel* water, being made to float in it; or two small iron balls made to float in mercury; when their *depressions* interfere, attract one another: but if a pith-ball and a ball of wax, or another pith-ball which has been held over the smoke of a lamp, be made so to approach that the depression of the one interferes with the elevation of the other, they are immediately repelled.

fig. 17.

105. The Attraction of Needles floating on Water.

If two needles be slightly rubbed with grease, and then placed with care on the surface of the water, they will float upon it, depressing it all around them. If, when thus floating, these needles be made to approach one another, so that their depressions interfere, they will immediately rush into contact.

106. Attraction and Repulsion of small Bodies by the Sides of Vessels.

It is on the principle just stated, that the sides of vessels, containing water, *attract* small bodies, floating upon it, when the material of the vessel and of the floating body are both capable or both incapable of

being wetted, and repel them when one is capable of being wetted and the other not.

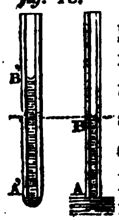
In the first case, the attraction takes place when the elevation round the body interferes with the elevation round the sides of the vessel, or the depression round the body with the depression round the sides.

In the second case, the repulsion occurs when the elevation round the one interferes with the depression round the other.

If the vessel containing the water be *brim*-full, so that the water stands all round above its edges, then, since its surface will be depressed or *convex* at its edges, instead of concave, as when it stood beneath the edges of the vessel, the *opposite* phenomena will occur. All bodies floating upon this surface will be repelled towards its centre, unless they be in their nature such as cannot be wetted.

107. When a Capillary Tube is taken out of the Fluid in which it has been plunged, a Portion of the Fluid which remains in it stands at a much greater Height than it stood before.

fig. 18.



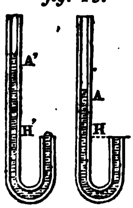
Thus, if A B be the height of the capillary column when the tube was immersed, then, when it is lifted out of the fluid, it will have become A' B', considerably greater than A B; and, if the sides of the tube be very *thin*, the suspended column, when thus withdrawn from the fluid, will even be *double* that which it was before.

This greater elevation of the capillary column, when the tube is withdrawn from the fluid, is produced by the drop, which, when it is withdrawn, is suspended from its lower extremity: the force of attraction between that drop and the end of the tube sustains the additional column. In the case of a thin tube, this drop has an extremely convex surface; in the case of a thick tube, it is spread out and flattened; it is on this circumstance that the greater elevation in the thin than the thick tube is dependent.

108. WATER WILL NOT, UNDER CERTAIN CIRCUMSTANCES, FIND ITS LEVEL IN A CAPILLARY SYPHON.

Water being poured into the larger branch of a bent tube (such as that shown in the accompanying figure), will immediately attain the same level in both branches of the tube until it reaches the extremity of the shorter branch: instead of then flowing out, it will accumulate, and its surface will rise in the longer branch; whilst, in the shorter, its surface will remain fixed, becoming, at the same time, less and less concave, until, at length, it is perfectly flat, as in the second figure; as yet more water is poured in, this flat surface will become convex, the water rising in a drop (as in the first figure), until, when this drop has become a hemisphere, the height A' H', having become double of A H, it will burst; and the surface A' will fall a greater or a less distance, according to the thickness of the tube.

fig. 19.



109. To make a Vessel full of Holes, which shall yet contain Water.

If a vessel be formed of wire-gauze, of iron or brass, then, the meshes being small, it will hold a certain depth of water; for to each mesh will be fixed, by capillary attraction, a drop, which, as in the experiment of the tube, will, by the force of its adherence to the mesh, be sufficient to support the weight of the water immediately above it, provided the height of the superincumbent column, that is, the depth of the contained fluid, do not exceed a certain limit, determined by the smallness of the mesh.

110. To make a Vessel full of Holes, which shall float.

If a vessel, constructed of wire-gauze (as above), be immersed in a fluid, the fluid will not enter it, unless it be sunk beyond a certain depth; for to each mesh will, as before, be made to adhere a small portion of the fluid, which, by the force of its adherence, will prevent the rest of the fluid from entering by that mesh.

111. Effects of Capillarity in the Barometer Tube.

The top of the column of mercury suspended in the barometer tube, should, evidently, be a convex surface, glass being repulsive of mercury. The remark was, however, made in 1780 (by Don Casbois), that if the mercury be for some time boiled in the mercurial tube before it is hermetically sealed, a

perfect vacuum may be obtained; instead of being thus convex, the surface will be plane, or even concave; a fact which seemed to indicate an anomalous attraction of the glass, which might interfere with the accuracy of barometric admeasurements. The circumstance has been recently explained by M. Dulong, who has shown that the surface of the mercury is chemically affected by the ebullition, and becomes an oxide whose capillary properties are no longer those of the mercury.

112. THE HEIGHTS TO WHICH A FLUID ASCENDS IN DIFFERENT CAPILLARY TUBES, ARE GREATER AS THEIR DIAMETERS ARE LESS.

This will be strikingly seen if a number of capillary tubes, having different diameters, be placed side by side in a coloured fluid: the fluid will stand at different heights in all, being *highest* in those whose diameters are *least*.

113. THE HEIGHTS TO WHICH THE SAME FLUID ASCENDS IN DIFFERENT CAPILLARY TUBES, DO NOT DEPEND ON THE THICKNESS OF THE TUBES.

If tubes be taken of different thicknesses, but the same internal diameter, and partly immersed in the same fluid, the fluid will be seen to stand at the same height in all.

114. THE HEIGHTS TO WHICH THE SAME FLUID ASCENDS IN DIFFERENT TUBES, DO NOT DEPEND UPON THE SUBSTANCES OUT OF WHICH THE TUBE ARE FORMED, PROVIDED ONLY THEY BE SUBSTANCES WHICH DO NOT REPEL THE FLUID, OR WHICH ADMIT OF BEING WETTED BY IT.

It is found, by experiment, that the heights to which the same fluid—water, for instance—ascends in tubes of glass, iron, lead, tin, wood, &c., are the same, provided the *bores* of these tubes be all of the same diameter.

115. THE HEIGHTS TO WHICH DIFFERENT FLUIDS ASCEND IN THE SAME TUBE, ARE NOT THE SAME.

A heavier fluid ascends to a greater height than a lighter; thus, water ascends to more than twice the height of alcohol, as will be seen by the table of Gay Lussac's experiments.

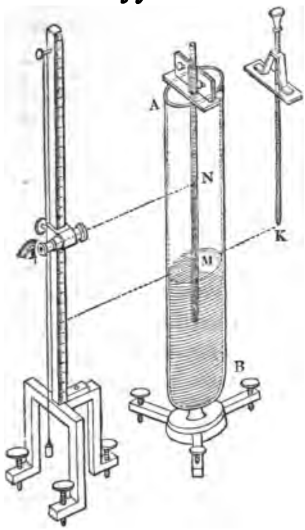
116.* THE HEIGHTS TO WHICH THE SAME FLUID ASCENDS IN DIFFERENT CAPILLARY TUBES, ARE INVERSELY PROPORTIONAL TO THE DIAMETERS OF THE TUBES.

That is, in whatever proportion the internal diameter of any one tube is less than another, precisely in the same proportion is the height of the capillary column in the first greater than the height of that in the other; so that a tube of $\frac{1}{2}$ the diameter will have a column of twice the height; one of $\frac{1}{3}$rd the diameter, a column of 3 times the height, &c. To determine this law accurately, the experiments must

be made with much care, and especially the tubes must be perfectly *clean;* for, as we have shown, the attraction may be converted into a repulsion by the slightest covering of any oily substance upon it. In order to get rid of all foreign substances on the surface of the glass, it is found that it must be *chemically* cleaned; and that upon this cleaning the precision of all experiments depends. Acids, or alcohol, must be passed through it, according to the nature of the impurities to which it has been liable; and it must throughout be wetted beforehand with the liquid in which the experiment is to be made.

The most accurate experiments on this subject are those of M. Gay Lussac. The apparatus used by him is represented in the accompanying figure. A B is a tall cylinder of glass, placed on a stand, capable of being levelled by screws; M is the surface of a liquid contained in it; and M N is a capillary tube, partly immersed in it, and supported by a piece which rests upon the edge of the vessel: beside the glass vessel is a vertical rule, with a divided scale, along which is moveable, by means of a micrometer screw, a small telescope. To measure the height M N, of the column suspended in the tube, the telescope is moved until the summit N of the column is seen on the cross wires of the telescope: the tube is then moved a little to the side of the vessel, and the screw K is made to rest, by means of a piece similar to that which supports the tube, upon the edges of the vessel, and turned until its point K just *touches* the surface of the fluid in the vessel; then, a little of the fluid

fig. 20.



having been removed with a tube *, the telescope is made to descend until the extremity K of the screw is just seen on its cross wires. The height on the scale at which the telescope before stood, having been observed, and the height at which it now stands, the difference between these two heights is that of the capillary column. The following table contains the result thus obtained by M. Gay Lussac: —

* This is a method commonly used for taking out from a large vase small quantities of a liquid it contains. The tube is plunged into it with both its extremities open, and then drawn out with the upper extremity closed with the finger, the pressure of the atmosphere from without keeps a large portion of the fluid, which had entered the tube, from flowing out of it.

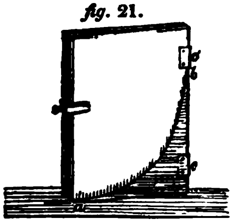
Fluid Expd. on	Specific Gravity.	Temp. in degrees of Centigral Ther.	Elevation in tubes, whose diameters were in millimeters.		
			1·2944	1·9038	10·508
			Milltrs.	Milltrs.	Milltrs.
Water -	1·000	8°·5	23·1634	15·5861	
Alcohol	0·8196	8°	9·1823	6·4012	
	0·8595	10°	9·301		
	0·9415	8°	9·997		
	0·8135	16°	7·078	- -	0·3835
Essence of Terebinthum	0·8695	8°	9·8516		

In these experiments the ratio of the diameters of the two first tubes is 1 : 1·474 ; and the ratio of the heights of the capillary columns is, for water, 1·486 : 1 ; and for alcohol, 1·434 : 1 ; which results, *practically*, coincide with the law.

117.* The Elevation of Water between Plates of Glass slightly inclined to one another.

If two plates be placed in water, in the position shown in the figure, the water will rise between them, its surface forming a curve, which, on examination, is found to be that which is called, by mathematicians the hyperbola. This is easily explained, the greater elevation of the water near the angle of the plates is caused by the less distance of the surfaces of the plates from one another there. This distance of the plates from one another, at different points, is, by geometry, proportional to the distance of those points from the angle of

fig. 21.



the plates: now, it follows from the experiment, article 97., and from the last article, that the heights to which the water is raised between the plates at different points, are inversely proportional to the distances of the plates there; they are therefore inversely proportional to the distances of these points from the angle. Thus, then, the heights of the different points of the surface of the water are inversely proportional to their distances from the angle; a property of the rectangular hyperbola between the asymptotes.

118. Of the Force with which Fibrous Substances imbibe Moisture by Capillary Attraction, and thereby increase their Bulk.

The following is said to be a method used in France for quarrying, in one piece, the large flat stones which are used as mill-stones.* A whole block of the stone being found, of sufficient dimensions, it is hewn into the form of a solid cylinder, several feet in height, and of the diameter required for a mill-stone; this block is destined to form several mill-stones. To cleave it into them, deep grooves are cut round it, where the divisions should take place; and into these grooves wedges of willow-wood, thoroughly dried in an oven, are firmly driven: these wedges, being sunk to their proper depth, are moistened, or left exposed to the humidity of the night: they take up the moisture by the

* See Montucla's Philosophical Recreations, Hutton's translation, vol. iv. p. 157.

capillary attraction of their contiguous fibres, and thereby swell out their dimensions with such prodigious power as to overcome the cohesion of the wide surface of the section of the stone, and divide it.

Another phenomenon, referable to the same principle, is, the lifting of great weights, by fastening a cord to them by one of its extremities, and to some firm attachment above them by the other, stretching it tightly, and then wetting it; the cord will imbibe the moisture by the capillary attraction of its contiguous fibres, and swell out its bulk with prodigious force; and in the act of swelling it will shorten its length, and lift the weight.

119. The Theory of Capillary Attraction.

Matter has been shown to be composed of elements which are inappreciably and infinitely *minute*.

It is between these infinitely minute elements that the greater number of the forces known to us have their only sensible action; and *there* we cannot follow them, to inquire into the *law of* that action. Although these forces — which are called molecular forces, and which include among their phenomena extensibility, compressibility, elasticity, the strength of materials, and capillary attraction — only thus operate sensibly at insensible distances; yet does their operation result in certain manifest and sensible properties of the matter in which they reside.

Thus, for instance, extensibility, elasticity, and cohesive strength, are sufficiently sensible qualities of matter, although the molecular forces — from

which they result, and into which they ultimately resolve themselves — lie hidden, far from our view, among the inexhaustible divisions of matter.

But are there not, it may be asked, in these sensible phenomena, numerous as they are, some indications from which we may reason back to the elementary forces of which they are complicated results? May we not resolve this complicated manifestation of force into others more simple, these into others, and so on, until the reason has thus followed them where the senses could not, and the eye of science seen their operation between *particles* of matter, and measured it through infinitesimals of space, where every appliance of physical sight has long lost it? It is not to be *despaired* of, that this may, in some state of philosophy, far advanced beyond that which belongs to it now, be effected. At present we do not approach that state.

That molecular force whose theory has been most successfully investigated is capillary attraction; and that theory assumes as its basis, an entire *ignorance* of the law by which one particle of matter attracts another; it supposes only that this law, whatever it may be, is the *same* when it is a solid which attracts a fluid, as when it is a fluid which attracts itself; the same *law* of attraction, but a different *intensity* of attraction.

Clairaut was the first, starting from this simple and almost self-evident hypothesis, to prove, by the inexhaustible resources of mathematical analysis, that this one condition was *sufficient*. If the *intensity* of the attraction of the solid on the fluid was

greater than *half* that of the fluid on itself, the fluid would *elevate* itself about the solid — if *it was less, it would depress* itself — and if it was *equal*, it would neither elevate nor depress itself.

La Place, — taking up this theory of Clairaut, and combining with his hypothesis this evident principle, that the unknown law, whatever it may be, causes the attraction to diminish so rapidly, that at *sensible* distances it becomes *insensible;* and, reasoning with admirable ingenuity on the principle, — has succeeded in explaining every one of the phenomena of capillary attraction which have been detailed in this work, with an accuracy which extends even to precise *linear admeasurement;* and may be considered as offering one of the most remarkable verifications that theory has ever received from experiment. This verification, however, unfortunately indicates to us the fact, that various as the phenomena of capillary attraction are, they are none of them sufficient to manifest to us the *real law* of the force on which they depend. That *law is the desideratum*, and they leave us in utter ignorance of it.

It may be mentioned, that some of the principles of the theory of Clairaut and La Place have been impugned by Poissou, in his recent work "On Capillary Attraction;" but it would seem without sufficient ground.

* See Professor Challis's valuable report "On the Theory of Capillary Attraction," in the third volume of the "Reports of the British Association of Science."

120. Application of Capillary Attraction to Assaying.

There is a very beautiful application of the principles of capillary attraction in the process by which the precious metals are separated from foreign ingredients. The method is used generally in assaying,— and is called *Cupellation;* we shall describe it, as we have seen it applied to the separation of gold and silver from the dust which is swept from the shops of working goldsmiths and jewellers.*

A portion of this dust is mingled with a certain proportion of an oxide of lead (red lead), and a small quantity of flux, and placed in a flat crucible, called a *cupel;* the material of which is finely powdered bone-ash, made into a paste, and moulded, by pressure, into a circular mould. The cupel, whose bottom and sides are of great comparative thickness, is then placed in a small earthen oven, called a *muffle,* which is so introduced into the assay furnace, as that a free admission of air shall be allowed to the contents of the *cupel.*

The mixture soon enters into a state of fusion; and the lead dissolving the dust and other foreign ingredients, and uniting with itself continually more and more of the oxygen of the air which has admission to it, becomes more and more liquid, until at length it has reached that state of liquidity in which the intensity of the attraction of its par-

* These sweepings are carefully preserved, they become an article of commerce, and the assaying of them is a separate trade.

ticles for one another, is not so much as double that of its particles for the solid material of the cupel. This limit of liquidity being passed, the whole fluid mass of scoria (composed of the oxide of lead, and the foreign ingredients dissolved in it) passes, by capillary attraction, into the porous material of the cupel, whilst the metallic substances, gold and silver, — which have been dissolved, but have not partaken in the oxidisation of the lead, and have not therefore passed the supposed limits of fluidity — remain behind, collected in a globule in the bottom of the cupel. When the whole is cooled, the globule of gold and silver is taken out; and the cupel being broken, the mass of scorea is found collected in a cake, in the substance of the bottom of it, there being no external indication of its presence there.*

The process, above described, is that used for *testing* the average quantity of the precious metals in the sweepings, before they are purchased. It is, however, an epitome of the process by which the separation is made on a larger scale; except that various contrivances are there introduced for economising it. Metallic lead, for instance, is made to supply the place of red oxide of lead; and, by an ingenious process, this lead, after being used, is separated from the scoria, and used again and again, to serve the purpose of the refiner. The cupel used is made shallow, and of large dimensions;

* The silver of the metallic globule, which remains in the cupel, is separated from the gold by solution in nitric acid, and precipitated from this solution by immersing in it bars of copper.

air is propelled upon the surface of the liquid mass; and, as in the process of oxidisation the lead increases its volume, a portion of it is allowed to run over the sides of the cupel.

121. The Agency of Capillary Attraction in Nature.

Let it not be supposed, that the phenomena of capillary attraction are limited to mere *experiments* in physics, or to its applications in art. Capillarity is one of the most active principles in nature. What is it but this ubiquitous power, which retains in the soil of the earth the moisture necessary to vegetation, ministering it, drop by drop, to the radicles of plants and trees; conveying it, at one time, beneath the surface, down the slope of a hill, to the valley below, or to some deep-sunken reservoir; thence lifting it up again to quench the thirst of the parched herbage; checking its progress to the streams, which it would otherwise swell *instantaneously* to *floods* — floods, whose waters, having uselessly deluged the land, would be lost as uselessly in the ocean. *Take away* capillary attraction, or *alter* it, so that the intensity of the attraction of the solid substances which compose the soil, for water, shall be *less* instead of *more* than half the intensity of the attraction of water for itself — and the earth must become a *desert*. Rain would fall upon it as mercury falls upon a piece of glass — it would roll off it in drops. There would be no fertilising influence in the shower; no moisture could reach the parched roots of plants and trees; vegetation would become extinct — and

animal life would gasp itself away in a thick atmosphere of dust.

The greater apparent elevation of water, and the greater force of capillarity in its operation in many natural phenomena than in artificial tubes, is to be explained by the extreme proximity of the surfaces between which it *there* acts. The elevation of water in a tube is inversely proportional to its diameter, or, between two plates, it is inversely proportional to the distance of the plates; thus, if we kept *halving* the diameter of a tube, or *halving* the distance between the plates, we should keep *doubling* the elevation of the *fluid*. *Artificial* tubes may thus be made to elevate water to a remarkable height: but nature provides tubes infinitely finer than any that art can reach; and to the capillary elevation of fluids in them, there seems to be no limit. The same is the case with particles of earth and sand; the close proximity of their surfaces to one another, gives them a power of capillary attraction, which is almost without limit.

122. Endosmose and Exosmose.

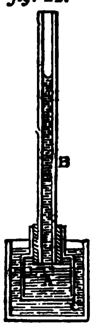
M. Dutrochet having introduced into the swimming bladder of a carp a thin mucillage, effectually closed up the aperture by which he introduced it, and placed the bladder in water, found, by weighing it, after it had remained there some time, that its weight had considerably increased: the water in which it was immersed had, in fact, made its way through the substance of the bladder, and mingled itself with the mucilage.

He then filled the bladder with water, and im-

mersed it in the thin mucillage, and found that the opposite phenomenon took place. The bladder and its contents *lost* weight: the water made its way through the substance of the bladder into the mucillage. These phenomena he afterwards developed, under a great variety of other circumstances; and called the first *endosmose*, and the other, *exosmose*.

His subsequent experiments will best be understood from the description of an instrument, which he calls the *endosmometer*; and which is represented in the accompanying diagram. It represents two reservoirs — an outer, C, and an inner one, A; which may be of glass: the inner one is open at the bottom, and is supported above the bottom, and away from the sides of the other: a vertical tube, B, is fitted into the top of it by grinding. Over the open bottom of the inner reservoir is stretched, tightly, a membrane of bladder, or there is cemented across it a piece of slate, or other porous substance, whose properties are the subject of experiment.

fig. 22.



Now, suppose water to be contained in the exterior reservoir, and alcohol in the interior; the two fluids will then be *divided* by the partition of bladder, or the porous solid plate forming the bottom of the inner reservoirs.

This division of the two fluids into two separate chambers will not however be sufficient to prevent them *mingling*. Through the substance of the partition the water will, in a few minutes, be seen to have made its way, by the rising of the alcohol in

the tube; and, if the tube be not more than 16 or 18 inches in length, in the course of a day the alcohol will have risen to the top of it, and flow over.

This remarkable fact is hitherto entirely unexplained. Dutrochet is said to have ascertained, by the most delicate experiments, that it is accompanied by no perceptible traces of change in the electrical state of the substances concerned.

If one of the vessels contain water and the other gum-water, or acetic acid, or nitric acid, or especially hydrochloric acid, the exosmose will be *from* the *water*.

Besides bladder, numerous other animal, as well as vegetable, substances present similar phenomena of endosmose, and partake of it in common with inorganic substances—such as plates of baked earth, of calcined slate, and of clay.

The extreme elevation of the liquid in the tube, marks the force of the action: that elevation is different for different liquids, and when partitions of different substances are used.

Dutrochet found water thickened with sugar, in the proportion of one part of sugar to two parts of water, to be productive of a power of endosmose, capable of sustaining the pressure of a column of mercury 127 inches in height. Dutrochet conceived, on no sufficient grounds it would appear, that endosmose was the immediate agent in all the phenomena of vegetable life.

123. Adhesion of Plates of different Substances to the Surfaces of Fluids.

If a plate of any substance be brought into contact with the surface of a fluid, it will immediately

be perceived that an adhesion has taken place between the two, which may be measured by attaching the plate, by means of a string, to one extremity of the scale of a balance, and adding weights in the other scale-pan until the adhesion is overcome.

The following table contains the results thus obtained by M. Gay Lussac, with a plate of glass:—

Fluids experimented on.	Specific Gravity.	Temperature.	Weight necessary to detach a circular disc of glass, diameter = 118·366 millimeters.
			Grammes.
Water - - - -	1·000	8·5	59·40
Alcohol - - -	0·8196	8	31·08
—— - - -	0.8595	10	32·87
—— - - -	0·9415	8	37·15
Essence of Terebinthum -	0·8695	8	34·10

A disc of copper or of any other substance, of the same diameter, and capable of being wetted by the fluid, gives exactly the same result. A circumstance which is easily understood; for the surface of the plate always brings away with it a thin film of the fluid: it is the adhesion of this film of fluid to the rest which is therefore broken.

M. Achard—by whom an extensive series of experiments on this subject was made, and their results published in the Berlin Memoirs for 1776—found, by varying the atmospheric pressure, under which his experiments were made, that the results were wholly independent of it. Varying the temperature, he found that as it was *increased*, the adhesion uniformly *diminished*.

When the substance of the disc is repulsive of the fluid, or incapable of being wetted by it, it is found

that an adhesion of the two still exists; which is, nevertheless, exceedingly variable, depending on the time during which contact has been allowed to take place. Thus M. G. Lussac found, that to separate the disc of glass used before, from the surface of mercury, the weight required increased, with the time of adhesion from 158 to 296 grammes. In this case the adhesion of the fluid to itself is stronger than its adhesion to the plate.

It is found that, under these circumstances, different metals have different forces of adhesion to the surface of mercury.

M. Guyton de Morveau (Élémens de Chymie, 1777,) found that the separation of a circular disc of pure gold, one inch in diameter, from the surface of mercury, required a weight of 446 grains; an equal disc of silver, 429 grains; a disc of tin, of the same size, 418 grains; of lead, 397 grains; of Bismuth, 372; of platina, 282; of zinc, 204; of copper, 142; of antimony, 126; of iron, 115; of Cobalt, 8.

These forces of adhesion appear to be in the proportion of the *chemical affinities* of mercury to the different metals experimented on; they were looked upon in that light by M. Guyton.

124. Adhesion of a Column of Mercury to the Internal Surface of a Capillary Tube.

The following fact was observed, in 1792, by Huygens:—A barometer tube, 70 inches in length, and a few lines in diameter, having been well cleaned with alcohol, filled with mercury, freed from all air, and then carefully inverted, it was

seen with amazement, that the column, instead of descending to the barometric height, remained suspended, until the tube had been several times slightly shaken, when finally it occupied its proper position of 28 inches. This phenomenon, which occurs under the same circumstances whenever the tube is thoroughly cleaned, is evidently a result of the adherence of the mercury to the tube.

125. Adhesion of Plates of Glass to one another.

When pieces of plate glass have received their last polish from the hands of the workman, it is customary to clean them, and to place them in a vertical position in the warehouse, somewhat like books on the shelves of a library. In this position they not unfrequently acquire, in the course of time, an adhesion, which renders it very difficult, and sometimes impossible, to separate them. Three or four plates have been thus absolutely *incorporated*, so that they might be worked as one piece, annd even cut with a diamond, like a single piece. M. Pouillet states that he had seen pieces of glass, thus united, from the royal manufactory of St. Gobin, which adhered as perfectly as though they had been melted together. An exceedingly great mechanical force was applied, to cause them to slip upon one another; and when at length they yielded, it was found, on examination, that the plates had not separated at their common surfaces, but that the thickness of the glass had been torn away; so that to the surface of one, still adhered a lamina of the other.

CHAPTER IV.

STATICS.

DEFINITIONS.—THE EQUILIBRIUM OF THREE PRESSURES. — THE EQUILIBRIUM OF ANY NUMBER OF PRESSURES IN THE SAME PLANE. — THE LEVER. —THE WHEEL AND AXLE. — THE COMPOSITION AND RESOLUTION OF FORCES. — THE CENTRE OF GRAVITY. — THE RESISTANCE OF A SURFACE. — FRICTION. — THE INCLINED PLANE. — THE WEDGE. — THE SCREW. — THE EQUILIBRIUM OF BODIES IN CONTACT.—PIERS.—ARCHES.

FORCE is that which produces or destroys motion, or which *tends* to produce or destroy it.

That which is the subject of motion, or a tendency to motion, is MATTER.

126. EQUILIBRIUM.

When the tendency of a force to communicate motion *does not take effect*, it is a thing of daily experience and observation, that there exists some other force or forces, having, one or more of them, an *opposite* tendency; which other forces are the *causes* of the quiescence. That state of a body in which, being acted upon by certain forces, it remains at rest, or as it may be termed, the state of its *forced* rest, is called its state of EQUILIBRIUM: and the forces which *constitute* that state are said to be *forces in equilibrium*, or PRESSURES.

127. Forces of Pressure, and Forces of Motion.

Pressures are, then, forces whose tendency to produce motion in a body *does not take effect;* and they are thus distinguished from those in which this tendency *does take effect,* and which are Forces of Motion.

The *laws* which govern the operation of these two great classes of forces are as different as are their phenomena, and the circumstances under which they act. Nevertheless there have been shown to exist certain *relations* between them, so that the phenomena of either may, in a degree, be deduced from those of the other.

The general laws which govern the various relations of forces of pressure will first be discussed in this work; and then those of forces of motion. The former discussion belongs to a science, called the science of Statics, from a Greek word (ιστημι), signifying to stand, or to be in a state of rest; and the latter, to a science called that of Dynamics, from a Greek word (δυναμις), implying force coupled with *motion.*

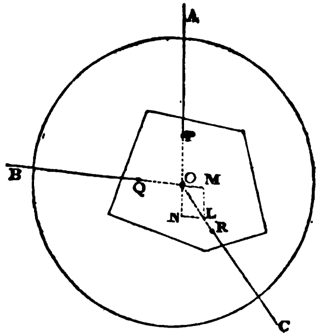
128. The Relation between Three Pressures in Equilibrium.—The Parallelogram of Pressures.

This fundamental principle of Statics will be readily understood from the following experiment:—

Let a board be made to float on the surface of water, in a vessel filled to the brim.

Let three strings, attached to different points, P, Q, R, in the surface of this board, be made to

fig. 23.



pass over pulleys, the heights of which are so adjusted that the strings may just lie *flat* upon the board: from these strings let different weights be suspended, and let the positions of the pulleys be so adjusted that the board may float free of the sides of the vessel, and that each string may run freely on its pulley. When the whole has, under these circumstances, come to rest, the following remarkable relations will be found to obtain, between the directions of the strings and the magnitudes of the weights attached to them:—

1st. If the directions of the strings A P, B Q, C R, be *produced*, they will all meet in the same point, O.

2d. If in A P produced, O N be measured off, containing as many inches as there are pounds weight in the weight acting on the string A P, and in B Q produced, O M be measured off, con-

taining as many inches as there are pounds in the weight attached to BQ, and if a parallelogram, ONLM, be then drawn, having OM and ON for two of its adjacent sides, then will the diagonal OL of this parallelogram be exactly in the same straight line with the third string, CR.

3dly. The *number* of *inches* in the diagonal OL of this parallelogram will exactly equal the number of pounds in the weight attached to this third string, CR.

The weights have been supposed to be measured in *pounds*, and the distances in *inches*. Any other unit of weight, an ounce or an hundred weight might have been used, and instead of inches the distances might have been measured in eighths, or in tenths, or in any other fractions, of an inch, or in feet or yards. The law which governs the equilibrium of weights is manifestly that which governs the equilibrium of *any other pressures whatever;* for a weight may be taken equivalent to *any* pressure: moreover, it will be found to be true for *any weights* whatever. Thus, then, it appears that this law of the parallelogram of pressures is true for any pressures, and is a *general* law. It is usually proved by *theory*, and is the foundation of the whole theory of statics.

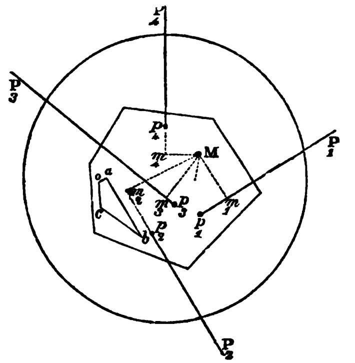
The board is floated, to neutralise its gravity; which force would introduce other forces into the system, and interfere with the equilibrium of the three, were the board laid upon a table, or only suspended between the pulleys. That the experiment may completely succeed, it is necessary that the pulleys should be of the best workmanship, and

of considerable size, that friction may, as much as possible, be avoided.

129. THE EQUILIBRIUM OF ANY NUMBER OF PRESSURES IN THE SAME PLANE.—THE PRINCIPLE OF THE EQUALITY OF MOMENTS.

Let us now suppose, that instead of the *three* pressures applied to the board in the last experiment, there were *any number*, as shown in the accompanying figure.

fig 24.



The pullies being adjusted as before, and the board having come to rest, the following relation will be found to obtain between the magnitudes and the direction of the pressures:—

If any point M be taken on the board, and perpendiculars Mm_1, Mm_2, &c. be drawn from M on the directions of all the strings, or on those directions produced, and if the number of inches in the length of each perpendicular be multiplied by the number of

pounds in the corresponding weight*, and this product be called the *moment* of that weight; then the *moments* of *all* the weights being thus taken, and it being observed that some of these weights tend to turn the board in one direction about the point M, and some in the opposite direction, it will be found that the *sum* of the *moments* of all those which thus tend to turn it one way, equals the *sum* of the *moments* of all those which tend to turn it the other.

This principle is called, that of the EQUALITY OF MOMENTS; it may be deduced from the principle of the parallelogram of forces; and, like it, it is perfectly general, applied to any number of pressures in the same plane, and to pressures of any kind. The units of weights and measurement have been taken to be *pounds* and *inches*,—they may be any other units whatever.

130. THE POLYGON OF PRESSURES.

If in the last experiment any point, *o*, be taken on the board, and if from that point there be drawn a line, *o a*, parallel to the string $P_1 p_1$, and as many inches in length as there are pounds in the weight attached to that string; if moreover, from the extremity *a* of this line, a second, *a b*, be drawn parallel to the string $P_2 p_2$, and as many inches long as there are pounds in the weight attached to that string; and if from *b*, a third line be similarly drawn, parallel to $P_3 p_3$, and so on, until a line is drawn parallel to the last string, then will it be found that the line parallel to this

* That is the weight attached to the string, on which this perpendicular falls.

last string will pass through the point *o*, from which the first line was drawn, so as to complete a geometrical figure, called a polygon. Moreover, this last line, *c o*, so terminating in *o*, will be found to contain exactly as many inches as there are *units* (i. e. pounds or ounces) in the last weight. This remarkable principle, called that of the POLYGON OF PRESSURES, and that, last described, of the EQUALITY OF MOMENTS, are *necessary* to the equilibrium of any number of pressures acting in the same plane; and constitute *all* that is necessary to that equilibrium.

131. THE LEVER.

A lever is a rigid bar, moveable about a certain fixed point, called its fulcrum, and acted upon by the resistance of that point and by two other forces applied to other points in it; one of which it is the use of the lever to overcome by the action of the other. A lever, then, if we put its own weight out of the consideration, is a body acted upon by three forces in the same plane—which three forces, *when it is on the point of moving*, are in equilibrium; the principle of the equality of moments must then obtain in respect to them. Take, then, in every one of the cases represented in the accompanying figures, the fulcrum C of the lever, for the point from which the *moments* are measured; the moment about this point of the *resistance* of the fulcrum will in each case be nothing, because the perpendicular *from* that point, upon the direction of the resistance which goes *through* it, is, of necessity, nothing: thus, the moment of *one* of the three forces which act upon the lever being in each case

nothing, and the principle of the equality of moments still obtaining, it must obtain in each case in respect to the other two forces.

The moment about C of the one A, called the power, is then in *every case* equal to the moment of the other W, called the weight. Thus in the case represented in the first of the accompanying figures, where the power and weight act at opposite extremities of the lever, and the fulcrum is *between* them; and where the weight, (which is 72 lbs.) acts at a distance of *one* division (representing an inch a foot, a yard, &c.) from C, whilst the power acts at a distance of *nine* such divisions; it follows, by the principle of the equality of moments, that, when there is an equilibrium, the first pressure, multiplied by 1, must equal the other multiplied by 9; these *products* being the *moments* of the two forces. Thus the pressure of the hand P must be such that the number of pounds in it, or equivalent to it, being multiplied by 9, shall equal 72

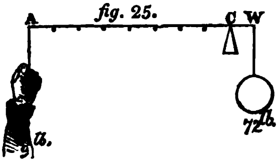


fig. 25.

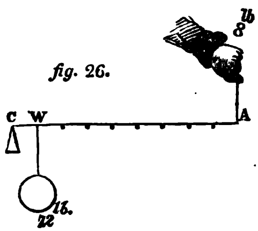


fig. 26.

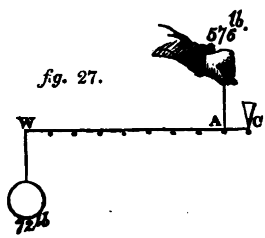


fig. 27.

multiplied by 1. Now that this may be the case, it is evident that P must be a force equivalent to 9 pounds. In the second figure, the perpendicular distance C A, of A, from the fulcrum is 9 divisions, and that of W, 1 division; A must then be a force of such a number of pounds, that this number multiplied by 9 shall equal the number of pounds in W multiplied by 1; or, W being 72 lbs., it must equal 72: that this equality may obtain, A must evidently be a force of 8 lbs.

In the third figure, the distance of A from the fulcrum is 1 division, and that of W is 8 divisions; A must then be such that, multiplied by 1, the product shall equal 72 multiplied by 8; an equality to make up which, A must equal no less than 576 lbs.

In the two first figures, the *power* A is nearer to the fulcrum than the *weight* W to be raised by it; and, for this reason, a power less than the weight yet has an equal momentum, and makes up the equilibrium.

In the third figure, the *power* is nearer to the fulcrum than the *weight;* to have a momentum equal to that of the weight, it must, therefore, be *greater* in amount than it.

In the two first cases, the power is said to act by the intervention of the lever, at a mechanical *advantage;* in the last case, at a mechanical *disadvantage.*

Levers, such as those represented in the first figure, in which the weight is on the opposite side of the fulcrum from the power, are said to be of the first class.

Levers, similar to those in the second figure, in

which the weight is between the fulcrum and the power, are of the second class.

Levers, like those in the third figure, where the power is applied between the weight and the fulcrum, are of the third class.

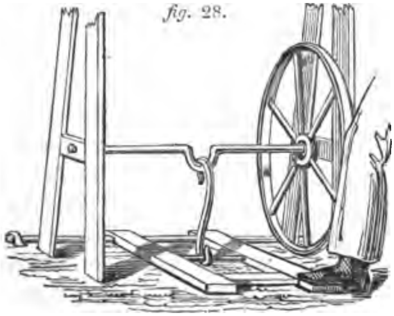
The relation above described manifestly obtains whether the lever be straight, as shown in the figure, or of any crooked form whatever.

Of the first class of levers are the hand-spike, the pump handle, the hammer when used to prise up a nail, scissors, shears, nippers, a common poker when used to raise the coals, &c. &c.

Of the second class, which support the weight between the fulcrum and the power, are the crow-bar, the wheel-barrow, nut-crackers, &c.

To the third class belong the treddle of a lathe, a pair of tongs, shears such as those used for shear-

fig. 28.



ing sheep, &c. The limbs of locomotion and prehension of all animals, are, moreover, levers of this class: the power applied to them is by means of tendons whose direction is near the joint, which is

the fulcrum of each; and the weight raised at a distance from it, whether it be only the weight of the limb, or something in addition to that weight, which it moves. Thus, applied near the fulcrum of the limb, the muscular force required is enormously great.

132. Could Archimedes have lifted the World with a Lever if he had had a Fulcrum to rest it upon?

In *reality* Archimedes would have had *no difficulty* in moving the world could he have brought his lever *to bear upon it.* It rests upon *nothing*, is suspended by *nothing*, rubs against *nothing*, and floats in space without being *buoyed* up. It is perfectly free to move in *any* direction; no force would oppose itself to any attempt which Archimedes might make to move it either upwards or downwards; the only forces which act upon it — its centrifugal force and that which attracts it to the sun — being exactly *balanced*, and, as it were, *neutralised.* So that, in point of fact, to move the earth, the mechanical advantage of a *lever* is a superfluous thing; it would yield to any, the slightest force, impressed upon it, and Archimedes had only to *stamp his foot* and the thing was done. These were not, however, the ideas entertained by Archimedes on the subject. His conception of the matter evidently was, that the huge mass of the earth rested upon some other mass based in the infinities of space, towards which other mass it gravitated as does a stone or a rock to the mass of the earth; and the question which presented

itself to his mind was, what, on this supposition, would supply a sufficient force to lift up and overthrow it. This sufficient force he found in his lever, his own arm moving it. "Give me," said he, "a place where I may stand, and I will move the world."* The principle on which his conclusion was founded was undeniable; the calculation was perfectly correct; but one element was probably omitted from it, it was the *time* requisite to give so huge a mass any *appreciable* motion by means of a lever, which should move it with so small a force as that which the arm of Archimedes could supply.

Taking the diameter of the earth at 7,930 miles, the number of cubic feet in it may be calculated to be 38,434,476,263,828,705,280,000: and assuming each cubic foot to weigh 300 pounds, which has been assumed as a probable amount†, we shall have for the weight of the earth, in pounds, the number 11,530,342,879,148,611,584,000,000. Now, supposing Archimedes to act at the end of his lever with a force of 30 pounds, one arm of it must be 384,344,762,638,287,052,800,000 times longer than the other, that he may move this mass with it. And, one arm of the lever being this number of times longer than the other, when it was made to turn round its fulcrum, the end of that longer arm must move exactly this number of times faster, or farther, than the end of the other: so that, whilst the end of the shorter arm was moving one inch, the end of the longer arm must move 384,344,762,638,287,052,800,000

* Δὸς μοί που στῶ καί τον κοσμον κίνησω.

† Hutton's "Mathematical Recreations," vol. ii. p. 14.

inches; and conversely, when Archimedes had made the end of the lever to which he applied his arm move this immense number of inches, he would only have prised up the *earth*, to which the other end was applied, *one inch*.

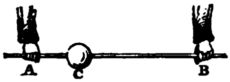
Now, a man pulling with a force of 30 pounds, and moving the object which he pulls at the rate of 10,000 feet an hour, can work continually for from eight to ten hours a day, and this is all that he can accomplish. Each day, then, Archimedes could, at the utmost, move his end of his lever 100,000 feet, or 1,200,000 inches; and hence it may thus readily be calculated, that to move it 384,344,762,638,287,052,800,000 inches, or to move the other end—that is, the earth—one inch, would require the continual labour of Archimedes for 8,774,994,580,737 CENTURIES.

133. TWO PERSONS CARRY A BURDEN BETWEEN THEM BY MEANS OF A LEVER OR POLE, TO FIND HOW MUCH OF THE WEIGHT IS BORNE BY EACH.

Three forces are in equilibrium on such a pole: the burden borne, and the two forces which bear it. Therefore, by the principle of the equality of moments, if we take *any* point in it, and take the moments of these forces, severally, about that point, the *sum* of the moments of those which tend to turn it one way about it, must equal the sum of those which tend to turn it the other. Take either extremity for the point. The moment of the force at that extremity will then be *nothing*, since the perpendicular upon it will be nothing. The moments

of the two other forces must therefore be equal. Thus, if C be the burden, and A the force with

fig. 29.



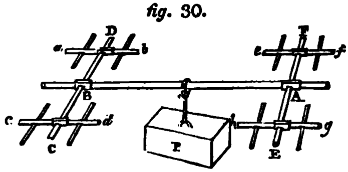
which the pole is supported at the extremity A, then A multiplied by AB must equal C multiplied by CB; and the value of A being found so as to make up this equality, will be the true force exerted at A. In the same manner the force at B may be found.

134. Method of combining the Efforts of a great Number of Men to carry a Burden.

The following method is said* to have been used in Constantinople for raising and carrying the heaviest burdens, such as cannons, mortars, and enormous stones; and the rapidity with which they were thus transported from one place to another is stated to have been truly surprising.

AB is a bar of sufficient strength to sustain the

fig. 30.



whole weight of the load P, which is attached to its middle point; CD and EF are cross bars fixed to this, near its extremities; and to the extremities of

* Hutton's "Mathematical Recreations," vol. ii. p. 8.

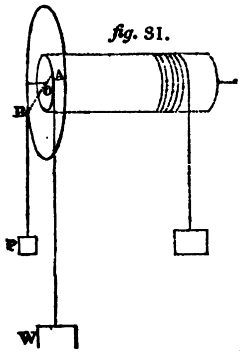
these cross bars are affixed others, *ab, cd, ef, gh;* to these last again, in like manner, others, whose extremities are borne upon the shoulders of the men who are to carry the load. Supposing one man's shoulder to support each extremity of these last mentioned bars, the whole number, whose effort will be combined to lift and carry the weight, will be 16. If other cross bars had been fixed in like manner to the extremities of *these,* the united effort of 32 men might have been applied. If other cross bars had been fixed yet again to the extremities of *these,* 64 men might have united their strength to the task; and so on for any number. If the bar A B, which supports the weight, carry it suspended accurately from its middle point; and if the point where each cross bar is fixed to the one preceding it in the series be exactly half way between the points where the two which follow it are affixed to *it*—then the weight will be *equally* distributed between all the bearers. A small deviation from this rule will produce great inequality in the distribution, and it would be easy to adapt this distribution, according to the principles explained in the last article, so that each should have any given share of the load. The inconvenience of the method is the increase of the load by the weights of the additional cross pieces.

135. The Wheel and Axle.

If we imagine a wheel moveable about a fixed axis O, and having cut in it two circular grooves A and B, whose centres are in O, and if we con-

ceive strings, A W and B P, to be wound round these grooves, carrying at their extremities, P and W, weights which just *balance* one another, then shall we have a system of three forces acting in the same plane and in equilibrium, and therefore subject to the law of the *equality of moments*.

fig. 31.



These three forces are the *weights* P and W acting in the directions B P and A W, and the *resistance* of the fixed axis O. Now let us take O for the point from which we measure the moments; the moment of one of the three forces—the resistance of the axis—will then *vanish;* for the perpendicular from O upon this resistance, which acts through O, is manifestly nothing, and therefore the *product* of the resistance by this perpendicular is nothing. The only moments which remain are those of P and W. These, therefore, by the principle of the equality of moments, are *equal*.

Now the perpendicular from O, upon the direction of W, is O A, and that upon the direction of P is O B. The number of lbs. or cwts. in W multiplied by the number of inches in O A, being its *moment*, is then equal to the number of lbs. or cwts. in P multiplied by the number of inches in O P, being *its* moment; and this is the relation which must exist between P and W, so that they may be in equilibrium. If, for instance, O A were

3 inches, and O B 11 inches, and if W were 132 lbs., then would the moment of W be 3 times 132 or 396; and that P must be such that, being at 11 inches distance, it may have the same moment, 396, that W has. P must therefore be 36 lbs., because 11 times 36 is 396.

By diminishing the distance O A at which the weight W is applied, in comparison with the distance O B at which the power P is applied, we may by this contrivance balance ever so great a weight by ever so small a power.

In the actual use of the wheel and axle, it is customary not to apply the weight to be raised in the plane of the same circle to which the power is applied, but to widen the circular groove A into a cylinder of considerable length, as shown by the dotted lines in the figure, and to cause the string which carries W to wind round this cylinder. It is evident that, applied any where to the circumference of this cylinder, which is supposed to be solid and rigid, it will produce exactly the same effect as though it were applied at A, and will have the same relation to the power P as though it were applied there.

Moreover, it is customary in the use of this instrument, not to apply the power P as shown in the figure, by means of a circle and a cord winding round it.

The power is usually the effort of a workman, and is applied by means of an arm fixed to the cylinder, and carrying at its extremity a handle. The instrument then becomes the WINDLASS. The power applied to it in this case by the workman is

not the same throughout each revolution. The *direction* in which he *pulls* or *pushes* the handle *varies* continually, and the perpendicular upon this direction from O varies therefore continually; so that, unless the force which he exerts continually vary in *amount*, its *moment* cannot remain the same, so as to equal to or a little exceed the moment of the weight which *does* always remain the same. Of this necessary variation of his effort dependent upon the direction in which it is made, the workman is perfectly conscious.

If the cylinder is placed vertically, instead of horizontally, and the force is applied by means of a number of bars fixed horizontally, like radii, in its upper extremity, which are pushed forwards by the workmen, the instrument becomes the CAPSTAIN, whose principal use is to elevate the heavy anchors on ship-board.*

136. Modification of the Wheel and Axle, by which any Weight can be raised by a given Power.

A limit is in practice fixed to the weight which a given power will raise on the common wheel and axle by the insufficiency of the cylinder, when its radius O A is diminished beyond a certain limit to bear the weight suspended from it. It is in diminishing this radius O A of the cylinder, or increasing the distance O B at which the power is

* See "Mechanics applied to Arts," page 84.

applied, that we increase the weight which a given power will raise; and both these methods of increasing it become, beyond a certain limit, impracticable. There is another form of the wheel and axle, of admirable ingenuity, which completely removes the difficulty. Instead of *one* cylinder, *two* of different diameters, as shown by the dotted lines on the figure, are fixed together, and moveable upon the same axle. The two ends of the cord which supports the weight, are wound in opposite directions round these cylinders, and this cord passes round a moveable pulley which carries the weight suspended from it. It is evident that, thus applied to the cylinder, the equal tensions of the two strings which support the weight will produce the same effect as though they were applied in the same plane as the power at B and C. Suppose them to be applied there; this plane will then be acted upon by four forces, the power P, at A, the equal tensions of the two strings which carry the weight at B and C, and the resistance of the axis at O. Now these are forces in equilibrium; the principle of the equality of moments obtains then in respect to them; and taking O for the point from which the moments are

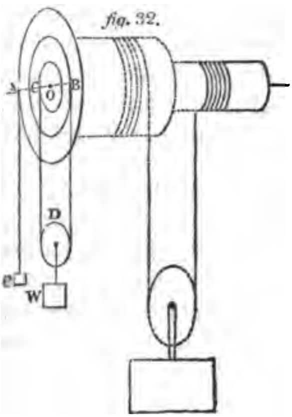


fig. 32.

measured, since the moment of the resistance of the axis vanishes there, it follows that the moments of P, and the tension at C, which tend to turn the system one way about O, are, together, equal to the moment of the tension at B, which tends to turn it the other way. The moment of P then must be such that, being added to the moment of C, it shall make up a sum equal to the moment of B, or, in other words, the moment of P must equal the difference of the moments of B and C; and the less the difference of the moments of B and C, the less need P be, to produce this equality and balance the system. Now the actual forces at B and C are equal, for the strings B D and C D support the weight *equally;* the difference of their *moments* depends then entirely upon the difference of their *distances* O B and O C from O, or upon the difference of the radii of the circles to which they are applied, or upon the *difference* of the diameters of the two cylinders; the less the difference of these diameters, the less the power required to maintain the weight in equilibrium, and to move it. Thus, by making the two cylinders more nearly of the same diameter, we can diminish the power necessary to raise any given weight, or increase the weight which any given power will raise, without limit.

137. When any Number of Pressures acting on a Body, in the same Plane, are not in Equilibrium, to apply to it another which shall produce an Equilibrium.

If we know all the pressures in a system of pressures in equilibrium, excepting *one*, we can, from the principles of the equality of moments and the polygon of pressures (see articles 129. and 130.), determine what that one must be; for that one must be applied at such a *distance*, and of such a *magnitude*, as to make up the deficiency in the *equality of moments*, and in such a *direction* that it may complete the *polygon of pressures*. Thus, then, to find a pressure which will cause any number of other pressures to be in equilibrium, we have only to take any point, and *thence* estimate the moments of all the other forces, and find how much is necessary to make up the equality of their sums, as explained in article 129. We shall thus know what must be the *moment* of the required force. Drawing then the *polygon* of pressures, as was also described, this polygon will be *complete*, all but *one side*, and we shall know the magnitude and direction of that side. The number of inches in its magnitude will tell us the number of pounds in the required force. Also, we before have found its *moment*, and we now know its amount; we can, therefore, tell what must be its perpendicular *distance* from the point we have assumed. Moreover, its direction is parallel to the last side of the polygon. These two facts guide us to the exact

position where it must be applied, so that thus it is fully determined.

138. The Resultant of any Number of Pressures.

The resultant of any number of forces acting upon a body is that force which would *singly* produce the same *effect*, as to the equilibrium or motion of the body, that they do *conjointly*. Now let us imagine any number of forces to be in equilibrium, and of these let us take all except one particular force, and let us consider what is their *resultant*. It is that force which would produce the same *effect* singly that they do conjointly. But what effect do they produce? They just *balance* the one remaining force: this is their effect. *Any* force, therefore, which would balance the one remaining force would produce the same effect that they do. But a force exactly *opposite* to that one remaining force, and *equal* to it, would balance it. That force is then the resultant we want.

And to find the resultant of any number of forces, we must first find a single force which will produce an equilibrium with them. Having found this, we know the resultant; for it is equal and opposite to this force.

Thus, for instance, if *two* forces act upon a *point* in directions *inclined* to one another, and we wish to find their resultant, we must, in the first place, find the third force which will produce an equilibrium with these two. This we may do at once

by the parallelogram of forces. The third force in question is the diagonal of that parallelogram. The resultant required is equal and opposite to that third force. And so, in every other case, to find the resultant force of any number of forces, we must examine what these want of the conditions which make up an equilibrium, and then find a force which would just make up these conditions. A force equal and opposite to this will be the resultant.

139. The Composition and Resolution of Pressures.

The forces of which any other is the resultant are called the *components* of that resultant. Since the resultant force produces the same effect *singly* that all its components do *conjointly*, we shall not at all affect the conditions of the equilibrium of a body acted on by any forces, if we conceive certain of its forces to be *taken away*, and their *resultant put in their place:* if it was in equilibrium *before*, it will be in equilibrium *now*, and under precisely the same circumstances.

This putting of a single resultant in the place of any number of component forces is called *compounding* them. The *process* is that of the COMPOSITION OF FORCES. *Conversely*, we may find a number or group of forces which shall be such as, if we found their resultant, would have for it a particular *given* force. These forces would then, *conjointly*, produce the same effect which that does *singly*. This *group* of forces might then be substituted for

that ONE without affecting the conditions of the equilibrium.

The process of thus substituting an equivalent *set* of forces for a *single* one, is called that of the RESOLUTION OF FORCES; the single force being said to be *resolved* into the others.

140. The Centre of Gravity.

Of all forces, that whose operation we are most conversant with is GRAVITY: it operates, under various modifications, in every thing around us, and in every part of that thing. Every material substance is thus acted upon by as many separate pressures of gravity as we may imagine it divided into parts. We can lay hold of nothing which is not a body acted upon by a system of gravitating forces infinite in number. Of these, this is the characteristic property, that their directions all tend accurately to *one* point—the centre of the earth—which is so distant that, although they thus in reality meet when continually produced, yet are they, as to all *practical* considerations, *parallel*, by reason of the great distance (nearly 4000 miles) of the point in which they meet.

These forces of gravity, thus infinite in number, acting upon the different points of any body which we take up, have always a *resultant;* that is, a single force may always be found acting in a certain direction, which shall *singly* produce the same effect that they do *conjointly*. This force is equal and opposite to the single force which would produce an *equilibrium* in the body on which they act, that is, which would *support* it.

As we turn a body about, the direction *through it* of the forces of gravity which act upon it will be continually changed; at one time they will traverse it *lengthwise*, at another they will traverse it *across*, at another *diagonally:* in short, every new position will cause them to traverse it in a new direction; and by turning it completely round, we shall cause them to traverse it in an infinity of different directions. In each position they will have a *resultant.* Thus they will have an infinity of different resultants; and their resultants will traverse the body, as it is turned round, in an infinity of different directions.

Now there is this remarkable relation (easily determined by *geometry*) between the directions of these different resultants through the body, that *they all pass through the same point in it:* that point is called the CENTRE OF GRAVITY. This is, I say, a *remarkable* relation; it *might* have been *otherwise.* Under other laws of force, and other *conditions* of equilibrium, dependent upon these, the properties of that point would have had no existence.

It is difficult—nay, it is impossible—to conceive the amount of change, the confusion of all the great elements of nature, which this one simple circumstance would have been sufficient to introduce.

Take away this one property of matter, which determines in every mass a single point through which the resultant of the gravitations of its parts, in all its proportions, passes, and the fabric of the universe would *reel* from its very foundations; the order and uniformity of the vast machine would

cease; the cycles which bring back its mighty motions in their appointed seasons would be *broken;* and, to bear its part in the universal wreck, each organised and existing thing on the earth's surface would have the stability of the form under which it exists converted into one of the greatest conceivable instability. An upright position of the human body would be impossible; no vehicle could move without being overthrown; and four-footed animals, when they sought to walk, would but totter on, from one fall to another.

The centre of gravity of a body is then that point through which the resultant of the gravities of its parts passes, *in every position in which we turn the body.* This resultant, producing the same effect as do the gravities of the parts, evidently acts in a *vertical* direction; for the effect of the gravities of the parts is in a vertical direction. The resultant is evidently equal in amount to the weight of the body; for, by the definition of a resultant, it is equal to the single force which would support the body. Thus, then, we shall, in reality, conceive this resultant to act alone, through the centre of gravity, and in its proper vertical direction, if we *conceive* all the gravity or weight to be *extracted,* by some chemical process, from the different parts of the body among which it is diffused, and *collected* and *condensed* into this one *single* point—*its centre of gravity;* and were it possible to make this change, all the conditions of the equilibrium of the body, so far as they are affected by its weight, would remain *unaltered.*

141. To determine the Centre of Gravity of a Body by Experiment.

It is evidently through its centre of gravity that any force which is intended to *support* a body must be made to pass; and, conversely, any *sufficient* single force which is made to act through the centre of gravity vertically, or in a direction opposite to the weight of the body, would support it. To be *sufficient*, this single force must, as has before been shown, equal the *weight* of the body. Thus, if the centre of gravity of a body of any shape, however irregular, were *found;* and if the resistance of the *finest point that can be conceived,*—that of a needle, for instance,—were applied, so that its direction should be vertical and accurately through the centre of gravity of the body,—it would support it.

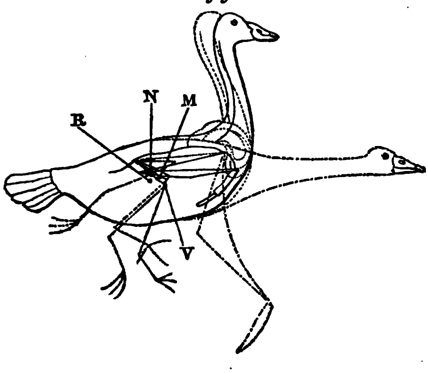
It would perhaps be impossible *practically* thus to cause the *resistance* of a point accurately to pass through a body's centre of gravity. If, however, a body be *suspended* by a single point from a *string*, it will of *itself* fall into such a position, that the direction of the tension of the string on that point shall be through the centre of gravity; and having assumed that position, it will be supported. This, in fact, furnishes us with a very easy practical way of determining the position of a body's centre of gravity. We have only to suspend it by a string from any point in its surface, and, waiting until it rests, to mark, by some means, what would be the direction of the line of the string through the body, if it were produced; then, hanging it from some other

point in the body's surface, to observe in like manner the line of the string's direction through the body, when suspended from that point. Both these lines pass (by what has before been said) through the centre of gravity. But the only point through which they both pass is that in which they intersect. Their intersection is therefore the body's centre of gravity.

142. The Attitudes of Animals.

When a body *alters* its form it *changes*, at the same time, the position of its centre of gravity. The accompanying diagram presents an illustration of this fact in the attitudes of a bird. The line drawn

fig. 33.



from R directs the eye to the position of the centre of gravity when the bird is *standing*, being then immediately above his foot. When he *swims*, the only alteration in his position is the elevation of his legs, accompanied by a corresponding elevation

of his centre of gravity, whose position is now shown by the line from N. When he *walks*, his head is thrown a little forwards, and his legs are alternately raised, but not so much as in swimming: a more forward and a somewhat lower position must therefore be assigned to his centre of gravity, pointed to by the line from M. When he *flies*, his neck is thrown forwards and depressed; his centre of gravity therefore advances and sinks, as shown by the line from V.

No heavy body can evidently be supported upon a surface on which it is placed, unless the vertical resistance of some point with which it is in contact passes through its centre of gravity.

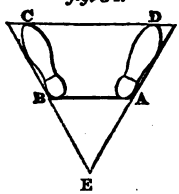
143. The best Position of the Feet in standing.

The human body has a different position of its centre of gravity, corresponding to each different attitude. All are, however, subject to this condition, that the centre of gravity shall remain vertically *over* some point or another in the base of the feet.* This *base* of the feet, or *pedestal* of the body, has for its boundaries, to the right and left, the outer edges of the soles of the feet; and before and behind, lines joining the toes and the heels. In the accompanying diagram it is represented by the trapezium A B C D. The attitudes of the body may evidently be varied, so as not to destroy the equilibrium, with

* For a variety of illustrations of this subject, the reader is referred to the author's treatise on "Mechanics applied to the Arts," p. 33, &c.

the greatest facility and with the fewest precautions, when this base of the feet is the *largest.* Thus the securest position of the feet in standing is that which causes the pedestal to cover the greatest surface, or the figure A B C D, as shown in the diagram to have the greatest area. Supposing the heels to be placed in a given position, and the feet turned round upon them, this greatest area will be found not in a parallel position of the feet, but in an inclined position, like that shown in the figure.

fig. 34.



Thus we see a sufficient reason for the military custom of causing soldiers on drill to stand with their toes turned out.

If the outside points A and B of the heels could be brought accurately to coincide with one another, then, when the heels thus touched, it is found (by a mathematical discussion of the subject) that this greatest base would be obtained when the feet were turned each *half-way* round, or when they made with one another a right angle. As these points can never, however, coincide, but must always be distant by at least double the width of the heel, it is certain that the feet never should be turned apart so far as a right angle.

If the distance A B of the outer edges of the heels exactly equals the length A D of the foot, the inclination of the feet to one another should equal sixty degrees. Or, imagining the lines D A and C B to be produced so as to meet in E, their inclination

should be such as to make the triangle C E D an equilateral triangle.

144. The Shepherds of the Landes.

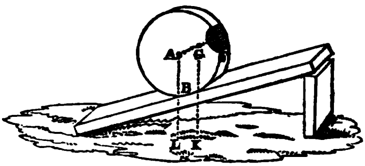
The vast plains of the Landes, in the south-west of France, are covered with a loose sandy soil, and overgrown with thick furze; moreover, during some months of the year, they are in many places *flooded.* This wild region, nevertheless, yields pasture to sheep. To traverse it in search of their flocks, the shepherds have, from time immemorial, adopted the singular custom of mounting themselves on high *stilts.* They are said on these to travel over the loose sand as through the water, with steps of eight or ten feet in length, and with the speed at which a horse trots. This is a remarkable instance of the power which the body possesses of varying its attitude so as to fix the position of its centre of gravity over the base which supports it, even when that base is, as here, of greatly less dimensions than the natural base of the foot, and the body elevated upon it greatly above its natural position.

145. To Cause a Cylinder to roll, by its Weight, a short Distance up an Inclined Plane.

If the vertical from the centre of gravity of a body do not pass through the base on which it rests, but have a direction on either side, then the body will turn over towards that side. The accompanying figure is intended to represent a cylinder placed upon an inclined plane. If this cylinder were not

loaded on one side, its centre of gravity would be in its axis A; and the vertical A L, from its centre of

fig. 35.



gravity, would evidently fall below the point B, where it rests upon the plane; so that, when left to itself, it would roll *downwards.* But by loading it near its surface at F (by pouring lead in a groove parallel to its axis), the position of its centre of gravity G may be *moved,* so that the vertical G K from it shall not be below, but *above* the point of support B. It will then roll for a short time *up* the inclined plane instead of down it, until, by the descent of F, the line G K is made to pass through the point of contact B of the cylinder with the plane. It will then rest.

* 146. Wheeler's Clock.

An ingenious person of the name of Wheeler, some years ago, conceived the idea of constructing a clock to be contained in a cylinder; the principle of whose motion should be, the tendency of the cylinder, when placed upon an inclined plane, to roll down it.

Let the cylinder represented in the last article be imagined to be hollow; and the weight F moveable in it round the axis A, by means of an arm A G F, to the extremity of which it is fixed:

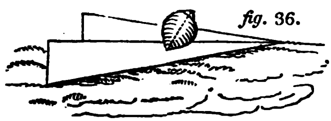
imagine that with this arm is connected a train of wheels similar to those of a watch, terminating in a scapement and balance-wheel, and giving motion to a hand moveable on the end of the axis, and showing hours on the extremity of the cylinder, which has its circumference divided like the face of a clock. The arm A G F being *turned*, motion is given to all this train of wheels, which motion is checked and regulated by the balance-wheel. But how is the arm to be turned? Thus: conceive the cylinder to be placed upon an inclined plane; it will seek for itself the position in which the vertical G K, from its centre of gravity, passes through B; in this position F will not coincide with B, being balanced about that point by the weight of the cylinder itself, and its wheels, which are so contrived that their common centre of gravity shall be in the axis A: the position of the equilibrium of the cylinder is then in an *inclined* position of the arm A F. Now, the axis being *supported*, and the arm A F *inclined*, it is evident that the weight F at its extremity tends to *turn* the arm about A; and being unopposed, except by the friction of the train of wheels connected with it, may readily have its size so adjusted that it *shall*, under these circumstances, *turn the arm*, and give motion to the wheels; but as the arm thus turns, the weight F *descends*, and the vertical G K, which before passed *through* the point of support B, now falls *below* it: the cylinder will now, therefore, roll *down* the plane. Now, as it rolls down the plane, it elevates the weight F again, so as to place the arm AF in the same inclined position as before, and give it the

same leverage precisely, to turn the wheels: thus the descent is continued, and the same power is continually supplied to give motion to the works of the clock, whilst the scapement and balance-wheel give a yet further uniformity to the motion which, the proper adjustments being made, may be regulated to keep any required time. The motion of the clock will only be stopped when it has rolled completely down the plane: that it may be begun again, the cylinder must be placed again at the top of the plane.

* 147. To cause a Body, by its own Gravity, to roll continually upwards.

Let a double cone, such as that shown in the figure, be made of wood; and let there be formed

fig. 36.



two inclined planes of boards of wood *, which, meeting at their bases, afterwards diverge from one another at an angle, as shown in the figure. If the double cone be placed between these planes, so as to rest equally upon each of them at the bottom, it will immediately put itself in motion and roll *up* them.

This apparent paradox is easily explained. The centre of gravity of the double cone is in the middle of the line joining its two extremities. Now, when it is placed between the two inclined planes,

* Pieces of string will answer the purpose.

the points on which it rests are, of necessity, nearer to the lowest point of the planes than this line is: the vertical from the centre of gravity is, therefore, on that side of the points of support which is towards the highest points of the inclined planes; it is in that direction, therefore, that the body has a tendency to roll. It does not, however, in reality ascend, although it appears to do so: the points on which it is supported, continually approach its extremities; so that, although by reason of the inclination of the planes, the points of support *ascend*, yet, for the above-mentioned reason, the thicker part of the mass between them, *descends;* and it is necessary to the success of the experiment, that, for this last reason, the centre of gravity of the mass should descend *more* than for the other it ascends. That this may be the case, the inclination of each plane must be such, that a distance equal to the length of the double cone being measured along it, its corresponding height shall be somewhat less than half the diameter of the double cone in the middle.

* 148. Stable and unstable Equilibrium.

When a body, being slightly moved out of any position in which it rests upon another body, tends to *return* to it; and being left to itself, will roll back of its own accord into it—that position is said to be one of STABLE EQUILIBRIUM: when the body will *not* thus return to its previous position, that position is said to be one of UNSTABLE EQUILIBRIUM.

Since the whole of the weight of a body may be conceived to be collected in its centre of gravity,

without affecting the conditions of its equilibrium, it is evident that if it be supported by the resistance of a single point, that single point must be either immediately above, or immediately beneath, or actually *in*, its centre of gravity; and if it be supported, not upon a point, but upon an extended surface or base, or beneath such a surface, then must the centre of gravity be either directly *above* some point in that base or surface, or directly *beneath* some such a point. If the position of a body, which thus rests, be so *changed*, that its point, or surface of support, shall no longer lie vertically above, or vertically beneath, its centre of gravity—then, no vertical supporting force acting *upwards*, through its centre of gravity, and the whole weight or gravity acting *downwards* through it (or, rather, acting as though it so acted), it is manifest that the centre of gravity will have a tendency to *descend*, and that, if the body be left to itself, its centre of gravity will descend. It is possible that, moving a body from its position of equilibrium, we may, at the same time, so alter the position of its point of support, that it shall still remain directly beneath, or above, its centre of gravity. Thus, if a *sphere* rest upon a horizontal plane, and we roll it out of the position in which it rests into some other, we shall, in the act of rolling it, so alter the position of its point of support, that it shall still be beneath its centre of gravity; for the centre of *gravity* is in the *centre* of the sphere; and the perpendicular to a plane, on which a sphere rests, drawn from the point where it rests upon it, necessarily goes through its centre. Thus, into whatever position we roll a

sphere on a horizontal plane, the vertical, from the point on which it rests, passes through its centre of gravity; and the centre of gravity is vertically above the point of support. When a body, being moved more or less from its position of equilibrium, will rest in any of the positions in which it is placed, and is indifferent to any particular position, its equilibrium is said to be one of INDIFFERENCE.

This state of *indifferent* equilibrium is, however, one of exceedingly rare occurrence, even in respect to *slight* deflections of a body from its position of rest; and no other body besides a sphere, or a body resting on a spherical surface, and having its centre of gravity at the centre of that spherical surface, can thus be indifferent to *all* the positions in which it may be placed. Every solid body, with the exception above stated, tends to *return* to the position of equilibrium out of which it has been moved, or to *recede* from it; and if left to itself, it *will* spontaneously either return to that position, or roll farther from it.

The centre of gravity being moved from under, or from over, its point of support, and being unsupported, of necessity *descends*. The question then, whether a body's position of equilibrium be stable or unstable depends upon this other; will the *descent of its centre of gravity*, when the body is thus left to itself, bring it into its former position, or deflect it farther from it? In the first case the equilibrium is STABLE, and in the other, UNSTABLE.

Now, if the centre of gravity of the body be *elevated* in the act of deflecting it from its position of equilibrium, it is evident that it must be de-

pressed to be returned to it; and conversely, that depressing itself, it will return to it. In this case, then, the position out of which it was disturbed was *stable*. But if, in the act of deflecting it from its position of equilibrium, the centre of gravity of the body be *depressed*, then, to return of its own accord, the centre of gravity (where all the weight acts downwards, and which is unsupported) must *elevate* itself. This is impossible. The centre of gravity *descends*; and the body continues, therefore, in this case, to deflect more and more from its former position of equilibrium, which was, therefore, *unstable*. Thus, then, when the position in which a body rests is such that, being deflected from it, its centre of gravity *ascends*, that position is one of STABLE equilibrium; when the body being thus deflected, its centre of gravity *descends*, the position of equilibrium is UNSTABLE.

* 149. THAT POSITION OF A BODY RESTING UPON ANOTHER IN WHICH ITS CENTRE OF GRAVITY IS THE LOWEST POSSIBLE, IS A POSITION OF STABLE EQUILIBRIUM; THAT IN WHICH IT IS THE HIGHEST POSSIBLE, ONE OF UNSTABLE EQUILIBRIUM.

If the centre of gravity of the body *descends* when you deflect it from its position of rest, in any direction, it is evident that the height of the centre of gravity, in that position, is greater than in any of the positions into which you deflect it; its position of UNSTABLE EQUILIBRIUM corresponds, then, by what is stated in the last article, to that position in which, being placed, *its centre of gravity is highest*

in respect to the adjacent positions. If, on the contrary, the centre of gravity *rises* when you deflect it from its position of rest in any direction, then is it in that position *lower* than in any of the others. A body's position of STABLE EQUILIBRIUM corresponds, then, to the *lowest position* of its centre of gravity in respect to the adjacent positions of the body.

*150. EVERY BODY, EXCEPT A SPHERE, HAS AT LEAST ONE POSITION OF STABLE, AND ONE OF UNSTABLE, EQUILIBRIUM.

For if a body be made to turn round on the surface on which it rests, its centre of gravity will not *continually* ascend or *continually* descend; there must be a certain position of the body, after it has passed which, its centre of gravity, *from ascending*, begins *to descend;* and another in which, from *descending*, it begins to *ascend*. The first is a position of *unstable*, and the last of *stable*, equilibrium.

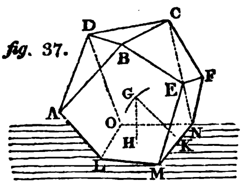
In a *sphere*, the centre of gravity (which is the centre of the sphere) continues always at the *same height* as you roll it. There is, therefore, no position either of *stable* or *unstable* equilibrium. Every position in a sphere is one of *indifferent* equilibrium.

A body's position of equilibrium may be *stable* in respect to a deflexion in one direction, and unstable in respect to a deflexion in another. It is then said to be a position of MIXED equilibrium.

151. A Body having plane Faces has all its Positions of Equilibrium, on those Faces, Positions of stable Equilibrium; and all its Positions of Equilibrium, on their Edges, Positions of mixed, and on their Angles, of unstable Equilibrium.

For it is evident, that if a body rest upon a plane face L M N O, and be inclined from its position of equilibrium, it must turn upon one of the edges of that face as M N, so that its centre of gravity G must *ascend** in a circle round some point K in that edge.

fig. 37.



The centre of gravity thus *ascending*, when the body is deflected round either edge of the plane on which it rests, it follows that, when resting on that plane, its centre of gravity is *lowest*, in respect to

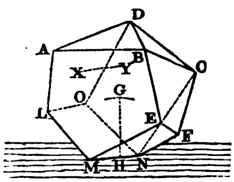
* It will be observed that the position upon the face L M N O being supposed to be one of equilibrium, the centre of gravity G is vertically *over* that face, so that the line G K is inclined downwards towards the face, and must be *elevated* in turning the body upon the edge M N. There is an exception in the case in which the point G is vertically *over the edge* M N on which the body is turned; the position is in that case *unstable*, in respect to a deflexion of the body round that edge, and stable in respect to a deflexion round every other. A position of equilibrium of this kind is said to be one of MIXED equilibrium, being stable one way and unstable the other.

M

the adjacent positions, and, therefore, that its position upon it is one of STABLE equilibrium.

If, now, the body be turned upon its edge M N, and be placed in such a position that the vertical G H, through the centre of gravity, shall pass accurately through some point H in that edge—this, too, will be a position of equilibrium, for the centre of gravity will be supported; but it will be a position of MIXED equilibrium, that is, a position from which the body being deflected in certain directions would tend to return, and being deflected in others would not, so as to be in respect to the first directions of deflexion *stable*, and in respect to the others *unstable*. For the centre of gravity G being vertically over the edge M N, it is clear that when the body is turned round that edge either way, the centre of gravity will be *depressed*; so that over the edge it is in its *highest* position, and the equilibrium is, in respect to deflexions round the edge, *unstable*. But if, instead of being turned round the edge, the body be lifted so as to turn about either of its extremities M or N, then its centre of gravity will be raised, so that, in respect to those deflexions, it is in its *lowest* position, and the equilibrium is *stable*. Thus the equilibrium about either edge is, in certain directions, *stable*, and in others, *unstable*; or it is MIXED.

fig. 38.



By reasoning precisely similar to the above, it is evident that, if the body can be made to rest on

either of its angles A, B, C, D, &c., so that the centre of gravity shall be vertically over that angle, the position will be one of *unstable* equilibrium.

152. A Body's Position is always one of stable Equilibrium, when its Centre of Gravity lies beneath the Point on which it is supported.

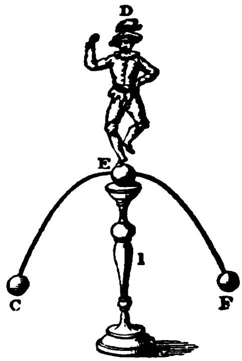
Thus, if the body represented in the last figure, instead of resting on a plane, had been suspended from a fixed point by either of its angles, or if it had been hung upon axis X Y passing through it *above* its centre of gravity, then it is clear that, deflecting its position, from that in which it rests with its centre of gravity vertically beneath the point of support, the centre of gravity will be *raised:* the position in which it rested was, therefore, a *stable* position.

153. To construct a Figure which, being placed upon a curved Surface, and inclined in any Position, shall, when left to itself, return into its former Position.

The accompanying cut represents a figure of any light substance, to which are attached, so as to hang beneath it, two heavy balls. The feet of the figure are fixed upon a piece of wood, the lower surface of which is curved, and this curved surface rests loosely upon a small table which is supported by a stand. If this figure be ever so far inclined in any

direction, it will immediately recover its position when left to itself, and with the *greater force* as it is *more* inclined. The explanation is as follows. The effect of the weight of the balls, which weight is much greater than that of the figure, is to bring the centre of gravity of the whole greatly beneath the point on which it rests. This being the case, it is evident that, in whatever direction the figure is made to incline in respect to its point of support the centre of gravity of the whole will be made to *rise*. In the position in which it rests, the centre of gravity is therefore in its lowest point, and the equilibrium is *stable*.

fig. 39.



154. To cause a Body to support itself steadily, on an exceedingly small Point.

A body may be made to support itself steadily on an exceedingly *small point*, if it be so loaded that its centre of gravity may be *beneath* this point. This is strikingly illustrated in the following very simple experiment.

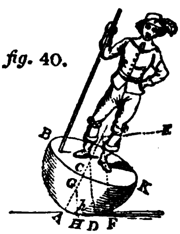
On opposite sides of a cork towards the top, let two forks be stuck, inclining downwards, and let the edge of the bottom of the cork be then made just to rest on the edge of a wine-glass, which must be held, if necessary, to prevent it from

falling. The cork may be brought, by pushing it gently sidewise, to rest upon so small a portion of the glass that it shall seem scarcely to *touch* it; and yet the whole will support itself steadily upon it: if slightly moved, it will return to its position, and the glass may be raised without causing it to fall. By the weight of the handles of the forks, the centre of gravity is brought far *below the point of support;* hence the steadiness of the equilibrium, and the facility with which it may be brought about, on so small a point.

155. A Body having a Portion of its Surface spherical, and resting by that Portion of its Surface on a horizontal Plane, has its Equilibrium stable or unstable, according as its Centre of Gravity is beneath or above the Centre of the Sphere of which that spherical Surface forms Part.

The figure in the woodcut is supported on a solid base whose curved surface B A K is part of the surface of a sphere having its centre in C. The common centre of gravity of the figure, and the mass which supports it, is G. On whatever point D the spherical surface B A K rests on the horizontal plane, the vertical through its point of support passes through C (by a geometrical property of the sphere); when it rests then on A, A C is the vertical, and this vertical passes then through

fig. 40.



its centre of gravity G. When it rests on A, therefore, the figure is in equilibrium. Now, when G is beneath C, this position is one of *stable* equilibrium; when G is above C, it is one of *unstable* equilibrium. For, let the figure be placed in the inclined position shown in the cut, so as to rest on D, and draw through G the vertical G H to the horizontal plane, then is G H the height of the centre of gravity in the present inclined position of the figure. But, in the upright position of the figure, when it rested on A, the height of its centre of gravity was A G.

Now G H is *greater* than A G if G be, as in the figure, beneath C*; but if G were above C, as, for instance, at E, then G H would be *less* than G A. In the first case, the centre of gravity is raised, then, by deflecting the body from its position of equilibrium; in the second case, it is depressed. In the one case, then, the equilibrium is *stable;* and in the other, *unstable*.

The same conclusion may yet more easily be drawn from the consideration, that when the centre of gravity is at G, the whole weight acting on that side of the point of support D, which is *towards* the former position of the body, tends to bring it back to it: and that when it is at E, this weight, acting on that side of D which is *from* its former position, tends to deflect it yet farther from that position.

A very ingenious toy is constructed on this principle. A hemisphere (or half-sphere) is rounded

* For by Euclid, Proposition 7, Book iii., G h, which is only part of G H, is greater than G A; much more, then, is G H greater than it.

of some very heavy substance, lead, for instance; (the half of a leaden bullet will answer the purpose). On this is fixed a figure cut out of some very light substance, such as the pith of the elder tree. This figure, if placed on the table, and inclined ever so much in any direction, will always regain its upright position. The explanation is contained in the principle stated above: the centre of gravity of the whole figure is beneath the centre of the spherical base; for the centre of gravity of the hemispherical ball is evidently within its mass, and therefore below the centre of the sphere of which it would form a part; and the weight of the figure placed upon it is so small, that it is not sufficient to raise the centre of gravity of the whole above that point, as it would do if it were heavy.

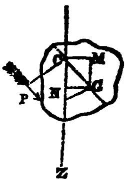
In this manner were constructed the French toys called Prussians. The figures represented soldiers: they were formed into battalions, and being made to fall down by drawing a rod over them, they immediately started up again as soon as it was removed.

Screens of the same form have since been invented, which always rise up of themselves when they happen to be pressed down.

*156. The Stability of a Body which is suspended from a Point, or a fixed Axis, is greater as the Centre of Gravity of the Body is lower beneath that Point or that Axis.

Suppose the body represented in the accompanying figure to be supported upon a point at C, or

fig. 41.



upon a fixed axis passing through it at that point. Let C Z be the vertical through C, and let the body be deflected from its ordinary position of equilibrium by the action of the force P, so that its centre of gravity G may occupy the position shown in the figure, instead of resting suspended beneath C in the vertical C Z.

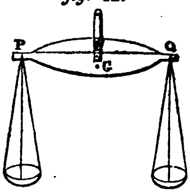
The body being thus held in equilibrium by the action of the weight in G by the force P, and by the resistance of the axis C, it follows, by the principle of the equality of moments, that, if we take C for the point from which we measure the moments of these forces, that of the last-mentioned force vanishing, those of the two others will be equal; that is, the product of P by C P will be equal to that, of the weight of the body, by C M; C P and C M being respectively perpendiculars upon these forces from C. Now, supposing P to be applied always at the same perpendicular distance from C, or C P always to be the same, it follows, from this equality, that P must be greater according as the product of the weight of the mass by C M is greater; or that, for bodies of the same weight, it must be greater as C M is greater. Now C M is equal to G N, and G N would evidently be greater if G were lower upon the line C G; or if the centre of gravity were lower beneath the point of suspension in that position of the body in which it rests of itself. Thus, then, the force P necessary to deflect the body from the position in which it rests of its own accord, into any inclination to that position, is greater as the centre of

gravity is lower. The body is therefore more stable as the centre of gravity is lower.

157. The Balance.

It is for this reason, that in the construction of delicate balances, whose degree of stability is required to be the least possible, that they may turn with the least possible difference of the weights in the scale-pans, precautions are taken by means of which the centre of gravity, G, of the whole moveable portion of the balance, including the beam, the scale-pans, and the weights they contain, shall lie, in every case, at an exceedingly small distance beneath the point of suspension, or fulcrum of the balance F. By making the scale-pans equal in weight, suspending them at equal distances, F P and F Q, from the fulcrum and from points lying at the extremities of a line, P Q, passing *through* the fulcrum F, their centre of gravity, and that of the weights they contain when equal, is brought, so that it would exactly coincide with the fulcrum if the beam did not bend; and it would then only be the centre of gravity of the beam itself, which would lie beneath the fulcrum, and produce the stability of the balance, bringing it back from its deflections so as to vibrate it. The beam, however, in reality always bends, whatever may be the care taken to give it rigidity. And thus the centre of gravity of the weights in the scale-pans, as well as that of the beam itself, is brought beneath the ful-

fig. 42.



crum; and this depression is greater as the objects weighed are heavier. The best balances are those made by Mr. Robinson; every precaution which science may suggest to ensure the accuracy of these balances, is taken in their construction and their adjustment.*

158. To make a Balance which shall appear true when empty, but yet weigh falsely.

Let a balance be constructed with unequal arms, and let scale-pans be suspended from them of unequal weights, so adjusted that they shall just equipoise one another, and make the beam to rest in a horizontal position. This balance will appear a just one when the scale-pans are empty, but it will not weigh truly; for any weights put in its scale-pans will be suspended at different distances from the fulcrum. They cannot, therefore, balance one another when they are equal—that suspended from the shorter arm must be greater than the other. The weights used being then put into this scale, and the commodities to be weighed into the other, the balance, appearing to be true, will weigh short weight.

The deception is easily detected by changing the scales in which weights and the things weighed are placed. If the balance be false, the equilibrium will then no longer exist.

* For a more complete discussion of the theory of the balance, the reader is referred to the author's treatise on "Mechanics applied to the Arts," p. 68.

159. To weigh truly with a false Balance.

Let the article to be weighed be placed in either scale-pan, and let the weight necessary to balance it in the other be found; place it then in the other scale, and let the weight necessary to balance it be found as before; take then the product of these two false weights: the square root of this product will be the true weight. Thus, if in one scale the article weigh 14 ounces, and in the other 16, taking the product of them we have is 224; the square root of this product is $14\frac{2}{3}$, which is the true weight in ounces.

160. Borda's Method of weighing truly with a false Balance.

A much simpler method than the above, of weighing truly with a false balance, has been conceived by Borda, and may be considered as in all cases the most certain way of ascertaining the weight of any substance. Let the thing to be weighed be placed in either scale of the balance, and any heavy but minute substance—leaden filings, for instance—accumulated in the other, until they precisely balance it; let the article to be weighed be now removed, and, in the scale which contained it, let weights be introduced until an equilibrium is again accurately produced. These weights will give the true weight of the body, independent of any error in the balance; for their weight produces exactly the same effect that its weight did—balancing identically the same load in the opposite scale.

*161. Under what Circumstances a Body, supported upon a horizontal Plane, is more or less stable.

In order that a body which rests upon another, and is therefore in a stable position of equilibrium, may be overthrown, it must be made to pass from that position of *stable* into one of *unstable* equilibrium. Thus, for, instance, to be overthrown, the body A B C D, from the *stable* position shown in the first of the accompanying figures, must be made to revolve into, and slightly beyond, the *unstable* position shown in the second.

fig. 43.

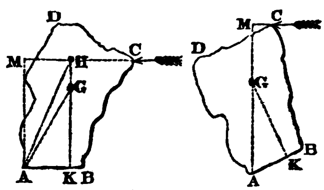


fig. 44.

The body is more or less stable in its position represented in the first figure, according as the *revolution* it must receive to bring it into the position represented in the second figure is greater or less, and according as the *force* required to produce this revolution is greater or less. Now, by this revolution, the line G A, in the first figure (G representing the centre of gravity) is made to pass, from an inclined to a vertical position, in the second figure; so that G may be vertically above the angular point A; on which the body turns. But that G A (*fig.* 43.) may revolve into a vertical position, it must revolve

through the angle G A M, which angle is equal to the angle A G K. The revolution, then, which the body must receive before it will fall over of itself, is greater or less, according as the angle A G K is greater or less. Now the angle A G K is greater according as G is *lower*, and according as A K is *greater*. For it is evident, that if G had been higher than it is—as, for instance, at H—then the angle G A M, or its equal A G K, would have been less than it is: moreover, if A K had been less than it is, then, also, it is evident that A G K would have been less.

Thus, then, we see one reason why it is that, as a body's centre of gravity is lower, and its base wider, it is more difficult to overthrow it—the body requiring, according as these conditions obtain, to be turned farther before it will pass into a position (one of unstable equilibrium) from which it will fall over of itself.

The amount of the revolution which must thus be given to a body, by the application of a sufficient force, before it can be overthrown, is not, however, the only element on which the degree of its stability depends. Another is the amount of the force necessary to produce this revolution.

The amount of the force depends upon the weight of the body, and the distance of the vertical G K through its centre of gravity from the point A, round which it is to be made to turn.

To make this appear, let us suppose that the force intended to turn it is applied at C in a horizontal direction; in which direction the line C M is drawn, meeting the vertical line A M in M. Suppose this

force C to be just upon the point of causing the body to turn on A, and very slightly to have raised it, so that the forces which act upon it are exactly in equilibrium; imagine, moreover, all the weight of the body to be collected in G, an allowable supposition: the weight acting in G, the force acting at C, and the resistance of the surface on which the body rests acting at A, then, are the forces in equilibrium. There must then obtain between them the relation of *the equality of moments.* (See art. 129.) If, then, from the point A perpendiculars be drawn upon the directions of the force at C, and the weight through G, then the products of the lengths of these perpendiculars, by the numbers of cwts., or pounds, or ounces, in their corresponding forces, must be equal.* Now these perpendiculars are evidently A M and A K. When the force C is just then upon the point of turning the body, it is a force equivalent to such a number of pounds, that this number of pounds being multiplied by the number of inches in A M, the product shall equal the number of pounds' weight in the body multiplied by the number of inches in A K. And the first product must be greater according as the last is greater; so that supposing the force C to be applied always at the same height, that force itself must be greater according as the last of the above mentioned pro-

* The perpendicular from A upon the resistance acting through that point, is of course nothing or of no length; the moment of this *resistance* is therefore nothing: thus this third force vanishes from the relation of the equality of moments, when we measure them from A, for which reason it is that A above all other points is selected to measure them from.

ducts is greater, and this last product is greater according as A K is greater. Thus, then, the force necessary to turn the body is greater according as the distance of the vertical through its centre of gravity from the point on which it is to turn is greater or less. The amount of this force has, however, nothing to do with the height of the centre of gravity; thus it is the same in the figure, however high G may be, provided it remains in the line K H.

So far, then, as the stability of the body is dependant upon the force necessary first to move it, it is independent of the height of the centre of gravity; so far as it is dependant upon the amount of the deflexion which will be sufficient to overthrow it, it is dependant upon that height.

It is because a *slight deflexion* will overthrow a body when loaded high, that it is then of little stability, not because a less force will then move it. As great a force is necessary at first to move a high body as a low one, but a less deflexion will overthrow it. Thus, when a body is of *necessity* subjected to certain deflexions, it should never be loaded high; a coach, for instance, which is of necessity deflected by the irregularity of the road, if it be loaded high, may be brought by some of these deflexions into, and beyond, its position of unstable equilibrium, and overthrown; whereas a tower, or a spire as high as that of Salisbury cathedral, stands firmly on its base.

If the vertical through the centre of gravity of a body do not pass through the *middle* of its base, the body is more stable to resist a force in one direction

than another. Thus in the figure the point K not being in the middle of the base A, it is evident from what has been said, that the body is more stable in respect to a force tending to turn it about A, than to one tending to turn it about B. There are structures whose centres of gravity are over points thus greatly nearer to one side of the base than the other, so as in one direction to possess but a slight degree of stability, which, by reason of their great weight, stand nevertheless firmly. Such are the hanging towers of Pisa and Bologna.

WALKING.

In the act of walking, the centre of gravity is raised, alternately, over the legs. The motion somewhat resembling that of a pair of open compasses, made to rest alternately on their points; the centre of gravity is over the fork of the legs, and may be imagined to be over the angle of the compasses. If, as the compasses are thus made to travel forwards, resting on their alternate points, these points are not placed in the same straight line, but alternately to the right and left of it, then the centre of gravity will describe a series of arcs to the right and left, and it will not be carried so far *forwards*, by a certain number of steps, as though these were made in the same right line; this corresponds to that ungainly motion in walking, which is called *waddling*. It is remarkable how nearly the footsteps of a person who walks well, are in the same straight line, as may be seen especially, if we trace them in the snow; this is, moreover, remarkably the case with

animals, horses for instance, and especially it is the case with birds, whose centres of gravity being for the most part very high, in comparison with the dimensions of their feet, they are taught instinctively to avoid those deflexions of their bodies to the right and left, by which they might be overthrown.

Taking the width of a man's foot at about three inches, and giving him an average stature, it may be calculated that a deflexion of his body of less than two degrees would, when he rests on either foot, be sufficient to overthrow him. How justly regulated then must be the effort which he makes at every step, to transfer his centre of gravity from above one of his feet to above the other, that his position may be kept within this narrow limit! Put upon his shoulders a burden, and you will raise his centre of gravity, and greatly increase the difficulty he will experience in balancing himself; yet how firmly and securely does he tread! A man carrying a burden as heavy as himself, and inclining his position as he steps on each foot, only half a degree to the right or left of the position in which he would rest on that foot, would be overthrown.

At each step the centre of gravity is *raised* and made to revolve over the foot. It is this raising of the centre of gravity, in which the whole weight of the body may be supposed to be collected, which constitutes the great *effort* of walking. It has been calculated that at every step the centre of gravity is raised a perpendicular height equal to about one eleventh the length of the step; so that a person who walks eleven miles, raises his centre of gravity and therefore the whole weight of his body, a succession of

lifts, equivalent to the direct raising of it, one mile. If six men, weighing each 182 lbs., and a boy of half that weight, walk at the rate of eleven miles in three hours, the aggregate of their labour, while thus walking, will be about equal to one horse's power; as the *amount* of a horse's power is usually estimated.

162. The Resistance of a Surface.

Resistance is a force which is lodged, like gravity, *universally* in matter. When it presents itself under the form of a *pressure*, or as one of a system of forces producing equilibrium, its characteristic property is this, that, at each point of its application, it is supplied precisely in that quantity and degree in which it is necessary, that motion may not be produced there, and in neither more nor less than that degree. It is by reason of this property of resistance, adapting its *energies*, as it were to the demand made upon them, that an infinite variety can (within certain limits) be introduced among the remainder of a system of pressures, of which a resistance is one, without, nevertheless, disturbing their equilibrium.

This property of supplying a force precisely equal of the amount required to counteract the tendency to motion is, however, in every case of resistance known to us, confined within limits, more or less extensive indeed, but yet *definite* and *fixed*.

Airs and liquids supply no resistance of *this kind* at all, or none that is appreciable, the bodies we call soft, but little; and all solid bodies, are subject to this law, that they *yield*, by reason of their elasticity,

more or less, but for the most part inappreciably, to *every* pressure, and that there are certain limits beyond which they resist no longer (or in other words do not supply that resistance which is necessary to prevent motion); motion then takes place, the structure of their parts is destroyed, and they *crush*, or break, or fly in pieces — these being all but so many terms used to express the insufficiency of their resisting power to supply the pressure necessary to equilibrium.

These remarks apply only to the *magnitude* or amount of the force by which the surfaces of solid bodies resist—its *direction* is another question.

163. The Direction of the Resistance of a Surface.

The *direction* in which a solid body resists was, when the theory of statics was first discussed, taken, hypothetically, to be a direction perpendicular to the surface of the resisting body.

It is difficult to assign any better reason for this hypothesis, than that desire to simplify the conditions of a question which is natural and, perhaps, necessary to the *first* discussion of it. The same reason does not, however, sufficiently account for the *preservation* of it. An abundance of examples will suggest themselves to every one, showing that the hypothesis is in no case true. Did the surface of the earth, for instance, on which we tread, resist only in a *perpendicular* direction, although we might *stand*, the first *step* we made would infallibly bring us to the ground; and, as to stretching our legs as

we do when we walk rapidly, inclining them at a considerable angle, and trusting to the *resistance* of the ground to counteract their oblique pressure upon it, it would be madness.

Resisting only in a *perpendicular* direction, the surface on which we trod could not possibly supply any opposite force to the *oblique* pressure which each leg in its turn would exert upon it—and falling, *where* we fell we must *lie*, unless some immoveable obstacle were at hand, by clinging to which we might regain an upright position; for to rise by the usual method, supported by our hands and knees, would be impracticable—every effort which we so made would be accompanied by an *oblique* thrust or pressure, and no such *oblique* pressure would be counteracted: at every effort our hands and knees would slip from under us; and it would be an equally useless task to attempt to rise ourselves, and to trust to the assistance of others. In short, under a state of things like this, to live the life of locomotive creatures on the solid surface of the earth would scarcely be possible; and had it existed from the beginning, we cannot but believe that all animated being, other than that which peopled the air or the seas, would have been rooted into the ground.

Whilst it is thus certain that the resistances of the surfaces of solid bodies are not confined to their perpendicular directions, it is by no means certain what that particular law is, by which, in each different body, the direction of its resistance is really governed. Nevertheless, the following is a very near approximation to that law.

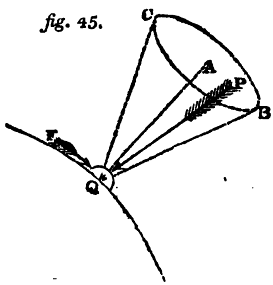
164. The Cone of Resistance.

The resistance of the surface of a solid body is *equally* exerted in *every* direction which does not make with the perpendicular an angle greater than a *certain angle*, called the *limiting angle of resistance*, which is always the same for the same surface, but different for different surfaces.

Or, perhaps, this law will be better understood under this other form of it.

If a cone be imagined to be taken, as in the accompanying diagram, having its axis A Q perpendicular to any point Q of the surface of a solid body, and having its angle, at the vertex, dependent, by a very simple law, upon its *friction* with the surface

fig. 45.



of another body pressed upon it at Q; then the pressure will be resisted, provided its direction be any where *within* the surface of the cone, as, for instance, in the direction of the arrow P Q; and it will *not* be resisted if its direction be any where *without* it.

The remarkable feature of this law is this, that it is true whatever may be the amount of the force P Q, within the limits of abrasion. Whether this force be great or small, it will be resisted if its direction be *within* this cone; and if it be *without* the cone, it will *not* be resisted.

The angle of the cone of resistance is dependant, by a very simple law*, upon this property of the friction of two surfaces in contact, that the amount of the friction is always proportional to the perpendicular pressure which produces it. We shall hereafter speak fully of this property. It will be here sufficient to state, that the angle of the cone being dependant upon the friction, there must always be the same cone of resistance for the same surfaces of contact, and different cones of resistance for different surfaces, inasmuch as the friction is the same for the same surface, and different for different surfaces.

165. Illustration of the Cone of Resistance in the Striking of a Hammer.

If a hammer be struck upon a polished mass of metal—an ANVIL for instance,—it will be found that there are an infinity of directions, besides the perpendicular one, in which the blow being given, it will be resisted; and this, however strong the blow may be, even although it were given with a sledge

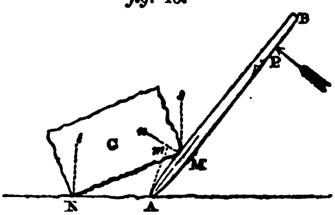
* The vertical angle of the cone is twice that whose tangent is the constant ratio of the friction to the pressure. See "Mechanics applied to Arts," p. 43.; also p. 47., where is a table of the angles of the cones of resistance for different surfaces.

hammer. If now, the direction of the stroke be continually, and very gradually, *inclined* farther and farther from the perpendicular, it will be found that there is a *certain* inclination up to which the resistance will continue *perfect*, and that *after* this inclination is passed, every blow will *slip*. The surface of the cone is, in point of fact, *beyond that inclination, passed*, and the principle above stated, is fully illustrated.

166. Illustration of the Law of the Resistance of a Surface, in the Use of the Crowbar.

There is scarcely any use to which the mechanical powers are put, in which the principle of resistance stated in the last article, does not find its application. In the crowbar, for instance, represented in the annexed engraving by A B, and used to lift up the

fig. 46.



mass G. Motion cannot take place, or the mass be lifted, except by its surface sliding on the surface of the lever, at the point where it rests on it, and in order that the two surfaces may thus slip upon one another, the direction M *n*, in which they are pressed upon one another, must be *without* the

surface of the *cone of resistance.* When by the action of the force P, which moves the lever and the resistance of the point A, on which it rests, the direction of the pressure M n, is made to assume a direction just *without* the surface of this cone; the surfaces begin to slip, and the mass to be elevated. Knowing the friction of the surfaces, we know what is the cone of resistance at M. Thus we know what must be the *direction* of M n, when motion is about to take place, and knowing this direction, we know the perpendicular A m. Knowing then A m and A P, we can compare the pressure of the lever in the direction M n with the force P, since by the principle of the equality of moments, the moments of these two forces about A, are equal. Proceeding then to the point N, and observing that, by the same principle, the moments about that point, of the force in M n, and of the weight of the mass supposed to be collected in G, are equal, we can determine a relation between this weight and the force in M n. Knowing then a relation between the weight of the mass and the pressure of the lever upon it in M n, and knowing also a relation between this last pressure and the force P, we can determine a relation between P and the weight of the mass, when motion is about to take place; that is, we can determine what force P, is necessary to raise the weight, when in any position. This problem is, however, a complicated one, and requires, to its complete solution, the application of considerable mathematical knowledge. It is merely described here, that the *nature* of such investigations may be presented to the mind of the reader. There are

other considerations, which yet more complicate the problem. It may be, that before P attains that amount which is thus shown to be necessary to lift the mass, it may produce a pressure upon the extremity A of the crowbar, whose direction is without the *cone of resistance* at that point, so that it may cause it to slip; or it may, before it reaches this limit, produce a pressure on N, in a direction without the cone of resistance at that point, so as to cause that point to slip. In either case, the elevation of the mass will be arrested.

167. THE MECHANICAL ADVANTAGE OF ANY MACHINE IS SUPPLIED BY THE RESISTANCES OF ITS PARTS.

Of all the different forms of force, that under which it most directly connects itself with practical mechanics, and with the operation of machinery—that without which no machine can act, and which every machine is indeed but a contrivance for applying, is RESISTANCE.

The *resistances* of the axles of its wheels, the fulcra of its levers, and of the various surfaces by which its parts move in contact with one another, are in point of fact but so many pressures, which it *borrows*, and which are made to co-operate in the effect it produces.

That which is known to us in a machine, by the name of a mechanical *advantage*, is no other than a contrivance by which we are enabled to *avail* ourselves of the *resistance* of some surface or surfaces entering into the construction of the machine

we use. Thus in the lever, the *resistance* of the fulcrum *aids* in supporting the weight; and by just so much as it resists, diminishes the pressure which must be made to act upon it, before the weight can be put in motion, or the work done. So too of the wheel and axle, it is but a contrivance by which the *resistances* of the points on which the axle turns, are made to contribute to the force which must be used before the weight can be raised, and which must be kept up during the whole time that it is in the act of being raised. And the inclined plane is but an *instrument* whereby the *resistance* of the surface of the plane is made to supply a certain portion of that pressure which would be necessary to raise the weight, *directly*, through a distance equal to the height of the plane.

Such in practical mechanics, and in the operation of machinery, is the essential part which belongs to the resistances of points of support.

168. FRICTION.

When a body is *pressed* upon the surface of another, it is moved along that surface with difficulty. If you attempt to cause one of the surfaces to slide on the other, a certain force opposes itself to the effort, which is found to be greater, as they are pressed together with greater force. By rendering the surfaces of contact more smooth, or by interposing unguents between them, the amount of this resistance, called *friction*, may be greatly diminished, but it can never be altogether got rid of.

The principal experiments which have been made

upon friction, have reference; First, To the proportion in which the friction *increases* with the *pressure*, on the *same surface*. Secondly, To the variation of the amount of friction, produced by the *same* pressure, upon equal surfaces of *different substances*. Thirdly, To its relation to the *size of the surface* of contact, the pressure being the same. Fourthly, To the influence of the *time* in which the bodies have been in contact, on the amount of the friction; and especially to the distinction between the friction which resists the first motion of a body from rest, and that which opposes itself to its motion during the continuance of that motion. The principal experiments on this subject have been made by Coulomb*, Professor Vince, Mr. G. Rennie†, M. A. Morin.‡ The following are among the principal results of these experiments; a more detailed statement of them is contained in tables in the Appendix.

169. The Friction is proportional to the Pressure.

Thus, if the surface of one body be pressed upon that of another with a certain force, and if that force be then doubled, the friction will be doubled; if the force pressing them together be tripled, the friction will be tripled, &c. &c.

Thus, for instance, a piece of cast-iron having a plane surface of 44 square inches was laid, by Mr.

* Mém. des Sav. Etrangers, 1781.
† Philosophical Transactions, 1829.
‡ Méms. de l'Institute, 1833.

Rennie, upon another larger plane surface of the same metal, and loaded with a weight, which, together with its own, amounted to 24 lbs.; and it was found that a force applied to it, parallel to the surface, by means of a string and pulley, just moved it, when it amounted to 3 lb. 3 oz.

It was then loaded, so as to be similarly pressed, with twice the first weight, or with 48 lbs. and a force of 6 lbs. 8 oz. was then required; indicating a friction in the last case, or under double the pressure, which only differed by 2 oz. from twice the former friction.

When, again, the surfaces were made to press upon one another with a weight of 36 lbs., being $1\frac{1}{2}$ times the first pressure, the force required to move the body, that is its friction, was found to be 4 lbs. 14 oz., differing by only $1\frac{1}{2}$oz. from $1\frac{1}{2}$ times the first friction.

By similar means, a piece of black beech, which had a surface of two inches square, being pressed upon another with a force of *one* hundred weight, was found to have a friction of 15 lbs. 5 oz.; and, being pressed with a force of *three* hundred weight, to have a friction of 45 lbs. 3 oz., differing from triple its previous friction, by no more than 12 ounces. A piece of Norway oak, of the same size, being pressed upon another with a weight of *one* hundred weight, exhibited a friction of 14 lbs. 5 oz.; whilst, under a pressure of *four* hundred weight, its friction became 56 lbs. 7 oz., differing from four times its former friction, by only 13 oz.

This rule is however only an *approximate* one, from which the actual friction varies but little, in the case of hard metals, for pressures less than 32 lbs. upon the square inch; but from which there is a

rapid deviation, for pressures exceeding that limit. For woods, the limit is somewhat higher; but, within this limit, the results are more irregular than in the case of metals.

170. Amount of the constant Proportion of the Friction to the Pressure in different Substances.

An extensive table of the results which have been obtained on this subject will be found in the Appendix. The following may be mentioned as general conclusions: 1st, That the ratio of the friction to the pressure in all hard metals is, for pressures less than 32 lbs. on the square inch, *nearly the same. For all these metals, the friction is very little different from one sixth of the pressure.*

2d. The friction of the soft metals is greater than that of the hard ones.

3d. The same relation obtains in respect to the friction of the soft woods and the hard ones. Thus two surfaces of yellow deal being pressed together, exhibited a friction equal to more than one third the pressure, whilst the friction of two surfaces of red teak was scarcely more than one ninth of the pressure. These were the two *extreme* ratios in the case of woods.

Whether the fibres of the two surfaces of wood be *parallel* or *perpendicular*, materially affects the amount of friction, and whether they be *wet* or *dry*.

Thus, when one surface of oak was pressed upon another, the fibres being *parallel*, the ratio of the friction to the pressure was from ·60 to ·65; when

the surfaces were so placed in contact that their fibres were *perpendicular*, the ratio rose to ·54; and when, the fibres remaining thus perpendicular, the surfaces were *wetted* it sank again to ·71. It is a practical fact of some *importance*, that the friction of surfaces of wood upon one another is thus so considerably diminished by wetting them.

171. THE AMOUNT OF FRICTION IS INDEPENDENT OF THE EXTENT OF THE SURFACE PRESSED, PROVIDED THE WHOLE AMOUNT OF THE PRESSURE REMAIN THE SAME, AND THAT THE SUBSTANCE OF THE SURFACE PRESSED IS THE SAME.

This is an important property of friction, which has been established by numerous experiments. By increasing the surface which supports the pressure, you diminish the amount of pressure upon every point of it, and you thus so diminish the friction upon every point, that although there are more points which rub, their aggregate amount of friction is only the same as before.

Thus, in one of the experiments of Mr. Rennie, a piece of cast iron, when laid upon its *flat* side, which had a surface of 44 square inches, and loaded, so as to press upon another surface of cast-iron, with a force of 14 lbs., required a force 2 lbs. 4 oz. to make it slide: when placed upon its *edge*, which had a surface only of 6¾ square inches, and subjected to the same pressure, 2 lbs. 2 oz., were found sufficient to move it. The friction, in the one case, was then 34 ounces, and in the other, 36.

172. The Friction of a Body when in a State of continuous Motion, bears a constant Ratio to the Pressure upon it, which is the same, whatever may be the Velocity of the Motion.

This fact results from the experiments of M. Morin, made in the years 1831, 1832, at Metz, on a very extensive scale, and under the sanction of the French government.

The force with which the cord was, at any time, pulling the various bodies which it put in motion (and whose friction this force was always equal to), was estimated by the deflexions of a steel spring to which it was attached; and these deflexions were made by a very ingenious contrivance, to register *themselves*, at every period of the motion. The principal facts resulting from them were those stated at the head of this article, that the friction in this case of con-continued motion, as well as when the body is moved from a state of rest, is always the same fraction of the pressure, however great (within certain limits), or however small, that pressure may be; and moreover, that its *amount* is wholly independent of the *velocity* with which the body is moving, being of the nature of that force which is called by mathematicians, a *uniformly* retarding force.

173. The Effect of Unguents upon Friction.

The general effect of *unguents* upon friction is, as it is well known, materially to diminish it. It is, however, important to observe, that in doing this,

they entirely destroy the constant ratio which, without unguents, friction is found to bear to the pressure.

Thus, Mr. Rennie found that the friction of an axle of yellow brass upon a collar of cast iron was, without unguents, in every case about $\frac{1}{4}$th the pressure.* When the surfaces were *oiled*, this ratio became under a pressure of $\frac{1}{2}$ cwt., only $\frac{1}{37}$th; but when the pressure was increased to 11 cwt., it rose to $\frac{1}{6}$th.

Axles of yellow brass, moving in collars of cast iron appear, from the experiments of Mr. Rennie, to exhibit when used with unguents, the least amount of friction; and that unguent, which is best adapted to them, appears to be tallow. With this unguent, the mean result of his experiments gives for a pressure of from 1 cwt. to 5 cwt., a friction of somewhat less than $\frac{1}{39}$th the pressure. With soft soap it becomes $\frac{1}{34}$th. It is a remarkable fact, that while, with the softer unguents, such as oil, hog's lard, &c., the ratio of the friction to the pressure increases with the pressure; with the harder unguents, soft soap, tallow, and anti-attrition composition, it diminishes.

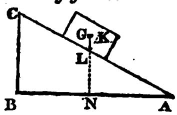
The question of time does not appear to have been sufficiently attended to, in these and other experiments on friction, and the subject is one in which much probably yet remains to be learned.

* The axle was revolving during the experiment over $4\frac{1}{2}$ inches of surface, in 90 seconds.

174. THE CIRCUMSTANCES UNDER WHICH A BODY WILL SUPPORT ITSELF UPON AN INCLINED PLANE.

Let the weight of a body, resting upon the inclined plane, represented in the accompanying

fig. 47.



figure, be supposed to be collected in its centre of gravity G, and draw the vertical line G N; it is in the direction of this line that the whole weight of the body will act, and it is in the direction of this line, therefore, that the body may be supposed to be *pressed* upon the plane at L. If, then, this direction lie without the cone of resistance at L, the body will slip down the plane; if it lie *within* it, it will not. Now, if L K be drawn perpendicular to the surface of the plane, from L, it will be the axis of the cone of resistance at that point, and the direction of G L will be within or without the cone, according as the angle K L G, is less or greater than one half the angle at the vertex of the cone. But the angle C A B is equal to the angle K L G; the body will therefore rest, of its own accord, or slip upon the inclined plane, according as the inclination C A B of the plane is less or greater than half the angle of its cone of resistance: and conversely, the inclination of the plane just equals half the angle of the cone of resistance, when it is such, that the body *begins* to slip upon it. Half the angle of the cone of resistance, is called the *limiting angle of resistance*, being that inclination of the

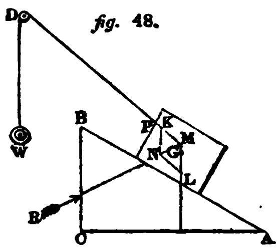
pressure to the perpendicular, which first, in any case, causes the body to slip. It is thus that the limiting angle of resistance, has, in respect to a great number of substances, been determined. Their surfaces having been made perfectly plane, have been placed upon one another, and then both bodies have been made to rest on an inclined plane; this inclined plane being moveable, so as to admit of receiving a greater or less inclination. It has then been gradually elevated, until the bodies were first observed to slip, and the angle of elevation, or, as it is called, the slipping angle, being observed, the limiting angle of resistance became known.

A table in the Appendix contains the results of experiments thus made by Mr. G. Rennie.

175. The circumstances under which a body may be supported upon an Inclined Plane.

Let the supporting force, be applied by means of a string D P, passing over a pulley D, and supporting a weight W. Let the direction, D P of

fig. 48.



this string be produced, so as to meet the vertical G L, through the centre of gravity in M. Mea-

sure off M K, containing as many equal parts as there are pounds or ounces in the *weight* W; and M L, containing as many as there are in the body to be supported. Complete then the parallelogram M K N L, and draw its diagonal M N. Then, by the principle of the parallelogram of forces, it is in the direction of this line M N that the *resultant* of the force P, and of the weight of the body to be supported, will act; it is, therefore, in the direction of this line that the body will be

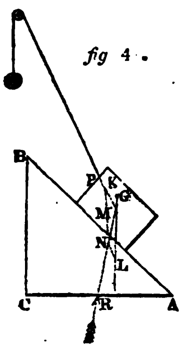


fig 4.

pressed upon the plane. If this direction be within the surface of the cone of resistance, this pressure will be counteracted by the resistance of the plane, and the body will rest; if it be without it, it will *not*, and the body will move. The direction of its motion, whether it be up or down the plane, depends upon the direction of the line M N; whether it be, as in the first figure, *upwards*, in respect to the surface of the plane, or as in the second, *downwards*.*

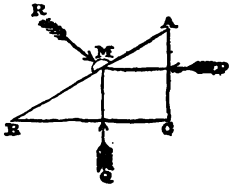
176. The moveable Inclined Plane.

Suppose an inclined plane A B C to be free to *move* along the surface on which its base B C rests, and let it be pressed along it by a force P, acting in a direction perpendicular to its back until it en-

* See "Mechanics applied to the Arts," p. 49.

counters and presses against a mass M, which resists its farther progress. The pressure of the surface of the plane upon M is produced by the resistance Q of the mass on which the base of the plane rests, and by the pressure P on its back, and it is equal to the resultant of these two pressures. Suppose that P is so increased, as to make it sufficient just to overcome the resistance of the mass M, and to cause the two surfaces to slide upon one another; the direction of this resultant pressure of the plane upon the mass must then be just without the cone of resistance, and in the same direction must be the opposite pressure R of the mass M, upon the plane. We know, then, what must be the *direction* of the pressure, whatever may be its amount, which causes the resistance to yield.* The amount of this force is dependent upon the nature of the resistance of the mass. In the cases about to be described, in which the moveable inclined plane is used under the forms of the screw and the wedge, the resistance commonly results from the cohesion of the parts of the mass, which must be overcome, before the plane can move.

fig. 50.



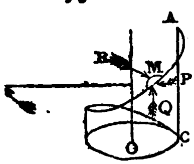
* 177. The Screw.

The surface of the plane, above and below that part of it M (see the last figure), on which the

* See "Mechanics applied to the Arts," p. 52.

mass rests, has nothing to do with its equilibrium upon that part, or with its pressure exerted upon it: thus, for instance, the parts A M and B M above and below M, might in any way be *altered*, — provided only that part were left on which M *rests*, — without at all affecting the circumstances of the equilibrium or the pressure. These, manifestly, only concern themselves with that portion of the plane on which the body is actually resting. Imagine, then, that, in the preceding figure, these two portions of the plane, A M and B M, are *twisted round* so as to convert the base B C into a circle, and make the two points B and C to meet; the plane will then assume the form represented in the accompanying figure, and the circumstances under which it exerts its pressure upon the mass M will be precisely those of the thread of a screw.

fig. 51.



The thread of a screw is, in point of fact, the surface of an inclined plane wound round a cylinder. It is pressed against the resisting mass M, which it is intended to move, by the leverage of a *screw-driver*, a *winch*, or an *arm*, which, giving to the screw a tendency to turn upon its axis, communicates to its surface a pressure P, which is parallel to its base, and therefore perpendicular to the back of the inclined plane, from the curving of which it may be supposed to result. The resistance Q perpendicular to the base of the plane, or parallel to the axis of the screw, is supplied by the resistance of the mass on which the extremity of that axis

turns. If this resistance be not sufficient to supply the requisite force to move the mass M, then the point of the mass on which the extremity of the axis turns *yields*, and the screw enters into the mass. Of this kind are the screws used by carpenters, and *tools*, such as gimblets and augurs, which make their way into timber by means of the screws at their extremities. In all these, it is necessary that the depth of their thread, and the distance of their consecutive threads, should be enough to cause the fibre of the wood, which represents the mass M, to oppose such a resistance as shall not be overcome, before the mass on which the extremity of the axis of the screw turns yields. Such screws should, therefore, have deep and distant threads.

The use of the common carpenter's screw is, commonly, to oppose itself to any force which may tend to tear asunder the pieces of timber which it screws together. To this tendency the adhesion of the fibres of the wood, which it receives between its threads, and the strength or tenacity of the screw itself, oppose themselves. If either of these fail, the attachment is broken,—in the first case by the *tearing out* of the screw, in the other by the tearing of it *asunder*.

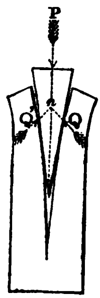
Now it is evident that the greatest economy of the material of the screw will be attained, when these two liabilities to failure are just *alike*, so that the screw is exactly upon the point of being torn *asunder* when it is on the point of being torn *out;* for any strength beyond this will not prevent rupture, nor have any tendency to prevent it, or to increase the strength. Screws are now commonly made

with reference to this proportion. With square threads, the inclination of the thread is about 7°, and with angular threads about 3½°. The depth of the thread is usually made equal to about half the distance between two threads.*

*178. The Wedge.

The wedge is a double moveable inclined plane, presenting two faces to two resistances to be overcome. In the accompanying figure, the points Q and Q′, are supposed to be the resisting points upon the wedge represented in it; and P is the direction of the force acting upon the back of the wedge, to drive it; and may be supposed to include the weight of the wedge. These three forces are in equilibrium. Moreover when the force P is on the point of driving the wedge, so that the points Q and Q′ of it are on the point of slipping upon the resisting surfaces, then the resistances at those points, have their directions accurately in the surfaces of the cones of resistance there. These directions Qn and $Q'n$ are therefore known. If therefore the force P be known, the amounts of the resistances may be determined by the principle of the parallelogram of forces. And conversely, if the amounts of the resistance Q and Q′ be known,

fig. 52.



* For a more complete discussion of the theory of the screw, the reader is referred to the "Mechanics applied to the Arts," p. 99.

the amount of the force P necessary to overcome them, will be known.

By applying the principle of the parallelogram of forces to this case, it will become evident, that the *sum* of the forces Q and Q′ is always essentially greater than P; and that in the case in which the angle Q *n* Q′ is greater than a right angle, *each* of these forces by which the wedge acts, from its two sides, upon the two resistances, is greater than the force P, by which it is impelled. This case occurs, when the vertical angle of the cone of resistance, and the vertical angle of the wedge, are together less than a right angle.

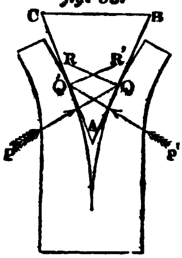
The great practical advantages in the use of the wedge, are, however, these, that it admits of being driven by *impact*, and that when its vertical angle is small enough, it *retains* every new position, between the resisting surfaces, into which it is driven.

It is especially the first of these properties which gives to the wedge its marvellous power. It will be shown in a subsequent part of this work, that *any* force of *impact* is infinitely great, as compared with *any* force of *pressure*. Now the resistances of the surfaces Q and Q′ are of the nature of forces of *pressure*, they necessarily therefore yield to any force of *impact* communicated to the wedge; and it is a second and scarcely a less useful property of the wedge, that every such yielding and separation of the surfaces between which it acts, it takes advantage of, and renders *permanent*.

* 179. THE CIRCUMSTANCES UNDER WHICH A WEDGE WILL NOT BE FORCED BACK BY THE TENDENCY OF THE SURFACES BETWEEN WHICH IT IS DRIVEN TO COLLAPSE.

Suppose the wedge to be in contact with the surfaces between which it is driven, at a great number of points. Let P and P′ be the pressures with which two of those points similarly situated, on its opposite faces, tend to collapse, and to drive back the wedge.

fig. 53.



The pressure P, being propagated through the mass of the wedge, will press the opposite face AB upon the surface with which it is in contact at Q; and the pressure P′, the face AC upon Q′. If, then, the directions PQ and P′Q′ be *without* the surfaces of the cones of resistance at those points, the wedge will be driven back; if they be *within* the cones of resistance, the forces PQ and P′Q′ will be wholly sustained by the resistances at Q and Q′, and the wedge will retain its position. The tendency of each surface to collapse being supposed to be exerted in a direction *perpendicular* to that surface, so that the forces P and P′ are respectively perpendicular to the faces AC and AB of the wedge*,

* It will be observed that the wedge being no longer supposed to be on the point of being *driven* either way, the forces P and P′ have no longer their directions necessarily upon the *surfaces* of the cones of resistance.

it may easily be shown (see Mech. app. to the Arts, p. 55.) that the directions of PQ and P′Q′ will be within the cones of resistance, and that these forces will not therefore expel the wedge, provided its vertical angle A be less than the *limiting angle of resistance*, or less than half the vertical angle of the *cone of resistance*.

A wedge will be of little or no use unless it be made, subject to this law. Thus, for instance, adopting the experiments of Mr. Rennie (which those of M. Morin do not however sanction), it appears that a wedge of oak to be driven into oak, and to keep any position into which it is driven, should not have a vertical angle of more than 8°. Adopting, however, the experiments of M. Morin, we may assign to it a vertical angle of 31°. It is greatly to be regretted that no experiments have been made, in this country, on a sufficiently extensive scale, or with sufficient precautions for accuracy, to enable us to pronounce on these opposite results.

* 180. Nails.

When the angle of a wedge is equal to its limiting angle of resistance, in respect to the surfaces between which it is driven, the tendency of these surfaces to collapse will be upon the point of expelling it; when it is *less* than this limiting angle, the application of a certain force will become necessary to expel it, it must be *drawn back*. The directions of PQ and P′Q′ being *within* the cones of resistance at Q and Q′, a force must act upwards

at A; or, which is the same thing, it must be applied to draw up the back of the wedge CB, so as, combining with the pressures P and P′, to give them a more oblique direction at Q and Q′, and bring them there without the cones of resistance.

The smaller is the angle A of the wedge, the further will the directions of P and P′ be *within* the cones of resistance; and the greater will be the force requisite to bring them *without* the cones, and to extract the wedge. Of this class of wedges, with exceedingly small vertical angles, are NAILS.

A table will be found in the appendix containing the results of experiments, made by Mr. Bevan, on the forces necessary to extract nails of different sizes, driven into different substances. It is evident that the length of the nail will greatly increase the force necessary to extract it, increasing rapidly the number of points P, by which it sustains the pressure of the surfaces into which it is driven.

Nails, as well as screws, are made with the greatest economy of their material, when they are made of such a thickness, that the force necessary to *tear* them asunder is exactly equal to that necessary to *draw* them. Any additional thickness would evidently have no effect in preventing the separation of the pieces of wood which they fix together, and would therefore be useless.

181. The circumstances under which an Edifice of uncemented Stones is overthrown.

An edifice built up with uncemented stones may fall, either by the *turning* of some of its stones on the edges of one another, or by their *slipping* upon one another.

These two cases are represented in the accompanying cuts. In the first, an arch is seen to be

fig. 54.

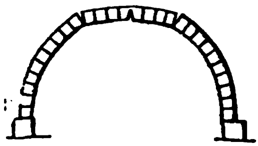
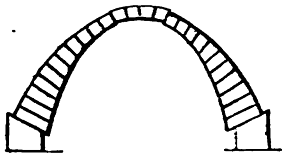


fig. 55.



falling by the *turning* of its voussoirs or arch-stones, at the crown, upon the *upper* edges of one another, and of those at the haunches, upon their *lower* edges. In the second figure, an arch falls by the *sliding* of the arch-stones near the abutment *downwards*, and by the sliding of those near the crown *upwards*.

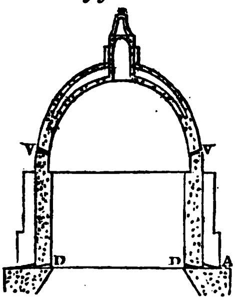
The last case is of rare occurrence; such is, for the most part, the *friction* of the surfaces of the stones used in construction, that their *slipping* upon one another is a contingency against which few, if any, precautions need be taken.*

It is by the *turning* of certain of its component masses upon the edges of others, that an edifice for

* The question of the *slipping* of the voussoirs upon one another, was a few years ago considered to involve *the whole* question of the stability of the arch.

the most part shows symptoms of failure. An example presents itself in the dome of St. Peter's at Rome. The walls of that mighty structure have in many places *yielded* under the outward thrust which they have to bear. Numerous cracks are apparent in them, and they have *especially* opened

fig. 56.



on the *outside*, about the haunches V V, and on the *inside*, about the springing D D of the dome. To counteract this tendency of the walls to turn at the haunches on their *internal*, and at the base on their *external* edges. Vanvitelli caused, in the year 1748, immense girdles of iron to be placed round the haunches of the dome at V V; to which others, of great strength, have since been added. It is by a similar contrivance that Sir Christopher Wren has strengthened the dome of St. Paul's.

* 182. The Conditions of the Equilibrium of an Edifice of uncemented Stones.

Let the extreme stone A D, of an edifice of uncemented stones be supposed, as in the accompanying figure, to have, impressed upon it, any given force P. Besides this force P, the stone is acted upon by *gravity*, which may be supposed to be collected in its centre of gravity. Let the *resultant* (art. 138.), of these two forces be imagined to be taken. This resultant will represent the *whole* force by which the

first stone is pressed upon the second. If this resultant have its direction anywhere *within* the edges, of

fig. 57.

the *joint* or surface of contact, of the first stone with the second, the one will *rest* upon the other; if not, it will turn over upon it. Let it be supposed to rest upon it, and let us proceed to consider the conditions of the equilibrium of the second stone. This second stone may be considered to have its upper surface acted upon by the resultant force just spoken of, and this to be the only force pressing it downwards, besides its own weight collected in its centre of gravity. If then a *second* resultant be taken, being that of two forces, of which the *first* resultant is one, and the weight of the second stone the other, then this second resultant will be that force by which the second stone may be supposed to be pressed upon the third. If its direction lie within the edges of the joint of the second and third stones, the second will rest upon the third; if not, the superstructure will turn upon the third stone. Similarly, if a third resultant be imagined to be taken, being that of two forces, of which one is the second resultant and the other the weight of the third stone, then this third resultant will be that force by which the third stone is pressed upon the fourth; and the conditions of the equilibrium of this third stone are, that this resultant shall have its direction within the edges of the joint of the third and fourth stones; and so on of the rest.

Thus then the *great* condition, that the structure

shall not be overthrown by the turning over of any one of its stones upon the edge of the subjacent stone, is included in this — that none of the *resultants* spoken of above, shall have its direction beyond the edges of the surface, by which the stone, to which it corresponds, touches the subjacent stone. Now let us suppose that the *intersections* of all these resultants, with the planes of the joints of the successive stones, are, by some mathematical investigation, *found ;* and let a line be imagined to be drawn, passing through all these points of intersection. That line is called THE LINE OF RESISTANCE. It is a curved line, whose form may be completely determined in every case, by the methods of analysis.*

If this curve, so determined, be found to have its direction anywhere beyond the joints of the stones, that is, if at any of those joints the curve passes *without* the mass of the stone, the edifice will, at that joint, be overthrown. If the curve nowhere lie without the mass of the edifice, it will nowhere be overthrown by the turning of its stones.

That none of them may slip, or that the second condition may be satisfied, it is further necessary, that none of the resultants spoken of in the commencement of the article, should have its direction *without* the cone of resistance of its corresponding

* For the analytical discussion of this curve, and of all the facts stated in this and the following articles, the reader is referred to a paper by the author, in the third volume of the "Cambridge Philosophical Transactions," part 3.; and to a a second, in the sixth volume, part 3. The theory stated above was for the first time given in the former of these papers.

joint. These two conditions include all that is required to the equilibrium.

* 183. The Line of Resistance in a Pier.

In an upright *pier* or *wall*, the line of resistance is the geometrical curve called the hyperbola.* The position and magnitude of this hyperbola may readily be determined by the following construction. Resolve the force P (see the last figure), which acts upon the summit of the pier, into two others, by the method explained in article 139, one of which two is in a vertical, and the other in a horizontal, direction.

Calculate the *height* of a mass, which being of the same substance, and the same thickness as the *pier*, shall have a weight equal to the *vertical* force of these two, and let this height be A T. Calculate, in like manner, the height of a mass whose weight shall equal the *horizontal* force, and let this height be A S.† Take B, the centre of the width of the pier, and set off B K, equal to A S. Draw then the vertical K C. C will be the *centre* of the hyperbola, and the vertical C K E will be its *asymptote*. Now the curve of an hyperbola always *approaches*, but never *touches*, its asymptote. The curve of resistance always then approaches, but never touches, the line C E; and if this line lie, as in the figure, *within* the mass of the sphere, then the line of resistance, never passing the line C E,

* Memoir on theory of equilibrium of bodies in contact, "Cambridge Philosophical Transactions, vol. vi. part 3.

† The dotted lines in the figure represent the two imaginary masses here spoken of.

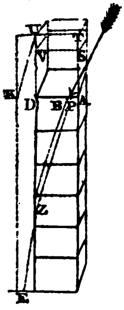
can never cut the outward surface of the *pier;* and however *tall* it may be, the pier can never be overthrown by the action of this force. Moreover (and this is a remarkable feature of the theory), the pier will bear this insistent pressure P, wherever, in A K, it is applied parallel to its present direction; the position of the centre of the hyperbola C, not being changed by any alteration in the point of application of that pressure, but only in its magnitude.

*184. The greatest Height to which a Pier can be built, so as to sustain a given Pressure upon its Summit.

If A S be greater than half the width of the pier, or if K lie beyond D, then there will be some point in the outward surface or *extrados* of the pier, where the line of resistance will *cut* it; and there will, therefore, be a certain height beyond which the pier cannot be carried, without being overthrown. This height is thus readily determined.

Let P be, as before, the point where the insistent pressure intersects the summit of the pier, and let A S, and A T, and B K, be taken as before; join U K, and through P draw P Z, parallel to U K. Z will be the point where the line of resistance cuts the extrados, and will indicate the greatest height to which the pier can be carried, without being overthrown; or, if it be carried higher, then is this the point to which an inclined buttress should be built to support it.

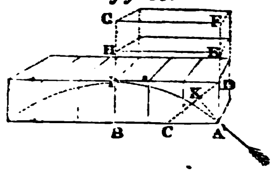
fig. 58.



*185. The straight Arch, or Plate Bande.

If stones be placed side by side, horizontally, and supported at their extremities, as in the accompanying figure, they constitute a *straight arch or plate bande.* If such a structure be supposed to rest by its inferior angle A, at either extremity, against an immoveable abutment, the following construction will determine the direction and amount of its pressure upon that abutment. Divide its length into two equal parts in B, and divide A B again into two equal parts in C; join C D, and draw A K perpendicular to C D, A K will be the *direction* of the pressure. Take D F equal to D C; the imaginary mass D C, shown by the dotted lines, having the same width and thickness with the straight arch, and half the length, and being of the same material, will then have its weight exactly equal to the amount of the whole pressure A upon the abutment. If D E be taken equal to A C, the weight of the mass D H will equal the *horizontal* portion of the force A, or the *outward* thrust.*

fig. 59.



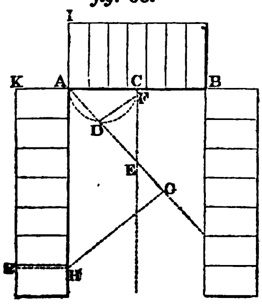
* For the analytical formulæ on which this construction, and that in the next article, are grounded, the reader is referred to the paper on the equilibrium of bodies in contact before alluded to.

*186. To find the greatest Height of the Piers, of a given Width, which will support a straight Arch of given Dimensions.

Let A I B be the straight arch to be supported, and A K the given width of the piers.

Divide A B into two equal parts in C: upon A C describe a semicircle, and measure off A D equal to A K, so as to cut the circumference of this semicircle in D: produce A D, and let it intersect the vertical line through C in E: measure off E F equal to A I, and A G equal to A B: join D F, and draw G H parallel to D F; then A H will be the extreme height of the pier. Being of any less height, it will stand firmly; being of any greater, it will be overthrown.

fig. 60.



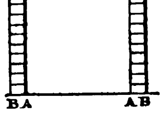
*187. The Arch.

The most useful and the most interesting application of the theory of the line of resistance, is that which may be made of it, to the conditions of the equilibrium of the arch. Any *detailed* discussion of a subject of so much difficulty, is, however, beyond the scope of this treatise.* It may, however,

* The reader is referred to the author's memoir in the Cambridge Philosophical Transactions, vol. vi. part 3., and to his elementary treatise on "Mechanics applied to the Arts," article 185.

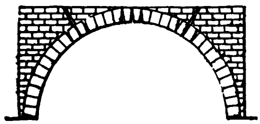
be stated as a general condition of the line of resistance in the arch, that it *touches* the intrados, or inner surface of the arch, on both sides at its *haunches;* and that afterwards at lower points, it *cuts* the *extrados*, or outer surface of the arch. If some resistance, of an abutment or pier, be not opposed at this last point to the pressure, the whole of which acts there, the arch will be overthrown. If it be supported there by a pier, the line of resistance passes into the pier, and assumes a new character and direction; that direction having a general tendency towards the *back* or outer surface of the pier. If by reason of the comparatively small height of the pier, the line of resistance does not any where reach the back of the pier, but intersects its base, then the pier will stand. If on the contrary the height be, as in the accompanying figure, so great, as to cause the line of resistance to cut the back of the pier at some *point above* its base, then the pier will be overthrown, and the arch will fall. When the arch falls, the line of resistance is made to *cut* the intrados at the points *m m* in the haunches, where before it *touched* it. These points are called the *points of rupture.* The line of resistance, thus *cutting* the intrados of the arch at *m, m*, the direction of the whole pressure is made, at those points, to act beyond the joints of the stones there; so that it causes the stones there to turn upon their *lower* edges, opening at their *upper* edges in

fig. 61.



the *extrados* at *n* and *n*. Besides touching the intrados at the haunches *m m*, it is another *general* characteristic of the line of resistance, in the *state of the equilibrium* of the arch, that it *touches* the extrados over the crown at N, and that when the arch is *falling*, it is made to *cut* the extrados there: so that the pressure, there also, acting beyond the

fig. 62.

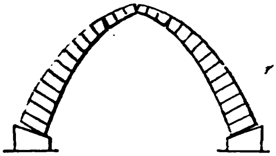


joints of the stone, causes them to turn, but in this case on their *superior*, instead of their *inferior* edges. The arch then opens at the crown, at its intrados in M; and thus it falls, separating itself into four distinct parts. These are the *general* conditions under which an arch may be understood to fall, by the too great height, or insufficient weight of its piers, in respect to the load it bears on its crown. There is yet; however, another condition which *may* bring about its overthrow. It may be so overloaded about its haunches as entirely to alter the direction of its line of resistance; to *flatten* this line at the top and give it two elbows on either side of the crown; so as to cause it to cut the *intrados* instead of the *extrados* at the crown, and the *extrados* at two points, a short distance on either side of the crown; the points where it *touches* the intrados, being by this process thrown much lower down upon the arch.

The arch will in this case fail, as shown in the

accompanying figure, by the rising of its crown, and the *falling in* of its sides. The great art of arch

fig. 63.



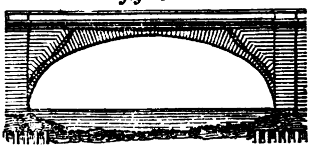
building consists in so loading the arch as to secure it against *either* of these contingencies. It is one of the most important and the most difficult problems of practical mechanics.

*188. The Settlement of the Arch.

Whilst the stones of an arch are being placed together, they are supported upon a frame of wood, whose upper surface is of the exact form of the arch to be constructed. This frame, called a centre, is supported upon wedges, and it is not until its removal, by the knocking away of these wedges, that the arch stones are allowed to *bear* upon one another. This process of removing the centre is called *striking* it. From a very early period in arch building, it was observed that, after the striking of the centre, when the *whole* pressure of the arch was, for the first, thrown upon the stones which compose it, certain motions took place among them. To ascertain what these motions were, at the bridge of Nogent sur Seine, Perronet caused three straight lines to be cut in the stones, upon the face of the arch, before the striking of the centre, one horizontally above the crown, and two others, equally

inclined to it, on either side, beginning from the abutments. These lines are represented by the *straight* lines in the accompanying figure. *After*

fig. 64.



the striking of the centres they altered their forms, and became the *curved* lines which are seen crossing the others.

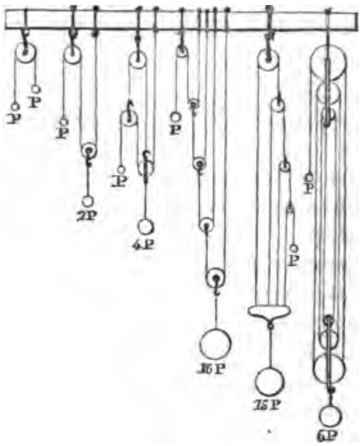
The curvature of these lines plainly shows, that, after the striking of the centres, the arch stones above the crown, and from the crown for some distance towards the haunches, *descended;* but that *beyond* a certain point in the haunches, and from thence to the abutments, they *ascended.* These points where the pressure of the arch *changes* from a pressure *downwards,* in respect to the faces of the voussoirs, to a pressure *upwards,* correspond to the *points of rupture* spoken of in a preceding article.

189. Pulleys.

The accompanying cut represents the different systems of pulleys which are commonly used. The pulley may be described as a circle of wood or iron, moveable round an axis which passes through the centre of the circle, and having a groove in its circumference or edge, round which is wound the string whose tension it is the use of the pulley to

direct and *apply*. This string passing round one half of the pulley (as in the first of the above

fig. 65.



figures), and fixing itself upon it by friction, so that it cannot be moved without turning the pulley, it is evident that its effect, upon each side of the pulley, must be the same as though it were actually fastened to its circumference at the points where it leaves it. Now, these points are equidistant from the axis of the pulley, and that forces, applied, at *equal perpendicular distances* from an axis, may balance one another about it, it is manifestly necessary (by the principle of the equality of moments), that they should be equal. Thus then it appears that the two weights which balance themselves on the first of the pulleys shown in the figure, must be

*equal.** When the two equal weights P thus balance themselves on this pulley, which is called the *fixed* pulley, it is evident that the hook from which it is suspended, must sustain a pressure equal to 2 P, together with the weight of the pulley itself.

The *second* system shown in the figure, is composed of two pulleys, one of which is fixed and the other moveable, and this last supports from its centre, the weight to be raised. The same string passes round both pulleys, and supports the power P. By the same reasoning as in the last case, it appears that this string must, on both sides of both pulleys, sustain a tension equal to P, so that at the point where, ultimately, one extremity is fastened to the beam, a force P, would hold it. Thus it is clear, that the moveable pulley and its appended weight are supported, on either side, by a force equal to P: now these together, will just support a weight equal to 2 P. Thus, then, in this system, called that of the single moveable pulley, a power can be made to support a weight which, including the moveable pulley, is equal to twice that power; or a given weight can be raised by the effort of a force equivalent to but little more than half that weight.

In the *third* system, called the Spanish Barton, there are two moveable pulleys, and one fixed. There are moreover two strings, one of which carries the power, and passing round that moveable pulley which carries the weight, is ultimately attached to

* The effects of the friction on the axle, and the rigidity of the cord, are not here considered. These, however, greatly influence the result in practice.

the beam. The other string suspends the two moveable pulleys, passing over the fixed one. The first string, having the power P suspended from it, acts exactly as in the last described system of the single moveable pulley, and thus it sustains, of the weight to be raised, a portion equal to 2 P. But this string, passing over the first moveable pulley, produces a tension in the string which suspends that pulley, equal to twice its own amount, or to 2 P, and this tension is ultimately applied to the last pulley, supporting an additional portion of the weight equal to 2 P. Thus on the whole, in this system, a given power will support *four* times its weight, or a given weight may be raised by a power equal to a little more than *one fourth* that weight.

In the *fourth* system there are as many different strings as moveable pulleys. The first, having a weight P suspended from it, produces a tension of 2 P on the second string, which holds down the first moveable pulley. A tension of 2 P thus being produced upon the second string, this going round the second moveable pulley, draws it upwards with a force equal to 4 P, and produces a tension of that amount in the third string; this third string, in like manner, drawing up the third pulley with a force equal to 8 P, produces that tension in the fourth string, so that, ultimately, the last pulley and weight, are supported by a force equal to 16 P. Or, by this system, a given weight could be raised by a little more than one sixteenth of that weight. Had there been a fifth moveable pulley, but one thirty-second of its amount would have been required to raise

the weight. If there had been six, but one sixty-fourth.

In the *fifth* system there are as many strings as pulleys. The first string, carrying the power P, supports a portion of the weight equal to P, and produces a tension in the second string equal to 2 P. This second string supports, by this tension, a portion of the weight equal to 2 P, and produces a tension in the third string equal to 4 P. The tension of 4 P, in the third string, causes that string to support a portion of the weight equal to 4 P, and to produce in the fourth string a tension equal to 8 P; this last tension, again, supports a portion of the weight, equal to 8 P. Thus, then, the four strings support portions of the weight, respectively equal to P, 2 P, 4 P, 8 P; and thus, together, they support a weight equal to 15 P. Had there been a fifth pulley in the system, it would have supported an additional portion of the weight, equal to 16 P; and the whole weight supported would have been 31 P.

In the *sixth* and last system, the same string passes round all the pulleys, and its tension is the same throughout. Thus the weight is borne by six distinct and equal tensions, which together will bear a pressure equal to six times any one of them; so that by this system a given power will support and raise nearly six times its weight. Had there been another pulley in each block, the weight raised would have been eight times the power.

This last system, although each additional pulley does not give, in it, the same additional amount of power as in the others, is yet much more convenient

in practice. In the other systems, whilst they raise the weight a given height, the pulleys move through different distances, and unless the strings be very long and the pulleys very wide apart at first, they soon become encumbered with one another. In the last system, the pulleys approach one another only by as much as the weight is raised.

CHAPTER V

DYNAMICS.

THE FORCE OF MOTION — ITS PERMANENCE — THE MEASURE OF IT — THE POINT WHERE IT MAY BE SUPPOSED TO BE COLLECTED. — MOTIONS OF TRANSLATION AND ROTATION, INDEPENDENT. — THE CENTRE OF GYRATION. — THE CENTRE OF SPONTANEOUS ROTATION. — THE CENTRE OF PERCUSSION. — THE PRINCIPAL AXES OF ROTATION. — THE FORCE OF A BODY'S MOTION IS NEVER GENERATED OR DESTROYED INSTANTANEOUSLY. — ACCELERATING FORCE. — GRAVITATION. — CAVENDISH'S EXPERIMENTS. — DESCENT OF A BODY FREELY BY GRAVITY. — ATWOOD'S MACHINE. — DESCENT OF A BODY UPON AN INCLINED PLANE AND UPON A CURVE. — THE CYCLOIDAL PENDULUM. — THE SIMPLE PENDULUM. — THE CENTRE OF OSCILLATION. — KATER'S PENDULUM. — THE COMPENSATION PENDULUM.

190. Certain Laws common to the Operation of all Forces.

The force, of which we trace the existence in the material substances around us, is presented under a variety of different forms and different circumstances.

Thus we find it in the *descent* of all bodies towards the centre of the earth — it is then called GRAVITY; we discover it a pervading principle in the material world, under another form, and

call it ELECTRICITY; — related to this is force under yet another form, which we call MAGNETISM; and there are forces of *Adherence*, of *Attraction*, and *Repulsion*, between the material particles of which all bodies are made up, which are known under the names of CAPILLARY ATTRACTION, COHESION, and CHEMICAL AFFINITY.

Whether the forces which we thus distinguish, by reason of certain differences in the manner and circumstances of their action, be or be not, different *modes of action* of the same principle of force, whether they be of the same family, or flow from the same fountain or source, we know not. This, however, we certainly know, that there are LAWS of force which are *common to all.*

The development of these LAWS, as they regard the *equilibrium* of bodies, constitutes the science of STATICS; as it regards their *motion*, it is the science of DYNAMICS. We have *now* then to inquire what are the laws which govern the MOTIONS of material bodies, and what relation exists between these and the FORCES in which they originate.

191. MOMENTUM, OR THE FORCE OF MOTION.

It is a matter of continual observation that a moving body becomes, *by reason of its motion*, capable of *communicating* motion to another body, or of destroying motion in that body.

Now that which *causes* or *destroys* motion is (by our definition, page 122.) FORCE.

A body, in the act of changing its place, possesses therefore a *principle of force*, co-existent with its motion, and dependant upon it. It is a force

wholly distinct and different from the force of pressure, which belongs to the state of the body's rest. Thus, for instance, the force with which a stone, falling to the ground, strikes it, is wholly distinct and different from that with which, resting upon the ground, it presses it; the one has wholly ceased, and has been destroyed, before the other begins to operate.

The *force* which thus exists in every moving body, which co-exists with its motion, and is dependant for its existence *upon* its motion, is called its FORCE OF MOTION, or, more frequently, its MOMENTUM.

192. THE FORCE OF A BODY'S MOTION IS PRECISELY EQUIVALENT TO THE FORCE EXPENDED IN PRODUCING IT.

This force of the body's motion is a *result* of the force which first gave it motion — an *effect* of that *cause* — and the effect and cause are equivalent — the force of motion in the body, and the force expended in producing it, are equal things. It is as though a *transfer* of the principle of force were made from the moving thing into the thing moved; thus, for instance, if a *ball* be put in motion by the recoil of a spring, the force with which the spring recoils is not lost, it is but *transferred* to the ball; and the ball is then ready to bring precisely the same quantity of force into operation on any other object which it encounters, as the spring did on it. So, too, if the ball were put in motion by the hand, the force expended in the production of its motion will not be lost; it will only be transferred from the hand to the ball, and the ball will be ready *to re-*

produce the whole of it, and to cause it to operate on the first obstacle which it encounters.

It is, of course, here supposed that there is no opposition to the free motion of the ball arising from the resistance of the air, friction, gravity, or any other of the causes which interfere with the motions of bodies on the earth's surface.

If there be such causes of retardation, their operation will continually destroy a portion of that force of motion in the ball, which was, nevertheless, *originally,* precisely equal to the force expended in putting it in motion.

Thus, a billiard ball continually loses a portion of the force with which it was originally struck — by reason of the friction of the baize, and the resistance of the air which, to move, it must continually displace; and, by this continual *destruction* of its force of motion, it may eventually be deprived of the whole of it, in which case it is said (improperly) to rest *of itself.* The same is true of a *bowl,* which continually loses the force of its motion as it rolls over the turf; and of a *cannon ball,* which, by reason of the resistance of the air, and frequent impacts, perhaps, on the ground, loses continually the force of its motion, until it becomes, at length, what is called a *spent* ball. In all these cases, at the *commencement* of its motion, before any opposing causes came into operation, the force of the body's motion was precisely equal to that expended in producing it; and it would have been found the longer to retain it, as these causes were more and more completely removed. Thus a smooth ball, rolled over the grass, *soon* stops; rolled over the cloth of

a billiard table, its motion, and force of motion, are longer continued; on a smooth plank, or iron plate, yet longer; on a level sheet of ice it suffers but little retardation; and, if the surface of the ice be continuous, and perfectly smooth, and no wind oppose the motion of the ball, it will lose very little of its force of motion for a great distance. Thus then we see, that, as the causes of the destruction of a body's motion, and force of motion, are *more* and *more* taken away, these approach more to the condition of permanence; and from this we conclude, that if they were completely removed, that condition of permanence would be *absolutely attained; so that if there were no causes of retardation* EXTERNAL TO ITSELF, *a body's motion and force of motion would continue for ever; hence the following law.*

193. THERE IS NO PRINCIPLE OF DIMINUTION OR DECAY IN THE NATURE OF MOTION ITSELF, OR IN THE NATURE OF THE FORCE OF A MOVING BODY.*

This principle is commonly known as the FIRST LAW OF MOTION.

The difficulty of conceiving or admitting it, lies in this, that we observe all those forces of motion which are produced *around* us, continually to *diminish*, and eventually to become *extinct*, as it appears to us, of themselves. Our own bodies when we have moved them, do not *of themselves* move on; fresh efforts must be continually made: our carriages require the continual draught of the horses, and even if we put them on a smooth road

* The body is here supposed not to be endued with vital power,

of iron, there is required a *force* continually to *impel* them: we move a stone with our foot, and but a few steps further on we find it at *rest*. It is a most wise provision of Providence by which the natural tendency of all these forces to permanence, is thus continually destroyed. Without it the world would scarcely be habitable. Were there no *friction* to check the superfluous force which we give to our bodies at every step, our state of existence would become one of incessant and involuntary motion: every thing we touched would, from that instant, become an ever-moving body; every thing not rooted in the earth, would be a sport of the winds, and men would soon desert the land, to dwell on the sea, as the more stable element. Could we diminish the resistance of FRICTION and the AIR to *any conceivable extent*, and if it were found that, as we diminished these, the motion of a moving body approached *continually* to a state of permanence, so that, by thus diminishing the causes of retardation, we could make the motion to differ from a permanent motion, by as little as we chose, this first law of motion would be completely proved. For if there were any sensible diminution of the force communicated to the mass, arising from *a failure in its own energies*, and independent of the *resistances* opposed to it, then that diminution would be *apparent* and *sensible* when the resistances were so far diminished as to be *insensible*.

Unfortunately, however, we cannot diminish the resistances of friction and the air beyond certain limits. As an absolute *demonstration*, this method therefore *fails*. Nevertheless, the fact that, dimi-

nishing the resistances to motion *as far as we can*, we find it continually approximating to a state of permanence, renders it in a high dgree *probable*, that, if we could carry this diminution on indefinitely, motion would approach indefinitely to a state of permanence, and that if these resistances could be absolutely *destroyed*, it would become permanent.

194. Illustrations of the Permanence of communicated Motion.

It is in the case of a revolving body that we can most effectually diminish these resistances to motion, by causing it to be supported and to turn on a very small surface, as compared with the dimensions of the body itself; as for instance, a large wheel round a slender axle, or a large spinning-top on a fine point, by which contrivance the resistance of friction is made to act at a great mechanical disadvantage, as compared with the force of the body's rotation; and we may, further, remove the resistance of the air, almost to any degree we choose, by placing the revolving body under the receiver of an air-pump.

Now, if we thus remove the air from the receiver of an air-pump, and then, without re-admitting it, by some mechanical contrivance, put rapidly in motion under the receiver, a large *wheel* with an exceedingly small axis, or, better, a large *spinning-top* with a fine hard point; we shall find that motion, which would, under other circumstances, soon cease, lasting, apparently unaltered, for hours. And a pendulum, delicately suspended on knife edges, and having thus yet greatly less friction to contend with

than either the axis of a wheel or the point of a top, when once a motion has been given to it, will retain the force of that motion, and continue to oscillate with it for more than a day. Mr. Roberts of Manchester, is said to have constructed a body which is of such a form and so truly balanced upon a fine point, that, having put it in motion round that point, it would not lose the force of its motion, but continue to spin with it for 43 minutes. These are all proofs of a tendency to the permanence of motion, and the force of motion which accompanies it, when causes of retardation from *without* are more or less removed; that is, of its tendency to absolute permanence, so far as any cause *within itself* is concerned. It does not die or diminish of *itself;* there is *within it* no principle of death or decay,—to cease, it must be operated upon by causes *external* to itself. The proofs hitherto given show, however, only the probability of this truth. It is *probable* that, since when we continually diminish the external causes of a body's retardation, its motion approaches to a state of permanence, if we were completely to take away those causes of retardation, that permanence of motion would be completely attained. But we *cannot* take away these causes of retardation—we cannot completely take away friction and the resistance of the air: we can therefore only speak of what would probably happen if these were completely removed.

To complete the proof, we must look out for some case of motion, in which there is no friction, and no resistance of the air. Such a motion we cannot find on or near the earth's surface, but we do find it in the heavens.

195. The Permanence of the Forces of Rotation of the Planets, and of their tangential Forces of Motion.

The PLANETS all roll in their orbits round the SUN, and their SATELLITES each round its primary planet, without friction, and unopposed by the resistance of any fluid atmosphere; and the motion first communicated to them, the velocity of their first projection, *remains*, in accordance with the first law of nature, *unabated*, *permanent*, from year to year, from century to century. It has remained the same from the period when they first went forth into space, at the mandate of God, to fulfil the designs of his providence, and it will remain the same until time is swallowed up and lost in eternity. That force by reason of which each planet moves not directly *towards* the sun which attracts it, but always nearly at right angles to that direction, is a force the principle of which resides within the planet itself: there is no *external* force to draw it from the path which it has a continual tendency to take towards the sun. The force, whatever it is, which produces this effect, does not emanate from *without*, but from *within* the planet itself; it is the *force of its motion*.

Were it not, then, in its own nature *permanent*, but such, that although unopposed, it would yet gradually, of itself, lose its original vigour and energy, then this force of motion in the planets, becoming from year to year gradually less, would continually be more and more controlled by the

attractive power of the sun, so that, from year to year, their orbits would alter their forms, becoming continually ellipses more *elongated*, until at length the deflecting force of motion in each planet being *extinct*, each elliptic orbit would resolve itself into a straight line, and each planet fall directly towards the central sun.

Now the very contrary of all this we know, by direct observation, to be the real state of things. There is no elongation of the orbit of any planet arising from any such cause. There is no alteration whatever in the orbits of any of the planets, except a slight one arising out of the influence of their mutual attractions; an alteration which of necessity returns perpetually in a cycle, and which, far from indicating an ultimate destruction of the existing system, supplies the most striking evidence of its permanence.

This is not, however, the only proof of the first law of motion which astronomy offers to us. In the system of the satellites of Jupiter, for instance, the astronomer beholds a beautiful epitome and model, of the great system of the universe. To disbelieve the revolutions of those satellites, he must disbelieve the direct evidence of his senses: and he finds their revolutions from month to month, and year to year, to be same, and the same as they were observed by other astronomers to be, two centuries ago; the effect of the primæval impulse, in which the motion of each had its origin, remains then in it unabated—unaltered from the beginning. The earth, too, *rotates* daily upon its axis by reason of a *first impulse*, given when the foundations of

the universe were laid, and not since renewed. No hand is *now* upon it; no cause *now* operates to turn it: it turns of *itself*, with its own innate force —the force given to it when it first came into the existing state of its being, the force of its motion. A question then arises—Does the effect of that impulse, the *force* of that original motion, remain *unabated*, *unimpaired*, to this day, or does it not? We have before us the evidence of 2000 years, and we thus know with certainty that the earth turns upon its axis now precisely in the same time that it did then: not the slightest appreciable fraction of the original force of its motion has in the intervening period disappeared. But this is not all. On this principle of the permanence of communicated force are grounded all the calculations of physical astronomy: these apply to all the phenomena of the heavens; they enter, for instance, largely into the calculation of eclipses, into those calculations of the positions of the moon in reference to certain of the fixed stars by which the navigator determines his longitude, and guides the course of his ship; and into an infinite variety of others which are every day submitted to the test of observation, and every day *verified*. Were motion not governed by *this law*, every one of these calculations would be *false*. That they are true is in itself, therefore, a sufficient proof of it. Such is the *direct* evidence of the permanence of unopposed force of motion.

The *indirect* manifestation of the existence of the same principle in the things around us, is not less remarkable.

196. Illustrations of the Permanence of the Force of Motion.

There is scarcely any case of motion in which it cannot easily be traced. The flying of the dust out of a *carpet* on one side, which is struck on the other, is but an effect of the force of motion communicated to the dust, in common with the carpet, by the blow, and an indication of its tendency to permanence.

When a man rides in a carriage or on horseback, with his motion, a *force* of motion is impressed upon him, which he does not indeed perceive, as long as his carriage or his horse moves with him; but which, if they be suddenly stopped, may throw him from his seat.

If he stands upright in a boat, as it approaches the shore, however slowly it may be moving, he will be in great danger of falling if it suddenly *ground*, because the motion which he before partook of, in common with the boat, has a tendency to permanence.

When a man jumps from a carriage in motion, unless, in the act of reaching the ground, he commence running, with a velocity at least equal to that of the carriage, he will certainly fall; for the force with which he was moving, in common with the carriage, will remain in the upper part of his body, whilst in his feet it will be arrested by contact with the ground.

It is by reason of this tendency to permanence in the force of communicated motion, that a race horse, whatever efforts he makes to stop himself,

cannot be brought up, until he has long *passed the goal;* that a man LEAPS farthest when he runs to make his leap; and that in a SHIP WHICH STRIKES when under sail, upon a rock, every thing is dashed forwards.

197. OF THE FORCE OF MOTION WHICH TENDS TO OVERTHROW A MOVING BODY, THE EFFECT OF THAT WILL BE THE GREATEST, WHICH EXISTS IN THE HIGHEST PORTIONS OF IT.

Because, there, the force acts with the greatest leverage, or at the greatest distance from the point or edge about which the whole is to be made to turn, in the act of being overthrown; and for this reason it is, that a tall person would be much more liable to fall, by reason of such a shock, than a short one: thus also, when a vessel strikes on a rock, a high mast is more likely to go by the board, than a short one — supposing its strength to be only the same. It is not uncommon for vessels thus striking to lose all their masts at once. It is for a similar reason, that a man, jumping from a carriage in motion, is in great danger of falling, the force of the motion, existing alike in all parts of his body, is suddenly arrested in his feet, whilst it carries forward the upper portion of his body, and with it his centre of gravity, beyond the limits of its natural pedestal; all that he can do to avoid this, is to *run* in the direction in which his body is thus carried forwards, so as to bring his feet beneath it; otherwise it will leave them behind.

198. Driving on the Head of a Tool.

Another illustration of the permanence of the force of motion may be found in that very common expedient of practical mechanics, by which, when they require the iron portion of one of their tools to fix itself into or upon the wooden parts of it, they put *both in motion*, and suddenly *stop* that part *into* or *upon* which the other is to be driven. Thus, to drive the head of a hammer firmly upon its handle, they place it loosely upon it, then strike the end of the handle upon the bench, arresting suddenly its motion by the intervention of the bench, by which means the force of motion in the *iron head* is made to take effect upon the handle, and the two are fixed together. The same expedient serves to drive a chisel into its handle; the handle is suddenly stopped, and by its acquired force of motion, the iron of the chisel drives itself into it.

199. The breaking of Bodies by Impact.

The force of motion exists in *every particle* of a moving body—hence, when such a body is to be brought absolutely to rest, the force of motion must be destroyed in every particle of it.

Now if a moving body be thus brought to rest by encountering an immoveable obstacle, the motion and force of motion in those parts of it immediately in contact with the obstacle will be destroyed *at once.**

* This expression is used relatively; it will be shown hereafter that force of motion can never be destroyed *at once*, according to the accurate meaning of that term.

The parts of the body immediately behind them retaining, however, the force of *their* motion, will press directly on the first—those behind these, on *them*; and so of the rest, until the momentum of each, in succession, is destroyed by the resistance of those before it.*

Of the parts which do not lie immediately behind the point or points of impact, each would, of necessity, at the instant of impact, separate itself from the rest by reason of its own proper force of motion, and move onwards, as do the particles of a mass of water dashed against an obstacle, were it not for that force, common to all solid bodies, which is called cohesion. If, moreover, the momentum of any one part of the solid be such, as the cohesion of that part to the rest is not sufficient to counteract, that part will separate from the rest, and a piece is then said to break out of it.

Sometimes the pressure which the destruction of the force of motion, in some interior portion of the body, produces in this way, overcomes the cohesion, and destroys the internal structure of that portion of the body, without affecting its *external* form and appearance. Thus, a stone after it has been several times struck against another, although there

* This entire destruction of the motion will not in reality obtain until after several oscillations of each particle for a certain distance on either side of its ultimate position of rest, to which it will continually be brought back by the elasticity of the mass, and carried through it by its acquired momentum; this last becoming, however, at every oscillation less, it will eventually rest. It is from these oscillations of the particles of bodies about their ultimate positions of equilibrium, that certain bodies become sonorous when struck.

be no external appearance of injury, will afterwards yield to a blow which would not before have broken it.

200. A Jar of the Body.

The sudden destruction of motion in the human body, is attended by effects analogous to these. Thus, a person walking carelessly, if he meet with some unevenness of the surface, and his heel come first in contact with the ground, will experience a very painful sensation of the kind called a jar; which is, in point of fact, but the indication his whole nervous system gives, of an unnatural pressure of the different solid portions of his body upon one another; resulting from a sudden destruction of the force of motion, first in his heel, then, by pressure upon that, in the bones of his leg, then in the successive vertebræ of his back, and lastly in his head—each of these having in succession its proper force of motion destroyed, by pressure upon that below it in the series. Of the same nature is the shock which a man feels whose seat is suddenly taken from under him, and it is thus that a man is stunned or perhaps crushed to pieces, who falls upon his legs from a great height.

201. The Phenomena which attend the sudden Production of Motion, are analogous to those of the sudden Destruction of it.

Thus, if a man be standing upright in a boat, which is suddenly pushed off from the shore, he

will probably fall, in the direction from which the boat is moving.

And the reason is this:—When the boat first moves, a certain force of motion is communicated to his feet which are in contact with it, and cannot slip along it, whilst no such force exists *in*, or is propagated to, the upper portions of his body.* Thus, then, his legs will be carried forwards by this force of motion, whilst his body retains its position, until by this relative displacement, the centre of gravity of the body is brought beyond the base of the feet, and he falls.

It is in the same way, that a sharp blow on a man's feet will strike them from under him; they receiving a motion in which the upper portion of his body does not partake.

Analogous to the process by which a body is broken in pieces when it is made to impinge upon an immoveable obstacle, is that by which it is broken, when, being itself immoveable, another body is made to *impinge upon it*. These are all instances of the *permanence* of the force of motion, once communicated to a body, except it be counteracted by the operation of some force from without.

Examples like these might readily be multiplied: no person, however, will be disposed to doubt the tendency of communicated motion, and the force of communicated motion to permanence, who has endeavoured *to stop himself when running*, or seen a

* Since he stands upright the force of motion in a horizontal direction could not propagate itself to the upper portion of his body without propagating itself in a direction at right angles to that in which it acts, which is mechanically *impossible*.

race-horse pulled up at the goal, or a *skater* by trusting to the mere impulse of a communicated motion, glide rapidly over fifty or sixty yards of the surface of the ice, or *a loaded carriage descend a hill*, and by the mere tendency to permanence of the force of motion communicated to it in its descent, ascend a considerable distance up the next hill, with scarcely any traction of the horses; or who has seen a *pendulum*, by the mere tendency to permanence of the force of motion which it acquires in the *descending* arc of its oscillation, complete its *ascending* arc against the force of gravity; which arc it does not terminate until, by the continual operation of that force of gravity, its force of motion is entirely destroyed, and it falls back, to re-acquire it in a second descent.

202. The Hammer.

The principle of the *permanence* of the force of communicated motion, so far as any cause within the moving body itself is concerned — that is of its absolute permanence, except in so far as it is counteracted by some external and opposite force — whilst it lies at the very foundation of all just views of the *theory*, is sufficiently shown, by the above examples, to be a most important element in the *practice* of mechanics. What is it, in fact, but this which constitutes the *giant* force of *impact*, and makes the HAMMER a weapon more powerful than any other — *irresistible* — in moulding and submitting the various objects around him to the uses and purposes of man. There is no machine comparable to the *hammer*.

The force of *heat,* indeed, insinuates itself between the pores and interstices of bodies, and operating there, *separately,* upon their particles, breaks them up in detail — but the *hammer* encounters the *accumulated* force of their cohesion and overcomes it. The *hardest* rocks and the most *unyielding* metals submit to it. If man reigns over inanimate matter, shapes out the face of the earth to his use or to his humour, and puts the impress of his skill and his labour upon the whole face of nature; it is chiefly with the aid which this mighty force of impact gives him. It is this that clears away for him the trees of the forest—that shapes for him the materials of his dwelling—that beats out for him the instruments of tillage—that digs and hoes up the earth,—that after having cut for him his corn, *threshes* it, and *crushes* it into flour,—that tames for him his cattle, shapes and binds together his waggons and carts, and makes his roads; in short there is no use of society for which this force of impact does not labour, and there is no operation of it which does not manifest this tendency of communicated force of motion to *permanence.*

Were there no tendency to *permanence* in the force of motion which his hammer acquires in its descent, its power on the substance which the artificer seeks to shape out, would only be the same as though he were to lay it gently down upon it; its impact would be no greater force than the *pressure* of its weight. So far is this, however, from being the case, that, as it is well known to the workman, a slight blow from the lightest hammer is sufficient to abrade a surface, which the direct pressure of a ton

weight would not make to yield. There is no force in nature comparable to that of impact.

203. If the Causes which tend to destroy the Force of a Body's Motion be continually counteracted as it moves on, then it will move uniformly.

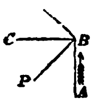
Thus, if the friction which would otherwise gradually destroy the motion of a CARRIAGE, be continually neutralised, by the traction of the horses, it will roll on uniformly. The friction of the road would not *instantly*, and at once, destroy the force with which the carriage moved, if left to itself, but by little and little. This friction is, therefore, at any instant, *less* than the force of the carriage's motion, and to overcome it, requires less effort of the horses, than to communicate, at first, its motion to the carriage. It is the *permanence* of this originally communicative force of motion which causes the carriage to move on, although the horses, at every instant, exert a much less force than that necessary to move it from rest.

204. The Tendency of the Force of Motion to Permanence is a Tendency to Permanence in that particular Direction in which the Body moves, or in which the Force acts.

Thus, if a man run rapidly, and, without at all abating his speed, so as to diminish the actual *force* of his motion, attempt to *alter* suddenly the

direction in which he runs, he will find that he has a considerable force to *resist* and *destroy* before he can do this — a force tending to carry him *straight forwards* in the path in which he was before moving — and the force which he thus has to counteract, he will find to be *greater* or *less*, as his turn is more or less *abrupt*, and the *previous force* of his motion greater or less. If he wish to turn directly at right angles to his former path, he will find that he must destroy absolutely *all* the force of his previous motion — an effort which is therefore precisely the same as though he were brought to a complete stand still; and if he has to proceed with the same speed in his new path, he will have to reproduce all this force of motion in that path.

fig. 66.



In the same manner, if his new path be in any way backwards, or making an angle less than a right angle, with the path in which he has been running, — as, for instance, if it be represented by B P, in the figure, A B being his previous direction, then, as before, all the force of his motion in A B must be destroyed, and reproduced in B P; and in point of fact, the quantity of force which he must destroy to take up a new direction is the same, whatever that direction may be, provided that it lies within the right angle A B C; being the whole force of the motion in A B.

205. Illustrations of the Tendency of Motion to Permanence, in respect to its Direction.

A man whose HORSE STARTS when he is riding rapidly, falls over his head, because his motion tends to permanence, in the direction in which he was moving. A SHRAPNEL SHELL, when it bursts, although, if it were at rest, it would scatter the bullets with which it is filled in all directions, being in motion, gives to each a force of motion, which, operating conjointly with, and modifying the forces, whose tendency is to disperse them, throws them all more or less *forwards*.

COURSING derives all its interest from the *doubling* of the hare, which finds a protection from the greater swiftness of the greyhound, in continually changing the direction of its motion — a change which the latter is less able to make, by reason of his greater weight and greater swiftness producing a greater force of motion, and the greater length of his legs rendering him the less able to check it. Independent of these causes, the principle furnishes, moreover, a protection to the pursued from the pursuer. The former may thus be made always to pass the point where the latter turns to him, unexpectedly.

206. The Measure of Momentum, or the Force of Motion.

Force being that which produces motion in a body, it is easy to conceive that that force must

be *double,* which produces *twice* the motion in that body, that *triple,* which produces *three* times the motion, that *quadruple,* which produces *four times* the motion, and so on ; — in short, that the force which produces motion in the body must be exactly proportional to the motion which that body receives. Now this force *producing* the motion, has been shown to be exactly equal to the force which the body *receives, with* its motion, and which accompanies it *, the force, in fact, *of* its motion, or its *momentum.* The force of a body's motion, or its momentum is then doubled when the body's velocity is doubled, tripled when its velocity is tripled, &c.; and by however many times you increase its velocity, or by however many times you make it less than it was, by so many times exactly do you in crease or make less its *momentum.*

Thus, in the *same* body, or in *equal* bodies, the momentum is proportional to the velocity. But how shall we compare the momenta of *unequal* bodies? Let the force of gravity be *imagined* to be extinguished, and let it be conceived that I have the power of propelling a number of equal balls with the same forces, so that they shall have exactly the same velocities. Let them all be propelled at the same instant, from different points, but in parallel directions, so as to form a *flight* of balls, all directed one way.

It is evident that these balls, all moving parallel

* It is, in fact, as though a transfer of the force took place from the moving body to the body moved ; as though it were *poured* into it like water from one *vase* into another,

to one another, with the same velocity, and towards one direction, will retain exactly the same relative distances; each ball will remain always at the same distance from the neighbouring balls, not at all altering its position amongst them as they all move forward together; so that if it be conceived that I could throw over these balls some hidden spell or power of resistance, which, without adding to their mass, should *bind* them altogether: if, for instance, I could *freeze* them into one continuous mass; then in the act of thus uniting them, since they had before no tendency to separate, I should not add to, or take away from, the force with which any one of them was moving; and the aggregate force of their motion in this *united* state would be the same as it was in their separate and divided state.

But what was this *aggregate* of their forces of motion, when they moved separately?

Their masses were all equal, and all moved with the same velocity; they moved, therefore, each with the same force of motion, and the aggregate of their force of motion was as many times the force of motion of one, as there were bodies. The aggregate of their forces of motion *now that they are united*, is therefore as many times the force of motion of one of the component bodies of the mass, as there are such bodies. Thus, if there were twice the number of the same component bodies in the mass, or if it were of twice the size, then would the aggregate force of motion in it be twice what it was before; if it were three times the size, its force of motion would be thrice, and so on.

Thus, then, if there be two masses, one of which contains double the quantity of matter that the other does, and they both move with the same velocity, then the one will have double the force of motion of the other; if the one have triple the mass of the other, it will have triple the force of motion, and so on.

On the whole, then, it appears that when *equal* bodies move with *different* velocities, their forces of motion are proportional to their *velocities*; and that when *unequal* bodies move with the *same* velocity, their forces of motion are proportional to their *masses*. From this it follows, by a well known principle of proportion, that when the masses of the bodies, and their velocities, are *both* unequal, their forces of motion are proportional to the *products* of their masses by their velocities. Thus, if there be two bodies, one of whose masses is represented by the number 12, and the other by the number 8, and the first have a velocity of 3 feet per second, and the other a velocity of 9 feet, then the force of motion in the first would be to that in the second as 12 multiplied by 3, to 8 multiplied by 9, or as 36 to 72. So that the lesser body by reason of its greater velocity, would have no less than twice the force of motion that the greater has, or move with twice the foree that it does. The *mass* of a body is proportional to its *weight*: thus then, the force of its motion is proportional to its weight, multiplied by its velocity. Thus, if there be a cannon ball of 20 lbs. weight, which flies with a velocity of 1200* feet per second, and a

* The velocity of a cannon ball when it leaves the mouth of

ship of 100 tons weight, which moves through the water at the rate 6½ feet per minute, it may easily be calculated that the force of motion in the ship, moving thus so slowly that its motion would scarcely be perceptible, would yet a little *more* than equal the force of motion in the cannon ball, thus flying with its swiftest motion, and bearing with it its most destructive force. The velocity of the ball is 72,000 feet per minute, and its weight being 20 lbs. its force of motion is represented by the number 1,440,000. The velocity of the ship is 6½ feet per minute, and its weight being 224,000 lbs. its force of motion is 1,456,000.

Great force of motion may be thrown into a *small* body or a *large* one; in the former case it will give *great* velocity, in the latter, *little* velocity. *Conversely*, if *great* velocity be thrown into a *small* body, although small itself, it will have *great* force of motion; and if *small* velocity be given to a *great* body, notwithstanding the smallness of the *velocity*, the force of the motion will be very great.

This fact of the dependance of the force of a body's motion, partly upon the velocity with which it moves, and partly upon its weight, is one of which almost every case of motion presents an illustration. A LARGE SHIP moving so slowly that it can scarcely be seen to move, yet by the great amount of the motion distributed through its *great mass*, crushes to pieces any obstacle that intervenes between it

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the cannon, varies from 1600 to 2000 feet per second, it loses by the resistance of the air about 800 feet in the first 1500 feet of its flight.

and the shore. A CANNON BALL of comparatively small dimensions, by reason of the great *velocity* of its motion, bears with it a force which, after struggling in a fierce and unceasing contest with the air in its path, and again and again striking and rebounding from the surface of the earth or the water over which it flies, hurls destruction on some spot which may be miles distant from the cannon's mouth.

If a *blow* were struck by a sledge hammer on a THIN PLATE laid on a man's chest, the force of motion transferred to the plate would, by reason of its small weight, give it a great velocity, and it would probably be driven into the man's body. But if the same blow had been struck on an ANVIL laid in like manner upon his chest, it would scarcely have been felt, for the same force of motion diluted over the great mass of the anvil, would produce in it a velocity as greatly less than that in the plate, as its weight was greater. Whilst force of motion may thus be so diluted, by diffusing it through a large body, as to produce no sensible effect, it may on the contrary be so condensed in a small body as to become irresistible in its action.

207. A Plate of soft Iron may be made, by the Force of its Motion, to cut through the hardest Steel.

If a circular plate of soft iron be made to revolve with great rapidity, the *force of motion* in each particle on its circumference will become so great, that if a piece of hard steel—a steel *file* for instance

—be held against it, the particles of this hard cohesive substance will be driven away by those of the soft iron, and it will be cut through as by a knife.

208. The Art of the Lapidary.

The lapidary, by means of a crank, moved by his foot like a lathe, causes a horizontal rod or tool, with a small circular disc or button of soft iron at its extremity, to *revolve* rapidly round its axis. On this soft iron disc, thus revolving, a mixture of fine *emery* and water, and in a certain stage of the engraving, *diamond* dust is continually dropping. The fine angular particles of this powder fixing themselves in the interstices, it would seem, of the iron, are swept round by it with great velocity and driven against the surface of the stone which is to be engraved, and which is held against the tool by the lapidary. It is thus cut with ease and engraved.

209. When a Body's Motion is arrested the whole Force with which it moves is made to act upon the Obstacle.

Thus the effect, to crush an obstacle, is proportionate to the force of motion in the moving body. A *heavy ship*, although it moves but slowly, would break down an obstacle, against which a *boat* might dash with violence without injuring it. On the other hand, a heavy mass of some cwts. may be *slowly* allowed to descend upon the surface of a table without indenting it, whilst the *blow* of ever so slight a hammer would be sufficient to *abrade* an equal surface to that on which the other rests.

210. The Impact of Bodies.

If one body, moving with a certain force of motion *inpinges* upon another at rest, but free to move, it *transfers* to it a portion of its own force of motion; so that *in the two together there is afterwards as much of this force as there was before in the one*, and the *force* of motion thus being, as it were, diluted through a larger mass, the *actual* motion of each body, must be in the same proportion less. If the two bodies after impact move on *together*, so as both to have the same motion or velocity, then the force of motion being the same now in the *two* that it was in the *one*, the product of the velocity now, by the quantity of matter in the two, must equal the product of the velocity *before* by the quantity of matter in the *one*. Thus if the bodies weigh respectively nine and eleven pounds, and the first impinge upon the other with a velocity of seven feet per second, or with a force of motion represented by the number 63; then, after impact, this force of motion being distributed through the two bodies, having together a mass of 20 pounds, the common velocity of this mass must be such, that its product by 20 shall equal 63; that is it must be $3\frac{3}{20}$ feet per second.

If a body in motion overtake another, also in motion in the same direction, and carry it along with it, then the force of motion in the two, after impact, will equal the sum of their two forces of motion before impact. Thus, if, in the last example, the second body had been *moving* with a velocity of 5 feet per second, so as to have a force of mo-

tion represented by 55, then, before impact, the sum of the forces of motion of the two would be represented by 118; and all this they will have *after* impact; only then it will be so distributed that they shall have a *common velocity;* this common velocity must then be such that, multiplied by the sum of their weights, or 20 pounds, it may equal 118. Their common velocity must then be $5\frac{9}{10}$ feet per second.

If, instead of one of the bodies *overtaking* the other, they had *met;* that which had the least force of motion would have destroyed the whole force of the motion of the other, losing, at the same time, itself, as much as it destroyed; so that, on the whole, after impact, there would only be, in the two, a force of motion equal to the difference of what was in the two before. Thus, taking the last example, and supposing the balls to move in opposite directions, and to meet; since *before* impact, their forces of motion were represented by 63 and 55, *afterwards* their remaining force of motion will be represented by the difference of these numbers, or by 8.

This *remaining* excess of the force of motion in the one body, will carry along with it the other, distributing itself equally through the *two.* Thus, then, the *two* whose united weight is 20 pounds, will after impact move with such a velocity that their force of motion is 8; this velocity must then be $2\frac{1}{2}$ feet per second.*

* The whole of the conclusions in this article depend upon the supposition of the entire absence of elasticity in the impinging body; the condition of elasticity greatly modifies them.

211. The Recoil of Fire-Arms.

The elasticity of an elastic fluid, such as the air or a gas, exerts itself *equally in all directions.* Thus, in the discharge of a cannon, which is but an effect of the elasticity of the gas liberated by setting fire to the gunpowder, this elasticity is made to act equally towards either side, and towards the muzzle and breech, of the cannon. The cannon does not *move* sideways, although an immense force is thus made to act sideways upon it, because the gas, expanding *equally* in all directions, acts with *equal* expansive forces on its two sides, and in opposite directions, so that these two equal and opposite forces *neutralise* one another, unless the strength of the cannon yields to either of them, and it bursts. The two forces acting towards the *sides* of the cannon being thus *neutralised,* there remain only those which act towards the muzzle and breech. These two would counteract and neutralise one another, if the mouth of the cannon were completely and effectually secured, but it is *not;* the ball and the wadding, however firmly driven, *yield;* the expansive forces of the gas towards the muzzle and breech do not counteract and neutralise one another, as do the other two; both of them take effect; and, being equal, they produce an *equal* effect; the one upon the ball, and the other upon the cannon. Thus the cannon and the ball

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The subject, however, under this form can only be discussed in theoretical treatises, to which the reader is referred for further information.

receive from the explosion equal* forces of motion; the one backwards, and the other forwards. The former is the force of the *recoil.*

If the weight of the cannon were only *equal* to that of the ball, having the same force of motion, it would have the same velocity that the ball has, and the two would, in fact, fly, in opposite directions, equal distances. But the cannon is greatly heavier than the ball; the same force of motion in it, produces, therefore, greatly less velocity, and the less as this disproportion is greater. Thus a light gun recoils greatly more than a heavy one. The effect of the recoil of the guns of a ship of war falls ultimately on the vessel herself. Thus a broadside causes her to heel towards the opposite side, and if she is chased, guns fired from her stern will accelerate her flight.

212. To fire from solid Cannon.

It has been proposed to replace cannon *balls,* by pieces of iron with cylindrical apertures cast in them, and *cannons* by solid cylinders of iron, on which these apertures fit. A cartridge being placed in this aperture, and the aperture then fitted on the solid cylinder, the cartridge would be fired through a touch-hole, and the missile thrown off by its *recoil.* The force of motion produced in this missile would certainly be the same as though the cartridge were exploded in a cannon, loaded with a ball of equal weight. The idea is exceedingly

* The cannon is here supposed to run without friction upon the wheels of its carriage.

ingenious, and the method presents advantages well worthy of consideration.

213. The Recoil of a Cannon does not become sensible until the Ball has left its Mouth.

This was first proved in an experiment made at Rochelle, in 1667, by order of the *Cardinal de Richelieu.*

A cannon was fixed in a horizontal position at the end of a long vertical shaft or rod, moveable freely about an axis, at its other extremity. The ball fired from it under these circumstances struck the object towards which it was directed, precisely as it would have done if the cannon had been fixed, showing that there was no sensible alteration of its position until the ball was discharged from it.

214. To determine the initial Velocity of a Cannon Ball.

It is evident that, by observing the velocity communicated to the cannon in the first instant of its recoil, the velocity with which the ball leaves it may be determined. For the weight of cannon, multiplied by the initial velocity of its recoil, represents its force of motion when the ball leaves it, which is equal to the *ball's* force of motion. And dividing the ball's force of motion by its weight, we evidently get its velocity. This method was used by Mr. Robins, and the results are given in his treatise on Gunnery. To determine the initial velocity of the recoil, it is only required to observe the *height* through which the cannon when

suspended, as described in the last article, is made by its discharge to oscillate. The velocity is that which a body would acquire by falling *freely* through *that height,* and is therefore easily determinable, as will be shown in a subsequent part of this work.

215. The Ballistic Pendulum.

To determine the velocity of a cannon ball *directly,* it is *fired* into a heavy mass of wood, suspended from a long iron bar. The *height* to which this mass is by the blow made to oscillate, is shown by an index on a wooden arc, which forms part of the apparatus, and determines the *velocity* with which the mass *first* began to move, when its force of motion was equal to that with which the ball struck it. From this consideration the latter is easily calculated, and the *force of motion* of the ball being thus known, as also its weight, its *velocity* is at once ascertained by dividing the former of these by the latter. There is sometimes used a simple contrivance by which the pendulum is made itself to register the height of its oscillation.

216. When a Body moves only with a Motion of Translation; that is, when all the Parts of it move with the same Velocity and in the same Direction, there is a certain Point in it, in which the whole Force of its Motion may be supposed to act. That Point is the Centre of Gravity.

If *all the parts* of a body move with the same velocity, or if it move only with a motion of *trans-*

lation and do not rotate, as for instance, a ball which flies through the air without turning round, or a heavy mass which falls to the earth without turning upon itself,—its force of motion will be distributed through its parts in proportion to their *weights;* for the velocities of all the parts being the *same*, it is evident that the quantities of the force of motion in the different parts, must be proportional to these weights. Now, the forces of motion are by supposition all *parallel* to one another, as the weights are, and it has been shown that they are all *proportional* to the weights; they are therefore a system of forces distributed through the body precisely as the weights of its parts are, and acting upon it precisely as they do. At whatever point then a *single* force would sustain the one system of forces, it would sustain the other: that is, a single force would support all *the forces of motion* of the parts of the body at the same point, where it would support all their *weights*, or at its *centre of gravity;* and therefore all these forces of motion produce the same effect, as a *single* force of motion equal to their sum would do, if it were made to act through that point.

THE CONVERSE OF THE PROPOSITION STATED IN THIS ARTICLE IS ALSO TRUE, THAT IS, "IF THE FORCE OF A BODY'S MOTION BE THE SAME AS THOUGH IT ALL ACTED THROUGH ITS CENTRE OF GRAVITY, THEN IT WILL MOVE ONLY WITH A MOTION OF TRANSLATION, OR IT, WILL NOT ROTATE AS IT MOVES ON." From this it follows, that a solid body, descending freely and exclusively by the action of gravity, will de-

scend with a motion of translation only, and will not turn upon itself, or rotate as it descends; for the force of such body's motion being the aggregate of the gravitations of its parts, must evidently have its direction through the body's centre of gravity. Thus too, a body to which its force of motion is communicated by *an impulse through its centre of gravity*, will move, only with a motion of translation, and will not rotate.

217. The Symmetry of Tools.

It is for the reason assigned in the last article, that *tools*, especially those of impact, are made *symmetrical*, about a certain plane passing through those points or surfaces by which, and parallel to the direction *in* which, they act. Thus, for instance, an *axe*, acting by its edge, is made symmetrical, about a plane passing through its edge: the handle of a *chisel* is symmetrical, about a plane in like manner passing through its edge; the heavy stone-mason's chisel is symmetrical, about the *end* by which it acts; and a *nail* about its point. A *hammer* is symmetrical, about a plane passing through its striking surface, and the same is true of a *cricket-bat*, *forge-hammer*, a *pile-driver*, &c.

None of these tools would strike straight, if this symmetry were not observed.

The reason of this is, that the *centres of gravity* of all those bodies (and all others), are in their planes of symmetry; their *forces of motion* producing the same effects as though they acted only in their centres of gravity, produce the same effect

as though they acted, therefore, exclusively in these planes of symmetry; that is immediately over the *cutting* or *striking* point, or line, or surface of the tool. If their centres of gravity were on either side of these *striking* parts of the tools,—that is, if the planes of symmetry did not go through the *striking* parts, the *force of motion* of the tool would produce an effect as though it acted on one side or other of the striking part. It would therefore *deflect* the direction of the blow, and its *effect*.

218. IF A BODY HAVE AN IMPULSE COMMUNICATED TO IT WHOSE DIRECTION IS NOT THROUGH ITS CENTRE OF GRAVITY, THEN WHEN MOVING FREELY BY REASON OF THIS IMPULSE, ITS MOTION WILL PARTLY BE ONE OF TRANSLATION, AND PARTLY OF ROTATION, BUT SUBJECT TO THIS REMARKABLE LAW: "THAT ITS MOTION OF TRANSLATION WILL BE THE SAME AS THOUGH THE IMPULSE HAD BEEN COMMUNICATED THROUGH ITS CENTRE OF GRAVITY, AND THERE HAD THUS BEEN NO ROTATION; AND ITS MOTION OF ROTATION THE SAME, AS THOUGH ITS CENTRE OF GRAVITY HAD BEEN FIXED, AND IT HAD REVOLVED ROUND IT THUS FIXED, SO THAT THERE COULD BE NO TRANSLATION.

These remarkable properties are proved by analysis, and can only here be *enunciated.* (See *Pratt's Mechanical Philosophy*, pp. 458, 459.) The following are illustrations of them: —

CHAIN SHOT. — Two cannon balls fastened to-

gether by a strong chain, and fired from the same cannon, are found to constitute a fearfully destructive missile, sweepng down at once whole ranks of men. This is easily explained. The *impulse* which the balls receive on leaving the cannon does not, except by a rare accident, pass through the common centre of gravity of the two balls and the chain; by the property stated at the head of this article, *two* motions are therefore of necessity communicated to the system, one of *rotation* about its centre of gravity, and the other, of *translation;* and these do not *interfere* with one another; so that the balls fly forwards as far as though they did not revolve*; and they revolve as they would do if they did not fly forwards. The *rotation* thus produced in the balls causes them to recede from one another, by what is called centrifugal force, (art. 233.) and *distends* the chain. Thus, as they fly forwards, they sweep continually with a rapid revolution over a circle, whose diameter is equal to the length of the chain, increased by the diameters of the two shot; and over the whole of this space they carry destruction with them.

Double headed shot, instead of being joined by a chain, are connected by a strong iron bar. The theory of their motion is the same. These have now, we believe, *superseded* chain shot.

* The effect of the resistance of the air is not here taken into account.

219. THE DOUBLE MOTION OF THE ROTATION AND TRANSLATION OF THE EARTH.

Every analogy of nature points to an *ecomony* of creative power. Reasoning, therefore, on the two motions of rotation and translation, which we find in the earth's mass, and of which the origin is to be traced to the period when it first moved through space, in the path in which it now moves, and God "divided the day from the night;" we must come to the conclusion that the mighty *event* of this epoch, was the result of a *single impulse* — of *that* single impulse which having its direction, *not* through the *centre of gravity* of the earth, would have been *sufficient* to produce the amount of these two motions of rotation and translation which we find the earth to have.

The distance from its centre, where this single impulse must have been communicated, has been calculated by John Bernouilli. Supposing the earth's mass to be homogeneous, he finds it to be 165th part of the radius, or about 24¼ miles from its centre. Similar calculations applied to the other planets, give, for the distances from their centres, at which the single forces which have given them their existing motions of rotation and translation must have been struck — for Mars $\frac{1}{412}$ths of his radius, for Jupiter $\frac{7}{19}$ths, for the Moon $\frac{1}{160}$ths.

220. To cause a Ball to move forwards a certain Distance upon a horizontal Plane, and then, although it meets with no Obstacle, to roll backwards.

This remarkable effect will be produced, if the ball, lying on a perfectly horizontal table over which a cloth is tightly stretched, be struck downwards, not *through* its centre, but on that side of it which is *from* the direction in which the ball is first to move. To explain this, let it be observed, that the ball in being thus struck, not through its centre of gravity, but on one side of it, receives two motions, (art. 218.), one of rotation, and the other of translation, the latter being the result of the *displacement* of the ball sideways, by the descent of the hand, and the former the *direct* effect of the impulse — moreover that the direction of the bodies rotation, is the opposite of that which it would have, if it *rolled* in the direction in which it is *actually* transferred by its motion of translation, so that it in fact *slides* forwards, *rotating* as though it would roll back. To both these motions, of sliding and rotation, the *friction* of the table opposes itself, and whichever of the two is destroyed by it first, will leave the other to take effect *alone*. Now, it is pretty evident, that since the rotation is the effect of the *direct* blow, whilst the translation is only that of the *indirect* displacement: the force of the former must, if the blow be properly struck, be much greater than the latter; so that the body's

force of *translation* will be destroyed by the friction much sooner than its force of *rotation* is destroyed.

The ball's motion of translation, or sliding motion, being thus destroyed, and its force of rotation remaining, whose direction is *backwards*, it will evidently roll back.

221. The Radius of Gyration.

When the motions of all the parts of a body are not *equal* and *parallel*, the *resultant* of all their forces of motion passes no longer through the body's centre of gravity. If the motion be round a fixed axis, so that all these parts describe *circles* about that axis, their velocities, and therefore their several forces of motion, will be proportional to their several distances from it. All these forces of motion will produce the same effect as a *single* force acting in a circle round the same axis, whose radius is called the *radius of gyration*, the discussion of the properties of which belongs to analysis, but whose length may be determined from an experiment hereafter to be described.

If a *straight line* be made to revolve about its centre, its radius of gyration is equal to half its length, divided by the square root of 3.

If a *cylinder* be put in motion about its axis, its radius of gyration will equal its radius, divided by the square root of 2.

If a *circular plate* be put in motion round one of its diameters, its radius of gyration will equal one-half the radius of the circle.

If a *sphere* be put in motion about one of its tangents, its radius of gyration will equal its radius multiplied by the square root of the fraction $\frac{7}{5}$.

222. A Body in Motion about a fixed Axis which encounters an Obstacle at a Distance from its Axis, equal to the Radius of Gyration, will expend all the Force of its Motion on the Obstacle. If it encounter it at any other Point, the Force will be divided between the Obstacle and the fixed Axis.

This is evident, since the whole force of the motion of the parts of the body produces the same effect as though it were *collected* at the distance of the radius of gyration. If the *obstacle* be *not* at this distance, this collected force will evidently act both upon the fixed axis and upon the obstacle, and will be divided *between them*, on the principle of a weight supported between two props. Thus also it appears that a body revolving round a fixed axis, and encountering an obstacle at the distance of its radius of gyration, does not in the act of impact produce any impulse or percussion upon its *axis*.

223. A Cricket Bat.

A cricket bat, when the ball is struck by it, may be considered to be revolving round an axis near the shoulder of the player. The whole force of its motion will therefore expend itself on the ball, only when the latter is struck at the point in it, which is at a distance from the shoulder, equal to the radius of gyration of the system, made up of the bat and the arms of the player; and it is at this point, and at this point only, that when the ball is struck, there

will be no *reaction* of the blow upon his shoulder. Expert players soon learn to know about what point of a bat they thus strike most effectually, and in this consists a great secret of good batting.

224. Tools of Impact.

In speaking of its centre of gravity as the point where the impact of a hammer, &c. may be considered to take place, we have supposed all its parts to move with the same velocity. In reality they do not. In the act of impact the instrument is turning round an *axis*, the points more distant from which are revolving more rapidly than those nearer to it. The point in which all the force of its motion may be supposed to be collected, is in reality at the distance of the *circle of gyration* from the axis. Thus a carpenter's *mallet*, in which the parts farther from the handle evidently move faster than those nearer to it, so that the greater portion of the force of motion is collected about the *end* of it, if he strike with it at its middle point will not produce its greatest *effect*, and will *sting* his hand; the true point is determined by the radius of gyration. A similar remark applies to the use of the forge or *tilt* hammer.

225. THE FORCE OF A BODY'S MOTION DEPENDING UPON ITS VELOCITY, IT IS EVIDENT THAT WHEN THE BODY IS MADE TO REVOLVE A CERTAIN NUMBER OF TIMES IN A MINUTE, ROUND A FIXED AXIS, ITS FORCE OF MOTION WILL BE GREATER, AS IT REVOLVES AT A GREATER DISTANCE FROM THE AXIS, OR IS CONNECTED WITH IT BY MEANS OF A LONGER ARM.

If, for instance, an axis be revolving a certain number of times per minute, and two equal balls be connected with it by arms of unequal lengths, then that which is attached to the longer arm will have the greater velocity, and therefore the greater force of motion.

From this it follows, that by varying the distances of the parts of a revolving body from its axis of revolution, we may vary greatly the force of its motion, provided we do not vary the velocity of its revolution; and conversely, that if we vary the distances of a body's parts from the axis of rotation, not varying its force of motion, we of necessity vary its velocity.

226. THE DIMENSIONS OF THE EARTH HAVE NOT DIMINISHED FOR THE LAST 2500 YEARS.

For, no obstacle being opposed to the force of motion with which the earth rotates, that force must be the same now that it always was. But if by the contraction of the earth's mass, its parts are brought now nearer to the axis about which it rotates, than

they were formerly, it is clear that these, revolving at a less distance, must, to have the same force of motion, revolve *faster*. So that if the earth's dimensions had contracted, the *day* would now be shorter than it was. Now we have observations which show, that the day is now, precisely of the same length that it was, 2500 years ago. None of that diminution of bulk from the cooling of its mass, of which geologists speak, can therefore have taken place, with any perceptible influence within that period.

227. The Compensation Balance Wheel.

The balance wheel of a watch is that which supplies in it, by the isochronism of its vibrations, the place of a pendulum. It receives by means of a contrivance connected with it, called the 'scapement successive impulsive motions, through a train of wheels from the main spring of the watch; and after each impulse it is brought back by a fine hair spring, which is fixed to the axis about which it turns, and may be seen coiled round in its centre. The escapement is so contrived, that no second impulse can be given to the balance wheel, until it has vibrated back into the position where it received its first impulse, and until this second impulse is given, the watch cannot go on. Thus ultimately the whole regularity of the motion of the watch is made to be dependant upon the regularity of the vibrations of this balance wheel. Now it was found by theory and confirmed by experiment, that the vibrations of a body to which a spring was attached, as in the case of this wheel, were performed in the same time, however

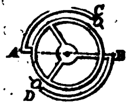
great they were, within certain limits,—that is, however great, or however short, the distance through which the body was drawn back from its position of rest before it was left to itself, it yet returned to its position of *rest* in the same time; the greater *force* of the spring, when farther uncoiled, exactly making up, for the greater space through which it had to move the wheel; so that in the watch, whether the impulse given to the balance wheel was small or great, it would yet vibrate back, always in the same time. Thus then, whatever irregularity there might be in the action of the *main* spring of the watch, and in the working of the train which connected it with the balance wheel, so that this should receive at one time a more violent impulse than at another; yet none of this irregularity would find its way into the actual going of the watch, governed as it was by the duration of the vibrations of the balance wheel, which, under all these circumstances of irregularity, would yet be of equal duration.* The actual length of each one of the isochronous vibrations of the balance wheel, is dependant first upon the *length* of the spring, and secondly on the *dimensions* of the balance wheel; the *force* of the spring being dependent upon the former cause, and the *velocity of the motion,* communicated to it at each impulse and to be destroyed by the action of the spring, on the latter. By varying either of these elements, the time of the vibrations may be varied as we like. That which is usually varied is the

* To this isochronism of the vibrations of the balance wheel, it is necessary that the length of the spring should be so great, that at each vibration it should not be *greatly* uncoiled,

length of the spring, of which, more or, less is set free to vibrate, by a contrivance which is generally visible, and which is easily understood.

Both the length of the spring, and the dimensions of the balance wheel, are however, of themselves, made to vary, by variations in the *temperature*; and both these are, for the reasons we have stated, causes of error in the going of the watch. The compensation balance wheel is a contrivance, by which they are made to compensate one another. It consists of a wheel, to two extremities, A and B, of a diameter of which, are fixed two curved arms or branches, A C and B D. These curved arms, A C and B D, are each formed of two bands of metal, soldered together; one of which bands, that forming the *convex* surface, is of brass, and the other, forming the *concave* surface, of steel. Now, an increase, of the same degree of temperature, causes brass to expand much more than steel; any increase of temperature will therefore, cause the outside surfaces of these two arms to lengthen, much more, than the inner surfaces of them; this can only happen by the *curling* up, as it were, of each arm, at that extremity which is free to move. Thus, by the turning in of these extremities of the arms, the material of the wheel is brought nearer to the centre, about which it revolves; an alteration in its form, which produces an immediate change in the time of its vibrations, compensating for the elongation of the arm A B, and the increased length and diminished elasticity of the hair spring.

fig. 67.



228. THE CENTRE OF SPONTANEOUS ROTATION.

If a force be made to act impulsively, on a body at rest, but free to move in any direction; in the instant of impact, certain forces of motion will be communicated to all its parts. If the blow be struck through the centre of gravity, it has been before shown, that all these forces of motion will be *equal* and *parallel;* but if it be not struck through the centre of gravity, the body will move, partly with a motion of translation, in which all its parts partake equally, and partly with a motion of rotation about its centre of gravity. Whilst, by this rotation, some of the parts of the body have a tendency to be carried *backwards;* by the motion of translation, these, and all the other parts of the body, are carried with a direction *forwards.* And if this rotation of any of the parts backwards, *exceed* their motion of translation forwards, then, whilst the rest move *forwards,* these parts will actually move *backwards* in space; a fact which any body may verify, who strikes near one of its extremities, a piece of wood, lying on a smooth surface, or floating in water. Tracing the different parts of the body, from those which thus move forwards after the blow, to those which move backwards, we shall evidently arrive at a point, or rather an axis, where the motion *passes,* from the *one* direction to the *other;* which point does not, therefore, move, either forwards or backwards; this axis is called the axis of *spontaneous rotation.* It is that about which the body tends, in the first instant of its motion, of its

own accord, to revolve. An analytical expression for the position of the axis of spontaneous rotation is easily found.* From this expression, it results that the axis is more remote from the point where the disturbing force is applied, as the centre of gravity is more distant from that point, and as the great mass of the body is more distant from its centre of gravity. Thus, in a body of considerable length, from the point of application of the disturbing force, it is more distant than in a shorter body; and in a body, the greater part of whose mass is collected near its extremity, it is more distant than in one whose mass is *uniform*, or which is lightest at its extremity.

229. THESE FACTS EXPLAIN THE EASE WITH WHICH A LONG POLE OR A LADDER MAY BE BALANCED ON ITS EXTREMITY, AND WHY EITHER OF THESE WILL BE YET MORE EASILY BALANCED IF IT IS LOADED AT THE TOP.

The axis of spontaneous rotation is that line in the body which *rests* when it is slightly displaced, or made to revolve through a small angle. If the action of the disturbing force be *continued*, after this small angle is passed, this axis will be carried forward with the rest of the body. Now, in balancing a body on its extremity, when we move its lower extremity to preserve the equilibrium, we cause the whole to revolve about its axis of spontaneous

* A practical method of determining the centre of spontaneous rotation will be given when we come to speak of the centre of *oscillation*.

rotation; and the higher this axis is, the farther we can move the lower extremity, without inclining the body round its axis, beyond this small limiting angle; so that in fact, when the axis of spontaneous rotation is *very* high above us, it remains stationary, or nearly so, notwithstanding that we give considerable motion to the lower extremity of the body, thus greatly facilitating the efforts we make to preserve its equilibrium.

Thus is explained that common feat of posture masters, by which they balance a long ladder, with the lowest stave resting on their chins; they even move about with it thus balanced; and have been known to do it, carrying one of their companions at the top.

230. The Centre of Percussion.

Since a blow communicates no motion, and therefore no force of motion to those particles of a body which lie in its axis of *spontaneous rotation* (in reference to that blow), it is evident that if a fixed axis were made actually to pass through the body, where its axis of spontaneous rotation passes through it, the blow would communicate no tendency to move, and therefore no *percussion*, to that axis. Now the position of the axis of spontaneous rotation is evidently dependant on the place in the body where the blow is struck; and, since the blow may be struck in an infinite number of different places, the axis of spontaneous rotation may be made to occupy an infinite number of different positions, and thus may be made to coincide, in one of these positions,

with a particular axis, before determined upon: a particular point of impact thus becoming necessary to cause the axis of spontaneous rotation to coincide with a particular axis.

This point of impact is called the *centre of percussion*, in respect to that particular axis. If the body be suspended from that axis, and struck upon that point, there will be no re-percussion on the axis, and it is the only point in the body possessing this property. The ballistic pendulum (art. 215.) presents an application of this principle. If the ball strike the mass against which it is fired at any other point than its centre of percussion, the blow will tend to tear away the axis.

232. The Centres of Suspension and Percussion are convertible.

If the centre of percussion of a body about a certain axis be found, and that axis be then changed for one passing through what was its centre of percussion, then its new centre of percussion will be in what was before its axis of rotation, so that the two are *convertible*. This is a very remarkable property, of which many important applications may be made, as will hereafter be shown.

232. The Tilt Hammer.

The tilt hammer is that used in the forging of steel (see art. 82.). It is of great weight, and is fixed to a strong arm, commonly a beam of wood, of considerable length, near whose opposite extremity, is a horizontal iron axis moveable in collars,

which are firmly bound down to a solid mass of iron and masonry, deeply imbedded in the earth. The hammer is raised by the action of a wheel commonly turned by water-power, on the circumference of which are fixed at equal intervals *cogs*, which are made to strike on a projection of the extremity of the arm of the hammer. The hammer is thus made rapidly to rise and fall, and a rapid series of impulses is given to the bar of steel which is placed on an anvil beneath it. The expense of erecting and maintaining one of these hammers is exceedingly great: they are extremely liable to break their axes, and to tear away their collars. That there might be no percussion upon the axis, when the hammer receives the blow which lifts it, it would evidently be necessary, by what has already been said (see art. 230.), that this blow should be struck at the *centre of percussion* of the hammer. That there should be no *re-percussion* upon the axis and collars, when the hammer *gives* its blow to the steel, it is necessary (see art. 222.) that it should give this blow at a distance from the axis, equal to the radius of gyration. The hammer might be made of such different *forms*, as to satisfy these conditions under a great variety of different circumstances, and some of these might be such, as not at all to interfere with the usual method of working it. Some practical knowledge of the expediency of such an arrangement appears to have been arrived at by the workmen, and there is professed to be much skill exercised in the erecting of these hammers. In reality, however, they appear all, to expend a large portion of the power which

raises them, in beating about their axes, and in perpetual efforts to tear away their collars.

233. Centrifugal Force.

The tendency of the force of a body's motion is to carry it forwards, in the same straight line in which at any time it is moving (art. 204.); if therefore it do not continue to move in that line, there must be some force or another controlling that tendency, and deflecting it from that path. That portion of the body's force of motion which is subdued by this deflecting force, is called its centrifugal force.

It is subdued by an effort perpendicular to the straight direction in which the body has a tendency at each instant to move, that is, to the tangent to the curve in which the body is moving: its direction is therefore perpendicular to the tangent, at the point where the body is at that instant moving; that is, it is perpendicular to the line of the curve itself at that point.

234. The Amount of Centrifugal Force.

The centrifugal force being equal to that which subdues the force of a body's motion from a straight to a curvilinear direction, must manifestly be greater as the force of the motion is greater, and less as it is less.*

There is a striking example of the effect of high

* It varies as the square of the angular velocity, and as the radius of curvature conjointly; thus, if α represent the angular velocity, and R the radius of curvature, the centrifugal is represented by α^2 R.

velocities in increasing the amount of centrifugal force in the frequent rupture of the GRINDING STONES used for the grinding of cutlery. These, although the stone which composes them is of great cohesive power, yet by reason of the rapid revolution which is given to them, are often shattered to pieces by the centrifugal force which results from it. Large fragments of the stone have been known to be carried through the roof of a building and hurled to a considerable distance from the spot where it was worked. The wreck produced by the disruption of one of these stones, resembles nothing more than the bursting of the boiler of a steam-engine.

235. A Sling.

If a stone is *whirled* rapidly round, at every instant of its circular motion, it tends to *continue* to move in the straight line, in which, during that instant, it may be considered to be moving; of straight lines similar to which, the whole circumference of the circle may be considered to be made up, and of which any one, being produced, is a tangent to the circle. To keep it from moving in that path, a certain other force must be combined every where with the force of its motion, the two together having a different direction from either separately. That other force is supplied by the tension of the string. This tension of the string is thus a force necessary to keep the body from moving in that straight line in which it continually *tends* to move, and if the tension of the string be taken away, it *will* move in that line. Thus when

the string is *unloosed,* at the instant when the stone is ascending in one of its gyrations, it ceases, at once, to move in a curved line; and by reason of the tendency to permanence of its force of motion, pursues the right line which is a tangent to the curve at the point in which at the instant of its release it was moving; and this right line in which it was moving, being directed upwards, it describes the same sort of curve as a stone thrown upwards by the hand, or a ball fired upwards from the mouth of a cannon. The mechanical advantage of using a sling, rather than the hand, is this, that by the interposition of the sling, it is possible to communicate to the stone a very rapid motion, and a proportionately great force of motion, with a comparatively small and slow motion of the hand; whereas, to throw the same stone from the hand itself, you must necessarily give to the hand at the instant when it discharges the stone, a motion as great as the stone is to have, and a force of motion much greater. Thus, by means of the sling, you produce the required force of motion in the stone with much less effort than would otherwise be necessary.

236. A Man running in a Circle.

Illustrations of the fact that if a body move in a curved line, some other force than that of its motion, or than any force in the direction of its motion, must act upon it, might be multiplied almost without number.

Let us take the following:—If a man runs in a curved line, he becomes at once conscious that a certain muscular effort of that foot which is on

the convex side of his path, greater than that of the other, is necessary. Thus it is that, in respect to the lower portion of his body, the force requisite to deflect it from the rectilinear path, in which it every where tends to proceed, is supplied. But, the upper portion of his body has also a certain force of motion tending to carry it forward in the same right line, and some other force must combine with this, in order to produce its deflection from that line, otherwise, although his legs might accurately enough proceed in the curve, the upper portion of his body would pass off in a tangent to it, and thus the man would be overthrown. He might supply this deflecting force to the upper portion of his body, by a direct muscular effort, propagated from the base of the feet. But he is taught instinctively to economise his muscular efforts, and does so by every conceivable means; and in this case he does it, by causing the weight of the upper portion of his body, to become the deflecting force required for its curvilinear motion. He inclines his body inwards, so that its centre of gravity is brought beyond the base of his feet. Thus the weight of his body tending to cause him to fall over inwards, constitutes every where a force at right angles to the direction in which he moves, acting inwards; and this force, combining with the force of his motion, deflects it from the rectilinear direction, and causes it to move continually in the same curve, into which by a slight muscular effort, he causes his feet, and the lower portion of his body to move. By this arrangement, it is wonderful with how small an exertion he is able to deflect himself

from a straight path, and move in a curve even of the greatest curvature. By a most perfect and beautiful adjustment, he causes his body to incline just so far as is necessary to supply the requisite deflecting or centripetal force, as it is called; the nicety of which adjustment will be understood when we consider that for every variation, even the slightest, in his forward motion, and therefore in the force of his forward motion, there must be a corresponding adjustment of his inclination.

237. THE CENTRIFUGAL FORCE OF A BODY'S MOTION MAY BE SUPPOSED TO BE COLLECTED FROM ITS DIFFERENT PARTS, AND MADE TO ACT THROUGH ITS CENTRE OF GRAVITY.

For if it be supposed to be moving in a straight line, all its parts moving with the same velocity and in parallel directions, it has before been shown (art. 216.) that the whole force of its motion may be supposed to act through its centre of gravity: any force therefore, which is to control this rectilinear force of motion without causing the body to turn round upon itself, must be made to act through this same point. Now opposite to this force thus necessary to deflect the body, is manifestly its centrifugal force. The centrifugal force acts then, as though it acted, through the *centre of gravity*.

238. It is by reason of the Centrifugal Force that a Carriage, rapidly turning a corner, is liable to be overthrown.

This force, acting horizontally as though it acted at its centre of gravity, and being greater as the velocity is greater and the deflexion greater, or the turn sharper, may be sufficient to overbalance the weight, which acts as though it acted vertically at the same point, and especially this will be likely to be the case, as the centre of gravity is higher. It is for the same reason that a horseman who gallops rapidly round a sharp corner, is liable to be unseated. It has been objected that the high velocities given to railroad carriages might produce sufficient centrifugal force on certain curves, to overthrow them. It is easy, however, to show satisfactorily by calculation, that this cannot be the case on any of the curves, or with any of the velocities, contemplated. The only danger which the centrifugal force might produce, is that of the carriages running off the rails, and this seems to be obviated by the conical form which is given to the surfaces of their wheels.

239. Feats of Horsemanship.

The horseman who would ride in *a straight line*, standing upon his saddle, must so alter the position of his body, with each motion of the horse, as to keep the centre of gravity of his body, continually over the narrow base of his feet. This is probably an impracticable task. If, however, instead of riding in a

straight line, he rides in a *curve*, a new force is lent to him to support his weight, acting too as if it acted at the same point where his weight may be supposed to act, viz. his centre of gravity; this new force is his centrifugal force. His centre of gravity has now no longer any occasion to be brought over the base of his feet, another horizontal force joins in supporting it, and poised between the horizontal force and the resistance of his feet, its equilibrium is easily found. To the action of the centrifugal force, which would otherwise overthrow him *outwards*, the horseman slightly opposes the weight of his body by leaning inwards: and does he find his inclination too great, he urges on his horse, and his centrifugal force, thus increased, raises him up again. By thus varying his velocity and the inclination of his body, the conditions of his equilibrium are placed completely under his control, and he can perform a thousand evolutions, that, moving in a straight line, he could not; he can leap upon his horse, stand upon his head or his hands, whilst he is performing his gyrations, or jump from his horse upon the ground, and *running to accompany its motion*, vault again upon his saddle: the conditions of his stability, and even the force of his gravity appear to be *mastered*. There is in fact given to him a third invisible power, by the act of his revolution, which is a certain modification of the force of his onward motion; this acts with him in all the evolutions he makes, and is the secret of all his feats.

240. A Glass of Water may be whirled round so as to be inverted, without being spilt.

This is a well-known feat. A tumbler of water is usually placed in a wide wooden hoop, in the circumference of which is a handle, round which it may be turned. The hoop is then whirled rapidly round in a vertical direction; the *centrifugal force* is sufficient to prevent the glass from falling from the hoop, and the water from the glass. Instead of being placed in the hoop, the glass may be tied to a string.

241. To make a Carriage run in an inverted position without falling.

Let a bar of iron be turned round so as to form a circle, as shown in the accompanying figure, the

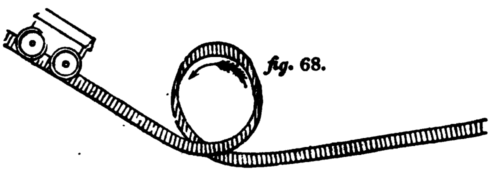


fig. 68.

two ends being brought out into two inclined planes, and the two curved portions of the bar being made to lie a small distance apart at the point where they pass one another. This bar being now placed with the curved portion of it in a vertical position as shown in the cut, let a small heavy carriage be placed at one of its extremities, with wheels, on the outside of which are *flanches*, to keep it, as it rolls,

upon the bar. Descending the inclined plane, this carriage will ascend the curve, and if the point from which it has descended be high enough, the velocity it will have acquired will cause it to ascend, in the direction of the arrow, to the *top* of the curve, and give to it sufficient centrifugal force at that point, to overcome its gravity, and cause it to run on in that inverted position without falling. It will thus descend in safety on the opposite branch of the curve, and will again be brought to rest as it ascends the opposite inclined plane towards the other extremity of the bar. This ingenious illustration of the effect of centrifugal force was devised by Mr. Roberts of Manchester.

242. The Governor.

This instrument, long used for the regulation of mill-work, is most generally known by the beautiful application which Mr. Watt has made of it, to the steam-engine. It consists of two heavy balls B B, suspended from crooked levers, B E F, which turn upon a common axis, at E. The extremities F, of these, are jointed to short bars F H, which last at their opposite extremities are also jointed upon a moveable piece, D H, which slides upon the upright rod, A C. This slide, by means of two shoulders

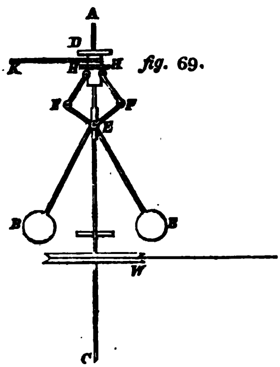


fig. 69.

worked upon it, carries with it the extremity of a lever H K, whose opposite extremity acts to open or close a valve in the pipe which conveys the steam from the boiler of the steam engine, to the cylinder; or, when the governor is used in the water-mill, it acts to raise or fall the sluice, which admits the water to the wheel; so that in either case the motion of this lever *governs* the moving power of the machine.

Now, the action of the balls is such, as to cause the *machine itself* thus to govern and control the power which moves it, so as itself to *temper* and *equalise* its own action; for the shaft A C is connected, by means of the wheel W, and the cord which passes round it, with the working part of the engine, by which the wheel and shaft are made to revolve, carrying with them the balls B. As the engine moves *faster*, these balls therefore revolve *quicker*, and their centrifugal force is *greater*; this centrifugal force, causing them to fly farther apart, causes them at the same time, to rise, causing the levers B E to revolve about E, and the points F, therefore to descend. These bring with them the slide D H, and the extremity H of the lever H K, by which means the steam valve, on which this lever acts, is more closed, less steam is admitted to the cylinder, and the machine slackens its action, and corrects its too rapid motion. An opposite action of the governor opens the valve, and throws more power into the engine, when its action is too slow.

243. The Pressure upon the Axis of a revolving Body.

When a body revolves round a fixed axis, the parts of it, situated at different distances from that axis, having different velocities, have different centrifugal forces; and a yet greater difference in the centrifugal forces of different parts is introduced, if they have different weights. These centrifugal forces act all directly *from the axis;* since all the parts of the body are describing circles round it. If the axis pass *through* the mass of the body, to the centrifugal force of each part, there is that of some *other*, on the opposite side of the axis, *opposed.* It is a *possible* case, that all these opposite centrifugal forces may exactly *balance* one another; there will then be no pressure upon the axis. The *general case* is, however, that they will not thus balance one another, and that a certain *residuum* of force will have to be borne by the axis itself, constituting the *pressure* upon it.

244. The principal Axis of a Body's Rotation.

Suppose the fixed axis spoken of in the last article to become *free*, so that the body may move in any direction. Being pressed unequally in different directions, by the centrifugal force, it will then immediately alter its position, and the revolution will begin to take place about some other imaginary axis passing through the body; this again, in its turn, will give place to some other, and so on, until out of the infinity of axes, about which it may thus, in

succession, be made to revolve, it falls upon one, about which the centrifugal forces exactly balance one another, and this axis, it will have no tendency to change. In every solid body, there are three such axes, called its *principal axes*. They intersect in its centre of gravity, and are at right angles to one another.

Although, when made to rotate *accurately* about either of its principal axes, the body has no tendency whatever to alter the axis of its rotation; yet its rotation may, or may not, when slightly deflected from that axis, tend to return to it; and it is of importance to know whether this will, or will not be the case; for, practically, it is impossible by any impulse, to cause the body, at the *first instant* of its motion, to rotate *accurately* round either of its principal axes, so that, when *free*, it cannot rotate round either of those axes, unless of its own accord, the rotation tend to pass into it. Now of the three axes, there is only one into which the rotation thus tends of its own accord to pass, and it is the *shortest* of the three. If the body, being free to move, be put in motion, not round this or any other principal axis, its rotation will yet always tend to pass into this shortest axis, and will eventually settle into a rotation about it.

Although generally, any body, whatever may be its form, has *three* principal axes of rotation, it yet may have more. *Any* diameter of a *sphere*, for instance, is a principal axis of rotation. Of a cylinder, the axis or line joining the middle of one of its circular ends to the middle of the other, is a principal axis of rotation, being the longest it can

have, but *any* axis at right angles to this from its middle point, is *also* a principal axis, than which it can have none less.

So in a *prolate* spheroid, a solid, which may be supposed to be generated by an ellipse revolving round its *greater* diameter; this greater diameter is the longest principal axis of rotation, but any axis *perpendicular* to this from its centre is also a principal axis of rotation. These last axes are all of the same size, and are the body's least principal axes of rotation.

fig. 70.

In an *oblate* spheroid, which is generated by the revolution of an ellipse about its *shorter* diameter, this shorter diameter is a principal axis, and it is the *shortest* of the principal axes of the spheroid; whilst any axis at right angles to this, from its middle point, is a principal axis, and these are its *greatest* principal axes.

fig. 71.

245. The Planets rotate about their shortest Diameters.

The shortest diameter of an oblate spheroid, being its *shortest* principal axis, is that about which, if any motion of rotation be communicated to it, it will tend to rotate, and into a rotation about which its motion, if left to itself, will ultimately settle (art. 244.). We have a striking example of this fact in the system of the universe. The planets are all oblate spheroids, and it is about their least diameters that they all of them rotate. Whether any cause have ever tended to interfere with this rotation, such as the shock of some comet, or whether such a cause ever shall operate, we know not; but this we know, that what-

ever disturbance may be, for a time, produced, in respect to the axis round which the rotation of any planet takes place, if its form remain unaltered, it will ultimately return to a rotation about its present axis. There are, indeed, various minute natural causes, always in operation, which might long ago have changed the existing axis of the earth's rotation*, had it not been *that*, into, a rotation about which, it *tends*, from all other axes, to pass. This change would involve a perpetual change in the seasons of every place on the earth's surface.

Had its form been that of a *prolate*, instead of an oblate spheroid, this case, of a perpetually changing axis of rotation would have occurred. The least axes of the rotation of such a spheroid, are any of those, at right angles to its greatest diameter, from its centre; about *some* of these it would always tend, from all others, to rotate; but it would have no tendency to rotate about one of them rather than the other; and the slightest disturbance, arising from a change in the condition of the earth's mass, the mere effect, indeed, of the tides of the air and sea, would be sufficient to make its rotation pass from one axis to another—a change which, once commenced, would never again cease.

246. Experimental Illustration of the Tendency of a body's Rotation about any other axis, to pass into one round its shortest principal Axis.

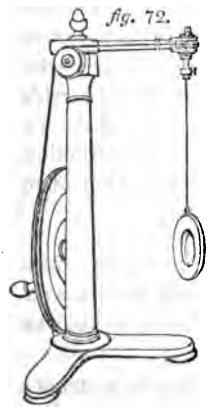
Let a body be suspended, hanging by a string; freely from any point which is not the extremity of its

* Any such change, once commenced, would go on for ever, even when the first cause of it had ceased.

shortest principal axis of rotation; and let the string then be rapidly turned round, which may be done by twisting it, and allowing it to untwist; the body will thus be made to rotate about an axis which is not its shortest principal axis of rotation; its rotation will therefore tend to *leave* this axis, and to pass into a rotation about its shortest principal axis; and it will do this with so great a force (if the motion be sufficiently rapid) as to overcome the bodies weight which tends to keep it in its first vertical position, so that it will gradually lift itself up, bringing its rotation continually nearer to its shortest principal axis; until, with a sufficiently rapid rotation, it will (so far as the eye can perceive) find that axis, and will rotate about it.

A very ingenious instrument is constructed by Messrs. Watkins and Hill, of Charing Cross, by which this rotation is made easy. It is represented in the accompanying figure. A very simple combination of wheels, which will be easily understood, from the cut, communicates a rapid rotation to the string, from which bodies of various forms are suspended, from any other axes than their shortest permanent axes. With different increasing velocities they alter their positions, continually approaching to a rotation about their shortest principal axes. In the progress of this change, a remarkable optical phe-

fig. 72.



nomenon presents itself. The body beginning to revolve obliquely, the place to which each part of it returns, after the interval of a revolution, is, in the intermediate time, left vacant; so that the sensation of vision is from that place received, not continuously, but impulsively. So rapid, however, are the impulses, that one sensation remains until it is replaced by the next; and the body appears at one and the same time, to fill the whole space, whose parts it in reality occupies in succession; — a phenomenon analogous to that of the continuous circle of flame shown by a fire-brand which is whirled rapidly round.

247. The Force with which a Body moves is never generated instantaneously.

The force with which a stone falls, in the very first instant of its fall, would scarcely be *perceptible*; it continually accumulates as the stone descends, and if it were allowed continually to descend without resistance, would soon become irresistibly great.

The *force of a cannon ball* is not communicated to it *instantaneously*, but by impulses of the air liberated from the gunpowder, which impulses are continually repeated until it finally leaves the barrel. The longer the barrel is, the longer these impulses are continued, and therefore the greater is the accumulated force. In some parts of the eastern Archipelago, and in South America, the savages are accustomed to propel small poisoned arrows, to the shafts of which, tufts of feather are attached, through long slender tubes, by blowing into them. The

velocity is thus accumulated in the arrow by continued impulses of the breath, until it leaves the tube, as in the bullet by a continual expansion of liberated gas. If a rope be attached to a ball fired from a cannon, as in Captain Manby's apparatus for saving shipwrecked mariners, the rope will almost always be broken, for the rapid motion of the ball cannot *instantaneously* be communicated to the parts of the rope, nor so rapidly as the ball moves. The elasticity of the rope has a tendency to prevent this rupture, because it allows of a certain motion of one part whilst the rest does not move, and during the time of this motion, it operates, to communicate the motion of the first part to the second.*

The proverbial *velocity of an arrow* is due to the *continued* action of the bow-string upon it, as the bow expands; and it is for this reason that the distance of the flight is greatly dependant upon the length of the bow; the string remaining in contact with the arrow, and impelling it longer, as the bow by reason of its length, admits of being further drawn back.

The *balistæ* of the ancients, with which they threw great stones and arrows, are similar instances of the accumulation of velocity; which was in these, produced by the elastic force with which ropes extended and doubled, and then many times twisted, tended to untwist themselves.

Rams and *goats*, when they fight, *recede* before they rush upon one another, that they may gradually *accumulate* a great velocity of impact.

* The motion is thus propagated through the rope like the undulation of an elastic medium.

The length of *neck* in some *birds* enables them to accumulate velocity in their heads through a great distance, and a blow from their beaks thus becomes of irresistible force; and to a like cause is to be attributed the extraordinary violence with which a *serpent* infixes its fangs.*

248. IF A GUINEA BE PLACED UPON A CARD AND THE WHOLE BALANCED ON THE TIP OF THE FINGER, A SHARP BLOW STRUCK UPON THE EDGE OF THIS CARD WILL CAUSE IT TO SLIP FROM UNDER THE GUINEA, AND THE LATTER WILL BE LEFT ALONE ON THE FINGER.

The force which tends to make the guinea move with the card, is its *friction* upon it; this is but a small force, a comparatively *long continued* action of which would be necessary, to communicate to the guinea a force of motion sufficient to cause it to move as fast as the card is made to move by the finger. This long continued action not being allowed, the guinea is made to move but very little, and the card passes from under it, leaving it nearly where it was.

249. THE EFFECT OF SWINGING, RIDING, ETC. ON THE CIRCULATION OF THE BLOOD.

Dr. Arnot has pointed out a remarkable effect, which the principles which govern the communication of motion, probably have, upon the circulation of the blood. It is well known that in all the veins,

* Serpents have been known, thus striking, to miss their prey, and unable to controul the violence of the blow, to drive their stings into their own bodies.

there are valves, which open towards the heart; now in some of the great veins which ascend from the lower part of the body to the heart, it cannot but be, that when the body is made to descend suddenly, the blood should as it were be left behind it in the vein, on the same principle that if a phial partly filled with a liquid be made suddenly to descend, the liquid will be made to strike against the top. This tendency of the blood to remain behind, when the vein, descends in swinging or riding, or even in walking, forces it through the valves of the vein, and thus probably quickens the circulation. The peculiar sensation felt in the descent of the body in swinging, is probably to be attributed to this cause.

250. A CANDLE FIRED FROM A MUSKET WILL PIERCE THROUGH A THICK BOARD.

When a body is struck, it is for the most part only a few of the points on its surface which receive the blow; to communicate the *effect* of this blow, (the motion of the parts immediately about the point of impact) to all the rest of its mass, certain *time* is required. Thus when a soft body is struck in *one* place, a certain *time* is required, before the *other* parts of it can be made so to feel the effects of the blow as to admit of the surface yielding, greatly, even in the place where it is struck; and *until* this is the case, the effect is the same as though the body were perfectly hard. It is thus that if the PALM OF THE HAND be struck with force on the surface of water, the blow will be resisted, at the first instant, almost as though by a solid body. Nay a MUSKET BALL

when fired against water is, it is said, *repelled* by it, and even *flattened*; and a CANNON BALL fired over the surface of a smooth sea, rebounds from it, as from a hard plane.

These circumstances sufficiently explain the perforation of a board by a soft body, like a candle, when fired from a musket. The parts of the candle cannot yield until after a certain *time*; until that time has passed, they are like the parts of a solid, and *before* it has passed, the candle has gone through the door.

251. A MUSKET BALL PASSES THROUGH A PANE OF GLASS WITHOUT CRACKING IT.

The explanation of this fact is the same with that of the last,—the indentation of the surface produced by the first impact has not *time* to propagate itself, so as to crack the pane before the ball has passed through it. Thus, although if thrown by the hand it would shatter the *whole*; being projected with this *velocity*, it carries away only so much, as will leave, room for it to pass; and were the pane suspended by a *thread*, it would not break the thread, or even cause it to oscillate. A sheet of paper placed edgeways may, for a like reason, be perforated by a *pistol ball*, without being knocked down; and a door half open, pierced by a cannon ball without being shut.

A *cannon ball* has been known to carry off the *extremity of a musket*, without the soldier who carried it feeling the stroke; as the head of a thistle may, by a rapid blow be struck off, without perceptibly bending the stalk. It is for a like reason to that explained above, that a cannon ball moving with

great velocity passes through the side of a ship, leaving a clear aperture, whilst a spent ball splinters it.

252. The Force with which a Body moves is never destroyed instantaneously.

A *cannon ball* which impinges against a wall causes a certain yielding of the substance of the wall, and of its own substance before it is stopped; this occupies *time*. A *bomb* enters the ground some distance before its descent is arrested, and that building only is bomb proof, the covering of which is of a thickness exceeding the distance to which the bomb will thus of necessity sink in it.

Bales of cotton have sometimes been placed to receive the impact of balls, and have *stopped them*, because by their elasticity they *continually* resist the progress of the ball as it enters them, and this *continual* resistance more effectually takes away their great force of motion (although in itself it is so small) than a much greater, but momentary resistance, would.

Dr. Arnot has given a very striking illustration of this fact, drawn from the comparative strength of iron and hempen cables.

A ROPE CABLE, being not far from the same in weight with an equal bulk of water, is so buoyed up by the water, as not to hang in any considerable curve from the ship to the anchor, but to be distended in a straight line, whilst an IRON CABLE being much heavier, hangs in a deep curve. Thus the force of the ship's motion, as when at anchor she is beat about by the wind, can only be counteracted by the *stretch-*

ing of the substance of the rope cable, whilst it is gradually counteracted merely by the *tightening* of the *curve* of the *chain* cable.

253. Accumulation and Destruction of the Force of Motion in a moving Body. Distinction between Force of Motion and Force of Pressure.

Let the force of gravity be imagined to become for an instant *extinct*, and a ball with a string attached to it, to be placed, in the void space, at some distance above a table, in the top of which is a small hole through which the string passes. Gravity being extinct, the ball will rest, unsupported, in the position in which it has been placed. Suppose now that the string is *pulled*, through the hole in the table, by a series of impulses, communicated to the ball: the force of motion communicated to it by each of these impulses the ball will retain, for nothing *opposes* itself to its motion, and the force of motion in a body is indestructible, except by the action of some opposing force. Retaining thus the force of motion communicated to it by each impulse, the ball will at length, strike the table with an aggregate force of motion, equal exactly to the *sum* of all the separate impulses which it has received. Moreover, this will manifestly be the case whether it receives *many* or *few* impulses before it reaches the table, whether each of them be *great* or *small*, and whether they be communicated at *longer* or *shorter* intervals of time. Thus it is true, if the impulses be *infinitely* near to

one another, or if the string is pulled *continuously*. Now, if the string be thus pulled *continuously*, the number of impulses which the ball receives before it reaches the table, must be *infinite;* so that the *sum* of all these must be infinite as compared with any *one* of them, and therefore *the force of motion with which the ball strikes the table, infinite as compared with the force with which the string is at any instant pulled.* Now, let the ball *rest* upon the table, and let the string be pulled with precisely the same force as before, each *separate* impulse of the force with which the string is pulled, will now be encountered by the resistance of the table, whilst before, it was the *accumulation* of these impulses which it had to encounter. Moreover, each separate force is infinitely small as compared with their sum. The table then, encounters in the one case a force infinitely greater than in the other. In the one case the force exerted upon the table by the ball was one of *motion*, in the other, one of *pressure:* this example points out the characteristic difference between them, and thus it is seen that any force of motion is infinite as compared with any force of pressure, every force of motion being the accumulation of an infinite number of elements, of which accumulation, every force of pressure is in the nature of *one* element.* It has, in fact, been shown in the immediately preceding articles, that force of motion in every case requires a finite *time*, and the operation of a *series* of im-

* If it be not one element of that actual sum, it is at any rate comparable to one of its elements, and bears a finite ratio to it.

pulses to its production, and is never generated *instantaneously*. It is in its nature an *accumulation*; these impulses are the *elements* of that accumulation, and this time is necessary to their aggregation. That force of *pressure* and force of *motion* thus stand in the relation of parts to a whole, sufficiently accounts for many remarkable analogies between the phenomena of these two descriptions of force, and therefore between the sciences of *Statics* and *Dynamics*.

Since force of motion is the sum of a series of impulses, it is evident that a *series* of such impulses in an opposite direction, is required in any case to destroy it; and thus it is sufficiently explained why force of motion is never *destroyed instantaneously*.

254. Gravitation.

Now, there pervades all material existences a force, analogous to that which we have been describing by the illustration of a string continually pulling a ball. Every portion of matter, impels towards itself every other portion of matter, at every instant of time, and under every variety of circumstances in which these portions of matter may be placed, whether of repose or motion. The earth is a huge mass of matter, every particle of which thus exerts a continual attraction upon every other particle of it. This attraction produces in all bodies on its surface, a tendency to descend towards its centre. If this tendency be resisted by any intervening obstacle, so that each impulse of the earth's attraction is *separately* counteracted,

there results a *pressure* upon the obstacle, called WEIGHT. If there intervene no obstacle, the body moves towards the earth's centre, continually *accumulating* the impulses of its attraction, increasing the rapidity of its descent, and acquiring a greater and greater FORCE OF MOTION; which force of motion, being the accumulation of an infinite number of separate gravitating impulses, is *infinite* as compared with the before-mentioned force of pressure or *weight*, which is in reality but *one* of these impulses. The wonderful force of an impact to overcome the resistance of the parts of any solid mass to rupture, is thus fully explained. Cohesive force is in the nature of a force of pressure, and therefore infinitely small as compared with any force of motion, so that it of necessity yields to any impact, however slight, at the moment of impact. Thus a weight which, resting upon a table, does not even indent its surface, being let *fall* upon it, *crushes* it.

255. No Force of Motion or Impact can be compared with, or measured by, a Weight.

We cannot, for instance, say that the force with which a body moves, is a force of so many pounds, or so many times a given weight, for it is infinitely greater than any given weight. How, then, shall we compare the different quantities of this hidden but mighty principle, in different but equal portions of the same moving mass, or of different moving masses? Clearly by the quantities of *motion* which it communicates to them (see art. 206.)

256. Uniform, accelerated, and retarded, Motion.

Motion is change of place. The motion of a body is said to be *uniform* when the distance, between the places occupied by it in any two successive instants of time, is always the *same*. It is *accelerated* when this distance, for any two successive instants, is *greater* than for the preceding two.

It is *retarded* when it is *less*.

The motion of a body, if it be uniform, is measured by the space it actually describes in a given time.

If it be accelerated or retarded, its motion at any instant is measured by the space it would have described in a given time, had the motion, which it had at that instant, been continued uniformly through that time. The time thus used as the standard of comparison is one second.

257. Velocity.

The space which a body, moving uniformly, describes in one second, or the space which a body whose motion is accelerated or retarded, would move through in one second, if its motion had continued uniform during that second, is called its velocity.

From the above it appears that the force with which a body moves, is proportional to its velocity, and conversely. (See art. 206.)

258. Accelerating Force.

A body acted upon by a series of different impulses, or by a force which *constantly* impels it, acquires continually more force, and therefore more velocity.

The quantity of force acquired from each impulse is proportional to the *additional* velocity which is communicated by it, and is *measured* by that additional velocity.

The *additional* velocity thus communicated in a second of time, is called the accelerating force.

Since the body retains all its *increments* of force, and therefore of velocity, its whole velocity after any number of seconds of time, from the commencement of its motion, will equal the velocity with which it first begun to move, added to the increments of velocity which it has continually received. That is added to the sum of its accelerating forces.

259. The Law of the Accelerating Force of Gravity.

Bodies moving to one another by reason of that attraction which pervades all matter, and is called gravitation, receive continually greater accessions of velocity in each second of time, as they approach one another more nearly. The accession of velocity or accelerating force at any one distance from the centre of either body, being to that at any other, as the square* of the second distance is, to the square of the first. This law is usually cited as that of the

* The square of number is the product of that number by itself.

inverse square of the distance. The accelerating force of gravity being said to vary inversely, as the square of the distance. Thus bodies falling at the surface of the earth, receive continually greater increments of velocity in each second as they approach its centre. Nevertheless the distance through which a body can be made to fall at the earth's surface being exceeding small, as compared with the whole distance to its centre, this variation in the accelerating force of falling bodies is exceedingly small, and indeed imperceptible.

For all practical purposes we may therefore consider the augmentations of velocity which a body falling at or near the earth's surface receives, in each successive instant of time, to be the same. This constant accelerating force or increment of velocity will subsequently be shown by experiment to be $32\frac{1}{6}$ feet.

260. Gravitation a Force inseparably and universally associated with Matter.

Gravitation is fixed in matter *eternally* and *inseparably*. No lapse of time wears it away, no modification of circumstances in which it can be placed — no appliance of artificial means — or power of other natural forces upon it, removes or can remove, the slightest conceivable portion of it. You may crush the parts of a body into a powder, apply to it the power of heat, and melt it into a liquid — or you may, by a yet intenser application of heat, dilate it into a gas; you may make of it a chemical solution; bring it again to its original form of a

solid — analyse it again and again — combine and recombine it: through all these changes you will not in the slightest conceivable degree have affected the gravity or weight of any one of its particles.

Not only is the power of gravitating thus *unalterably* infixed in matter, but it is infixed in it *universally*.

There is no place on the earth's surface where there is matter and not *weight*—there is no matter known to exist in *our* system of the universe, which does not *gravitate;* and if we carry on our inquiries *beyond* the limits of our system, into the fathomless depths of space, we find *there* the STARS gravitating towards one another. It is a recent discovery of astronomy, that those *multiple* stars which, being examined by powerful telescopes, are seen to revolve round one another, and of which there are many, are in their motions subject to certain laws, which prove them to be attracted towards one another by the force of *gravity*— or rather by a force subject to the same laws as that which attracts all things on the surface of our earth towards its centre, and our earth itself towards the sun.

Such is the *eternal, immutable* nature of gravitation, and such is its *universality*.

Although the force of gravity thus coexists universally with matter, yet does it not reside in the same manner and degree in all matter; there is not, for instance, throughout all matter the same quantity of the principle of gravitation (whatever it may be) associated with the same *volume*.

Thus the component materials of the planets are such, that were they all of the same volume or size,

they would not, nevertheless, all *weigh* the same. And it is scarcely possible to take up any two portions of the matter which composes the earth's surface of which, equal volumes, would be found to have the same weight or gravitating power.

261. The Gravitation of the Bodies around us to the great Mass of the Earth, is a sensible Force; their Gravitation towards one another, almost insensible.

Gravitation is the *aggregate* of the *attractions* of the elements of which the earth is composed, each such element attracting each other; thus the pebble under our feet is attracted, individually, by every other of the pebbles, that are scattered around it; and by all those that are strewed over the earth's wide surface—by every particle of fluid, air, or water, upon the earth, and by every atom of its solid substance. Whilst the aggregate of the attractions of the elements which compose the earth is a *finite* and sensible force, thus known to us as gravitation, the mutual attractions of any of these finite portions of it, which come within the scope of our immediate observation, are so small as to be insensible, except to the most delicate admeasurements. All the sensible objects around us, no doubt, gravitate towards one another, although we do not perceive it, by reason of the exceeding small amount of their gravitating power, and the forces which, in every case, oppose themselves to its taking effect. Thus, if I take up two stones which lie side by side, I immediately perceive the attraction of the great mass of the earth

upon them, but I remain wholly unconscious of their attraction upon one another.

If two balls of lead were suspended by strings from the ceiling of a room at the same height, they would gravitate towards one another, and did no force oppose itself to their motion, however small the force of their gravitation, they would *approach* to contact. But whilst they are thus attracted towards one another, each is attracted by all the elements which compose the great mass of the earth, and the direction of this much *greater* attraction must in each be disturbed, before they can, in obedience to their mutual gravitation, approach one another; their approach is thus rendered so small as to be *imperceptible.* If they were placed on a horizontal plane, their friction upon it would in like manner be sufficient to retain them apart, and if they *floated* in a fluid—the resistance of the fluid. Nevertheless there are cases in which attraction of gravitation may be rendered sensible, in comparatively small masses of matter.

262. The Attraction of Mountains.

The greatest mountain on the earth's surface is not the 59th millionth of its bulk; the attraction of such a mountain upon a ball of lead is, therefore, as nothing, compared with the attraction of the whole earth upon that ball of lead: yet would this attraction produce a deviation of the plumb line which might be rendered *sensible.* Bouguer was the first who traced a deviation of the plumb line from the attraction of a mountain. On the sides of Chimboraço, the highest of the Andes, by observations

made under circumstances of extraordinary difficulty, he ascertained that mountain to attract the plumb line 7″ or 8″ from the perpendicular. Chimboraço is *volcanic*, and its attraction is diminished by an immense cavity which it encloses.

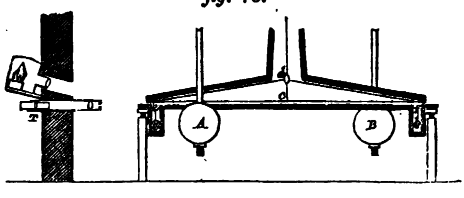
In 1772 Maskelyne, by observations similarly made at the foot of Mount Shehallian in Scotland, found a deviation of the plumb line of 54″.

In 1824 M. Carlini found the attraction of Mount Cenis to produce a variation in the oscillations of a pendulum, to correct which it was necessary to lengthen it by the ·0082677th of an inch. These experiments of Maskelyne and Carlini present a means of comparing the *attractions* of the earth and the mountain in each case, and therefore the *masses* or quantities of matter in the earth and mountain, which masses are proportional to their attractions; and the quantity of matter in the mountain being *estimated*, by observing of what material it is composed, and measuring its bulk; we are enabled to tell, from this comparison, what is the actual quantity of the *material*, or the *mass*, of the earth. Knowing thus the quantity of matter in the earth, and knowing also, from astronomical admeasurements, its bulk or volume, we can tell its mean *density*. The observation of Maskelyne gave 4·56 for this mean density, and that of Carlini 4·39. From the near agreement of these two observations, we conclude, with great probability, that the earth's mean density is about four times that of water.

263. The Experiments of Cavendish.

To the deviation of the plumb line the weight of the plummet opposes itself. It is evident that a much more delicate test of the existence of an attraction would be obtained, if the plummet could be *balanced.* No contrivance of this kind can, however, be made to show the attraction of a *mountain,* because the attraction of so great and comparatively distant a mass, would affect equally the ball and the counterpoise; but such a contrivance may be applied to show the attraction of a *less* and *nearer* mass; and, with this more delicate indication, the attraction of a mass very greatly less, has been rendered even more sensible than that of the mountain, upon the plumb line. The following is the admirable experiment of Cavendish. A and B are two balls of lead fixed to

fig. 73.



the extremities of a lever, and capable of being put in motion round an axis which coincides with *c d* produced. *a* and *b* are two smaller balls suspended, by slender silver wires, from the extremities of a rod *e f.* The wires which suspend these balls are afterwards continued to meet in *d*, where the whole is suspended by a third wire *c d*, about which, the least conceivable force is sufficient, to communicate,

motion to the rod and its suspended balls. The whole of this last-mentioned apparatus, of the rod and smaller balls, is contained, and separated from the rest, by a case, adapted to its form and to the motion which is to be given it. The section of this case is represented in the figure by the shaded line; it is intended to protect the motion of the balls from any impulses of the air. That the oscillations of the balls may be seen, small *apertures* are left in the case at *e* and *f*, at the extremities of the rod. Yet, more effectually to get rid of causes of disturbance, and to obtain a uniform temperature, Cavendish inclosed the *whole* of his apparatus in a room, without door or window, and into which was no other aperture than one for the admission of the reflected light and heat of a lamp, and a second in which was fixed a telescope T, through which the extremity of the rod might be seen. The lever, or arm, which carried the balls A and B, could be turned by a mechanical contrivance adapted to that purpose, to which motion was given outside of the chamber. When this arm was thus turned, its position was of necessity made to *cross* that of the light rod *ef*, carrying the lesser balls *a* and *b*; and the greater balls were thus placed in such a position that their attractions upon the lesser balls should both conspire to turn the rod *e f*; to which motion of the rod, no other force would oppose itself than the feeble resistance to *torsion* of the wire *c d*.

In the experiments of Cavendish, the large balls A and B were, in weight, somewhat more than 3 cwt. each; and their attraction upon the smaller balls,

when the arm carrying them was deflected, was sufficient immediately to cause a deflection of the rod *e f*, which, after a number of oscillations on either side, at length took up a position nearly in the line joining the centres of the greater balls, deviating from that position only by the amount due to the torsion of the wire *c d*. It appears from theory, that the *time* of each oscillation, before the rod eventually rests, is a *measure* of the attraction of the balls, and sufficient to determine it, allowance being made for the effects of the torsion. And it was thus that Cavendish determined the attractions of the greater balls upon the less; this he compared with the attraction of the *earth* upon these *lesser* balls; and thus he was enabled to compare the mass of the earth with the mass of the greater balls; and knowing the size of the earth and the size of the balls, he thence obtained a comparison between the densities of the two; that is, between the density of the earth and the density of lead. He thus found the density of the earth to be 5·48, or about 5½ times that of water.

The mass of the earth is a *unit*, in terms of which, the astronomer determines the masses of all the bodies of our system of the universe.

The apparatus of Cavendish is therefore, in fact, a scale in which the earth, sun, moon, and planets, may be considered to have been weighed.

264. The Attraction of the Earth would cause all Bodies, whether they were light or heavy, to fall towards its Surface with the same Rapidity, were it not for the Resistance of the Air.

If a light body—a piece of paper for instance—and a heavy, but less, body—a piece of metal—be let fall from any height, at the same time, the heavy body will soon be seen to have passed the other, and it will reach the earth before it. That the air is the principal cause of this difference may at once be shown, by doubling up the paper till it is nearly of the same size with the metal; they will then fall nearly in the same times. But the question may be submitted to the test of a conclusive experiment. It is a common experiment with the air-pump to adapt to the top of the interior of a high glass tube, a mechanical contrivance, on which a piece of money and a feather being placed, they can both be let fall at the same instant, by turning a screw on the outside of the tube. This tube is placed upon the plate of an air-pump, and the air having been extracted from it, the screw is turned, and the piece of money and the feather being let fall at the same instant, reach also the bottom of the tube *together*. If the experiment be repeated with the air only *partially* extracted from the tube, the coin will a little gain upon the feather; and if no air be extracted, the difference of the times of descent will be considerable.

265. THE VELOCITY WHICH IS COMMUNICATED TO A BODY FALLING FREELY BY GRAVITY.

Bodies falling freely, near the earth's surface, have communicated to them, equal additions of velocity in equal times; and since by the first law of motion (art. 93.) none of these increments of the velocity are lost *, but all accumulated in the falling body; it follows, that its actual amount at any time, must be proportional to the time during which the body has fallen. If, for instance, a body has fallen through ten seconds, since in each second the attrac- of the earth will have communicated to it the same addition of velocity, and since all these additions of velocity will be retained in it, its actual velocity must be five times that which it would have had after falling one second.

The velocity which gravity thus communicates to a falling body in *each* second of time near the earth's surface is $32\frac{1}{6}$ feet; so that after falling five seconds, its velocity will be five times this amount, after ten seconds ten times this amount, and so on.

This velocity is so great, that it would never have been possible to ascertain its amount by direct observations on the fall of heavy bodies.

Could we, however, by any contrivance *neutralise* the gravitating tendency of a body to any known amount, — reduce it, for instance, to *one-half*, or *one-tenth*, or *one-hundredth* of what it was, since we should diminish the *velocity*, communicated to it in

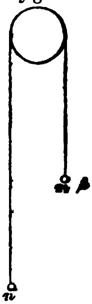
* The resistance of the air is here put out of the question.

each second, precisely to the same amount, we might thus render its motions so slow, that they might be *observed* and *measured;* we might thus find the amount of the additional velocity actually communicated to it in each second, and this multiplied by the known number of times by which we had previously diminished the force of its gravity, would give us the velocity which that force would communicate in each second, when *undiminished.* This is the object of Atwood's machine.

*266. Atwood's Machine.

Let m and n be two *equal* weights, suspended at the extremities of a string, which passes over a pulley, as shown in the figure, and *imagine* the pulley to be without friction, and the string to be without weight and perfectly flexible. It is clear that the weights m and n being equal, the tendency of each to descend, will be exactly neutralised by that of the other, and they will rest. Let now a small weight μ, equal to any known fraction of m and n, say the tenth of either or the twentieth of their sum, be added to m. No force whatever will act to counteract that with which μ tends to descend — for all the force in m and n is *neutralised* — no portion of the force with which μ tends to descend will therefore be *destroyed.* It will nevertheless not take effect in such a way, as to cause μ to descend as it would, if it descended freely; for μ cannot move without communicating

fig. 74.

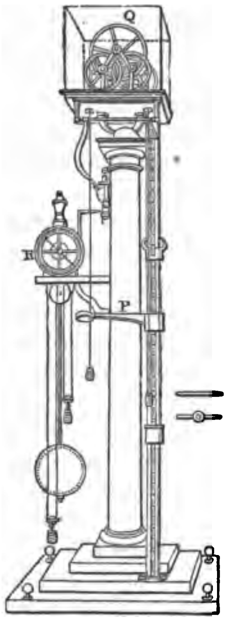


an *equal* motion to m and n. Throughout the bodies m n and μ, an equal force must therefore be distributed, to produce this motion; and that force can only come from the gravitating force of μ; this force, being that with which μ would actually descend if left to itself, is therefore, by this contrivance, made to be equally distributed through the bodies m n and μ, and to operate upon them in common, with an energy less, in proportion, as the mass through which it is thus diffused is greater. In the case we have supposed, the mass through which this force of μ is thus diffused, is equal to twenty-one times μ; the force actually existing in each portion of it, is therefore the 21st part of what it was in each portion of μ, and μ will, in this combination, descend with $\frac{1}{21}$st part of the force that it would, if it descended freely; that is with $\frac{1}{21}$st part of the ordinary force of gravity. This change being made in the *amount* of the force effective on μ leaves, nevertheless, the law under which that force takes effect the same, and reducing the velocity which it produces in each second to the 21st part, enables us to measure that velocity, and, taking it twenty-one times, to estimate what it would be if the body fell freely.

The conditions we have supposed of a perfect absence of friction in the pulley, and of weight and rigidity in the string, cannot be realized. They are nevertheless approached to, in the machine shown in the accompanying figure, which is called Atwood's machine, and which serves, when accurately constructed, to verify the law of the descent of heavy bodies with great precision.

The string is a slender thread of silk, and, to get rid of friction, the axle of the pulley Q is made to rest, at each extremity, upon the circumferences of two wheels, which turn with it, and thus offer a greatly less resistance to its motion than a collar would. These wheels being made with great care, and accurately balanced, and their axes being very small, the various resistances to the motion of the pulley, are in a great degree got rid of.

fig. 75.



A pendulum clock R, beating seconds, is affixed to the machine, and there is a mechanical contrivance connected with it, by which the pulley is set free, and the descent of the weights made to commence, at the commencement of a particular second. The gravitation of the weight μ, continually *adding* to the velocity with which the bodies move, in order to determine that velocity at any particular instant, it becomes necessary to remove, at *that instant*, the cause of acceleration, so that the motion may continue, for a time, the *same* as it was then. This is done by causing the descending body to pass through a ring P, moveable along a vertical scale. By trial, this ring is fixed in such a position, that the descending weight shall

pass through it, precisely at the instant at which the velocity is to be measured; after one, two, three, or any other number of beats of the pendulum; the weight μ, which is to produce the motion, is moreover made of the form of a small rod or bar, of a length greater than the diameter of the ring. Thus, whilst the two weights m and μ, are in the act of passing through the ring,—that is, at the instant for which the velocity was to be measured—the weight μ will be removed; and no force, thus, remaining to accelerate the motion, it will become uniform, and may be measured. In order to effect this measurement, a flat piece of wood, moveable along the scale, is placed, by trial, in such a position, that the descending weight shall strike it precisely after one beat of the pendulum from the time when it passed through the ring. The distance marked upon the scale between the position of P, and that of this second sliding piece, measures the space, which the descending body describes, uniformly, in one second, with the velocity which it had acquired at the instant of passing through P,—that is, after the given known number of seconds of its motion. Now it is found by these experiments that the velocity, thus acquired, in a descent of two seconds, is double of that acquired in a descent of one second; that acquired in a descent of three seconds, triple, that acquired in four seconds, quadruple, &c.

Thus then, the body, thus falling acquires in each second an equal amount of additional velocity, which (if, as we have supposed, μ is $\frac{1}{10}$th of m or n,) is $\frac{1}{21}$st part of the velocity which it

would have acquired, had it fallen freely: so that a body falling freely, would acquire *equal* additions to its velocity in each second. From experiments thus made, it is found that the addition made to a body's velocity in each second of its descent, when it falls freely, near the earth's surface, is $32\frac{1}{6}$ feet, or more accurately 386·28 inches.

This is its increase of velocity in each second, near the earth's surface. It would not be the same at greater distances from it: at twice the distance from the earth's centre that we are, it would only be $\frac{1}{4}$th what it is here; at three times the distance $\frac{1}{9}$th; and four times the distance, $\frac{1}{16}$th; at five times $\frac{1}{25}$th. The law of this variation, which is easily seen, is called that of the inverse square of the distance.

267. Descent of a Body by Gravity.

It has been shown (art. 266.), that a body, whatever may be its weight, descending freely by gravity, near the earth's surface, always increases the velocity with which it descends, by $32\frac{1}{6}$ feet, during every second, of its descent.

From this it may be calculated, that the space through which it descends in a given number of seconds, is equal to the square* of that number, multiplied by one half of $32\frac{1}{6}$ feet, or by $16\frac{1}{12}$ feet. Thus, for instance, a body which descends freely by gravity, during two seconds, will fall through a space equal to the square of 2, that is 4, multiplied

* The square of a number is the product of that number when multiplied by itself.

by $16\frac{1}{12}$ feet; or it will fall, in that time, through $64\frac{1}{3}$ feet. In 3 seconds, it will fall through 9 times $16\frac{1}{12}$ feet, or $144\frac{3}{4}$ feet. In 4 seconds, through 16 times $16\frac{1}{12}$ feet, or $257\frac{1}{3}$ feet. In 18 seconds, through 324 times $16\frac{1}{12}$ feet, or 5211 feet—that is, a mile within 57 feet.

From this relation, of the space to the time, and from the consideration that the velocity, after any number of seconds, is equal to $32\frac{1}{6}$ feet multiplied by that number of seconds, it is easily found, that the velocity acquired in falling through a given height, must equal the square root of the product, of $32\frac{1}{6}$ feet by twice that height. Thus, for instance, the velocity acquired in falling through $144\frac{3}{4}$ feet, must equal the square root of $289\frac{1}{2}$ feet, multiplied by $32\frac{1}{6}$; which multiplication being performed, and the square root extracted, there will be obtained the number $96\frac{1}{2}$, for the number of feet per second of the velocity of the body, after it has fallen that height. By a similar calculation, the velocity acquired in falling through $257\frac{1}{3}$ feet will be found to be $128\frac{2}{3}$ feet, and that acquired in falling through 5211 feet, 579 feet.

The velocity acquired by a body in thus falling through any given height, is called the *velocity due to that height.*

268. A Body projected downwards or upwards.

If the body have not acquired its whole velocity in falling, but has been *projected* downwards, it will retain the velocity of its projection, and acquire,

besides, an increment of velocity of $32\frac{1}{6}$ feet, in each successive second. Thus the whole velocity will equal that of projection, added to the product, of $32\frac{1}{6}$ feet, by the number of seconds, during which the body has descended.

If the body be projected upwards, instead of downwards, its velocity upwards, will be *diminished* by $32\frac{1}{6}$ feet in every second, until it is wholly destroyed. The body will then begin to *fall*, and its velocity, will from that time, increase continually by $32\frac{1}{6}$ feet per second as before.

Since, in its descent, the body will acquire, in each second, as much velocity as it lost, in its ascent; and that in the seconds which intervened, between any period in the ascent and the period of its greatest ascent, the body lost all the velocity it had at the first mentioned period; also, since it will acquire just so much velocity in descending, through that number of seconds; it follows that the body has, at any number of seconds after the period of its greatest height, just the same velocity which it had, at as many seconds before it attained that greatest height; and thus that, the velocities of its ascent and descent being in every successive instant (measuring the time from the period of its greatest height) the same, its motion in every respect, and the spaces it describes, will be the same.

Thus, the times of its ascent and descent, will be the same; and it will return to the earth's surface with the same velocity, with which it was projected from it.

269. To find the Depth of a Well by letting a Stone fall into it.

Let the number of seconds between the time when the stone is let fall, and that when the sound of its striking the bottom reaches the ear, be observed. This will best be done by counting the beats of a seconds pendulum. This time includes that of the falling of the stone to the bottom and the return of the sound to the ear. The velocity of sound being assumed to be uniform, and at the rate of 1130 feet per second *; a very simple algebraical calculation, gives us the following approximate rule: "Multiply the square of the observed number of seconds, by 565, and divide the product, by the observed number of seconds increased by 35, the quotient will be the depth of the well, in feet."

Thus, let it be supposed, that 5 seconds intervene, between the instant when the stone is let fall and that when it is heard to strike the bottom. The square of 5 is 25, and this multiplied 565, gives the product 14,125, which is to be divided by 5 increased by 35, that is, by 40. The quotient of this division is $353\frac{1}{8}$, which is nearly the depth of the well, in feet.

270. Velocity of the Descent of a Body upon an inclined Plane.

If a body be supposed to slide down an inclined plane, without any resistance, it will acquire, when

* The experiments of Flamstead and Halley give 1,142 feet for the velocity of sound. Recent experiments appear, however, to show it to be yet less than the number we have assumed.

it reaches the bottom, a velocity, precisely equal to that which it would acquire, by falling freely to the same level, from a height equal to that of the plane. Thus, if the point from which it fell be at a perpendicular height of $144\frac{3}{4}$ feet above the base, the velocity acquired by falling down the plane will be $96\frac{1}{2}$ feet per second; that being the velocity which it would have acquired by falling *freely*, or without the plane, through $144\frac{3}{4}$ feet (art. 267.). The velocity is thus, entirely *independant* of the *length* of the plane, and is the same, for instance, for a body falling down a plane which is twice the length of another, provided its *height* be the same. This may be easily understood. The resistance of the plane, tends to neutralise the gravitating force of the descending body, in a degree dependant upon the smallness of its inclination. If it be not inclined at all, or perfectly horizontal, it *entirely* neutralises the gravitating force, so that the body does not descend at all; if it be slightly inclined it takes away *some*, but not *all* the gravitating power, and the body descends *slowly;* if it be *greatly* inclined it takes away but *little*, and the body descends *rapidly*. Now if the inclined plane be but little *inclined*, it must be of great *length* to have a certain perpendicular *height*, and therefore the body, descending slowly, must be long in descending it; if it be more inclined, the length corresponding to that height is less, and the time of describing it, less; so that as on the one hand, by making the plane more and more inclined, less and less of the body's gravity is taken away *from it; on the other hand*, the time through which that gravity acts upon it in its descent, becomes,

with this increasing inclination, less and less; and, by a remarkable relation, it happens, that these causes just *compensate* one another. The greater *time* of descending the longer plane just compensates for the less *force* with which the body descends it; so that the whole velocity which that less force communicates, acting through that longer time, is just equal to the whole velocity which the greater force communicates, acting through the less time; and in both cases the same amount of velocity is ultimately produced.

271. Velocity of Descent upon a Curve.

When a body descends freely upon a curve, the resistance of the curve neutralises, not as in the inclined plane, the *same* portions of its gravitating force at *all* points of its descent; but *different* portions of it at *different points*. Nevertheless the velocity acquired in the descent is subject to the same law as that acquired on the inclined plane; it is the velocity due to the height, the velocity which the body would acquire in falling *freely* from a height, equal to that of the point *from* which it has descended, above that *to* which it has descended, upon the curve. This remarkable property obtains, whatever may be the form or the length of the curve —or rather it is a property which would obtain, if there were no resistance of the air, and no friction.

*272. Time of a Body's Descent upon a Curve.

The quantity of the descending body's gravitating force, which at each point of its descent is neutralised by the resistance of the curve, depends upon

the *inclination* of the element* of the curve, at that point, to the *horizon*. Now by varying the form of the curve, we can in any way vary this *inclination*; we can therefore in any way vary and modify the *un*neutralised or effective gravitation of the body, so that from a force acting with the same energy at all points (as in the case of free descent, or of descent upon an inclined plane), we can convert it into a force, varying, according to any law, from point to point. Now if the effective force on the descending body, could by any form of the curve on which it descends, be thus so modified as to vary directly as its distance *along the curve*, from the point where its descent is to *terminate*, then would the time of its descent to that point be the same, from however great a distance along the curve it had descended to reach it.

Thus if the form of a curve, A B, could be so contrived that, (its resistance neutralising, at every point, a certain portion of the force by which a body descending upon it, would otherwise be accelerated), that which *remained* should, at each point of its descent, be proportional to the distance of that point from the point A, to which the body is to descend; then the time required by the body to descend from any

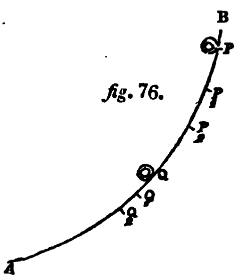


fig. 76.

* The curve may be supposed to be made up of an infinite number of exceedingly small straight lines, and the *element* here spoken of to be one of these lines.

point P on the curve, to A, would be precisely the same, as the time required to descend from Q to A, or from B to A.

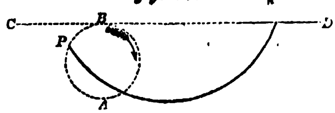
This may be understood without much difficulty. Let us suppose that the distance measured along the curve from A to P is twice that measured from A to Q, then the force accelerating a body which falls from P, is, by supposition, twice that accelerating a body which falls from Q; and in the first second the body falling from P, will fall, along the curve, twice as far as that from Q. Let P be the place which the one body reaches at the end of the first second, and Q_1 that which the other reaches. Therefore $P P_1$ is equal to twice $Q Q_1$, and hence it is easily seen, that since A P is twice A Q, $A P_1$ must be twice $A Q_1$. Since then after the expiration of the first second, one body is twice as far from A as the other, the force urging the one down the curve at the commencement of the second *second*, is twice that urging the other, so that during the second *second* the one will acquire, by the action of this force, twice as much *additional* velocity as the other will acquire. Also it begun that second with twice as much velocity as the other. The one *beginning* the second, then with twice the velocity of the other, and acquiring twice as much additional velocity *during* the second, must move during the second through twice the space. Thus, if P_2 and Q_2 be, respectively, the places of the bodies after the second *second*, then $P_1 P_2$ equals twice $Q_1 Q_2$. And reasoning in the same way, in respect to the third second *and* every succeeding second, we shall find

that during each second, the one body describes twice the space that the other does. After the number of seconds which will bring then the body which fell from Q to A, the body which fell from P, (having described in each second twice as great a distance as the other,) will on the whole, have described a space equal to twice Q A; that is it will have described the whole space P A. Or, in other words, *it* too will at that instant have arrived at A. The same reasoning applies whatever proportion the distances of the two bodies from A may have borne at the beginning of their motion. They will, on the supposition which has been made, arrive at the same instant at A. A curve possessing the property we have supposed, is called a *tautochronous* or *isochronous* curve.

273. The Cycloid is an isochronous Curve.

If, exactly on the circumference of a circular board A B, the point of a pencil P were fixed, and,

fig. 77.



the board being laid flat on a piece of paper, if it were then made to *roll* along the straight edge of a rule C D, the pencil would describe on the paper a curve called a cycloid; and this curve would possess the property of *isochronism* described in the last article. If a piece of wood or metal were

bent exactly to the form of this curve, and if, being placed upright, balls were allowed to roll from different points in it; then, if there were no resistance of friction or the air, these would be found all to reach the lowest point of the curve in the same time.

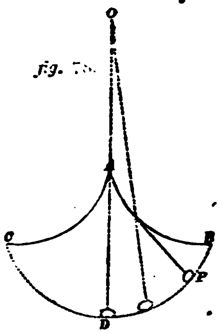
274. To make a Pendulum oscillate in a Cycloid.

A body cannot *roll* on a curved surface, such as that supposed in the last article, without *friction;* and this friction cannot but materially interfere with the equality of the times of its descent. The most effectual way of getting rid of this friction is to substitute, for the resistance of the curve, the tension of a string, to which the body is suspended; provided the tension of this string can be made to act at every point precisely as the reaction of the curve does. Thus, for instance, if the curve were a circle, this would be easy. A body suspended from a string and allowed to oscillate, would oscillate precisely under the same circumstances that a body sliding without friction on the surface of a circle would. The tension of the string, and the reaction of the surface, being both of them forces perpendicular to the circumference of the circle, and acting so as to keep the body in the circle.

To make a body, suspended from a string, descend of itself in a curve of the form of a cycloid, the direction of the string being always perpendicular to the direction of the descent, would, however,

seem to be nearly an impossible task; nevertheless by a remarkable property of the cycloid, it is easily effected. That property is the following.

If there be shaped out two surfaces, accurately of the form of half cycloids, as represented by A B and A C, and if they be placed together so that their extremities join in A, and their bases are in the same horizontal line; and if a body P, be suspended between them from the point A, by a string whose length is such that it will just wind over either of the half cycloids from A to B or from A to C; then, this body being left to itself, the string will, in the subsequent oscillations, so wind itself on the cycloidal cheeks A B and A C, and unwind itself from them, as to cause the body to describe a curve B D C, which is accurately a cycloid. Thus then, from whatever point it is made to fall, the body P will, under these circumstances, fall in the *same time* to D, and passing that point, to whatever height in the opposite curve D C it ascends, it will fall from that point back again to D in the same time. So that all its oscillations will be *isochronous*, or performed in the equal times. That great desideratum, a perfectly isochronous pendulum, would, by this contrivance, be obtained, were it not that it is impossible to find any substance of which the string A P can be formed, which shall be sufficiently strong, and yet so flexible, that no force shall be



required to *bend* it on the two cycloidal cheeks, and such, that no adherence shall take place between it and them. These causes of error, slight as they appear, are yet sufficient so materially to affect the oscillations of a pendulum thus formed, as to render it greatly inferior to the simple pendulum, which we are about to describe.

275. The Simple Pendulum.

If a line were drawn from D to A (see *fig.* 78. in last article), and continued to a point O as far beyond A, then a circle, described from this point, would accurately coincide with the cycloid B D C at D and for some short distance on either side of that point; so that a body, suspended by a string from the first-mentioned point, and oscillating freely in this circle, would, in point of fact, for some distance on either side of D, be oscillating in the cycloid B D C, and its oscillations would therefore be isochronous, so long as they were confined within that limited distance on either side of D; but if they exceeded that distance, then the path of the body thus suspended, deviating from the cycloid, the oscillations would deviate from their isochronism. Thus then we have a simple pendulum whose small oscillations are isochronous. Moreover, the cycloids A B and A C may be made of any size; therefore A D may be of any length, and the pendulum O D may be of any length. From which it follows that the small oscillations of a simple pendulum, of any length whatever, are isochronous.

This law of the isochronism of the simple pendulum, was one of the discoveries of Galileo. It

was first observed by him, it is said, when very young, in the oscillations of a lamp suspended from the roof of the metropolitan church of Pisa. He was struck by the equality of the times in which the lamp returned from oscillation to oscillation, as its motion gradually subsided; and this observation of a *child* became in the mind of *the man*, a principle of philosophy, on which some of the greatest discoveries of science have been founded.

276. To determine the Time in which a Pendulum of any given Length will perform its Oscillations.

The oscillations of a simple pendulum, which are made in a circle, coincide, if they be small, with oscillations in a *cycloid*. From this consideration it is shown by an easy process of the integral calculus (see *Pratt's Mechanics*, p. 370.), that the number of seconds occupied by each oscillation of a pendulum of a given length, in this country (where the force of gravity is such as to accelerate the descent of a falling body by $32\frac{1}{6}$ feet, or more accurately by 32·19084 feet in each second), may be found by extracting the square root of the length of the pendulum (measured in feet), and multiplying this square root by the decimal 0·55372.* Thus, if it were required to find what would be the

* This rule is represented by the algebraical formula $t = 0{\cdot}55372 \sqrt{L}$, where t is the time of an oscillation in seconds, and L the length of the pendulum in feet.

time between each beat of a pendulum 9 feet long, we must extract the square root of 9, which gives us 3, and multiply ·55372 by this square root; whence we have 1·66116 for the number of seconds

277. To determine what must be the Length of a simple Pendulum, so as to beat any given Number of Seconds.

By a simple transformation of the rule in the last article, which every one acquainted with algebra will understand, we obtain a rule to determine what must be the length of a pendulum, that its oscillations may be of any duration that we may require. "Square, or multiply by itself, the number of seconds which the pendulum is to beat, and multiply this result by the number 3·2616, the product will the required length in feet." Let it be required for instance, to find what must be the length of a pendulum, so as to beat once in every 2 seconds. Squaring 2 we get the number 4, and multiplying this number by 3·2616, we have 13·0464, or a little more than 13, for the number of feet. If it were required to find the length of a pendulum which would beat single seconds, we must square 1, which gives us 1, and this, multiplied by 3·2616, gives 3·2616, or somewhat more than $3\frac{1}{4}$ for the length in feet.

278. To measure the Force of Gravity at any Place, by observing the Beats of a Pendulum.

The force which causes the motion of a pendulum is gravity; its motion at any place must therefore be dependant upon the energy of the force of gravity at that place. The following is the very simple relation which connects them. "If the length of the pendulum were divided, by the additional velocity which gravity communicates to a falling body in each second at the place of observation, and the square root of this quotient being extracted, if the result were multiplied by the number 3·1415, that product, would equal the number of seconds in each oscillation."* From this relation, an easy process of algebra gives us this other. "If the length of a pendulum be divided by the square of the number of seconds which it requires to complete each of its oscillations, and if this quotient be multiplied by the number 9·8696, this last product will exactly equal the number of feet by which gravity will at that place, increase the velocity of the descent of a falling body in each second of time." This number of feet is what is called the measure of the force of gravity at that

* This relation is expressed by the mathematical formula, $t=\pi\sqrt{\frac{L}{g}}$, where t is the time of an oscillation in seconds, L the length of the pendulum, g the acceleration of gravity at the place of observation, and π the number 3·1415, which is half the circumference of the circle, whose radius is unity.

place. Suppose, for instance, it were observed at any place, that a pendulum whose length was 13·0464 feet, beat once every two seconds; and it were required to ascertain from this fact what was the force of gravity at that place. Dividing the length by the square of 2, or 4, we have 3·2616, which, being multiplied by 9·8696, gives 32·1908, which is very nearly the force of gravity in this country. In making observations with the pendulum, to determine the force of gravity at different places, it is usual, at each observation, to alter its length, until it is such as to make it beat *single* seconds; the above rule then becomes greatly more simple. "Let the length of the pendulum at which it beats seconds, be accurately measured. This length, multiplied by the number 9·8696, will be the measure of the force of gravity at that place.

279. The Force of Gravity diminishes as we approach the Equator.

By observations such as these, it is found that the force of gravity diminishes as we approach the equator; a less length being required to make a pendulum beat seconds there than here; so that a pendulum clock which went truly here, would, if carried there, go too slow, and would require to have its pendulum shortened. This striking phenomenon is explained by the flattened shape of the earth. Were it a perfect sphere, the force of gravity would be the same every where upon its surface. A table contained in the Appendix, and extracted

from the "Physique" of Pouillet, contains the result of the various observations which have been made with the pendulum, and a comparison of these results with those which are given by theory, on the supposition that the earth is accurately of the form of a spheroid.

280. To find the Depth of a Mine by observing the Beats of the Pendulum.

The force of gravity as we descend *into* the earth, does not vary by the law as it does when we descend towards the earth's surface from the regions above it.

A person descending from the top of a high mountain, and making observations from time to time with a pendulum, would find the force of gravity increasing continually until he reached the level of the sea; if, then, he descended a deep mine, observing his pendulum, as before, from distance to distance, he would find the force of gravity, instead of increasing, to diminish continually. The reason of this may be explained as follows: let the earth's mass be supposed, when he has descended to any distance, to be divided into two parts—one a spherical shell, extending over the whole of its surface, and having for its thickness the depth to which he has descended, and the other a solid sphere included in this shell and filling it. Now it is a remarkable fact, that the attractions of the different elements of a spherical shell, of whatever thickness, upon a body, any where situated in the interior or hollow of the shell, exactly counterbalance one another; so that the

body, being drawn in every direction alike, has no tendency to move in any one direction rather than another; and were the earth hollow, and its cavity a sphere, could we descend into it, we might float about in the void space, without, any effort — every muscular exertion would, indeed, be a source of *inconvenience* and *danger* to us, and the principal anxiety of our lives would be to guard ourselves against these continual collisions, upon the opposite walls of our prison-house, which each effort would tend to produce.

Since, then, this shell of the earth above him exerts no attraction upon a person who descends into it, the whole force by which he is attracted must be that of the solid sphere which it encloses. Now this sphere, beneath him, diminshes its diameter perpetually as he descends; whilst his position remains, in respect to this lesser sphere, precisely the same as it was in respect to the greater, when he was at the *surface;* he may, in fact, be considered as standing continually, in his descent, on the surface of a diminishing sphere; being then attracted continually, under the same circumstances, but by a *less* quantityof matter, it is clear that he must be *less* attracted.

It is found that this diminution of the attraction, is exactly proportional to the diminution of the distance from the earth's centre; and applying this principle to determine the effect of the diminished attraction on the motion of the pendulum, we have the following rule to determine the depth of a mine.

Observe the number of beats which the pen-

dulum loses in one day, by being carried into the mine; $\frac{5}{54}$ths, or nearly $\frac{1}{11}$th of that number of seconds, will give the depth of the mine in miles.

281. The Centre of Oscillation.

A simple pendulum is supposed to be a material point, suspended from a string without weight. Such a pendulum can have no *real* existence. *Every* material body which we can cause to oscillate is, in reality, a *combination* of material points, and therefore a *compound* pendulum. If each of the material points of which it were composed, were *free* to oscillate alone, each would have (art. 276.) its own different time of oscillation, dependant upon its own distance from the point of suspension. By reason of the *connexion* of these points into one mass, they are all made to have the *same* time of oscillation; the times of oscillation of some being thus made longer by their union with the rest, and those of others shorter. Now, between those points whose times of oscillation are made longer than they would be if they were free, and those whose times are made *shorter,* it is evident that there must be some point, where one of these states passes into the other, and where, therefore, the time of oscillation is neither made longer nor shorter, but is the same as it would be if the particle were free; this point is called the CENTRE OF OSCILLATION.

Its position may be determined for any body, whose parts are of geometrical forms, by certain rules of analysis. Having thus calculated the dis-

tance of this point from the point of suspension, we know at what distance a *single* point, suspended freely, would oscillate in exactly the same time, that the whole of the compound pendulum does. Now, knowing this distance, we can tell what would be the *time* of oscillation of this single point, since it would be that of a *simple* pendulum (see art. 276.): we can therefore tell the time of oscillation of the *compound* pendulum. Or, conversely, observing the time of the oscillation of the compound pendulum, we can calculate where its centre of oscillation must be. Both these calculations require, however, a knowledge of the force of gravity at the place of observation.

282. Practical Method of determining the Centres of Percussion and Gyration.

It is a remarkable fact, dependant upon some dynamical relation, which has not, we believe, hitherto been traced out—that the centre of *oscillation*, in respect to any given axis of suspension of a body, is also its centre of *percussion* in respect to that axis; determining the *one*, therefore, we, in fact, determine the *other*. Now it has been shown (art. 281.), that the centre of oscillation of a body may be found in respect to any axis, simply by observing the *time* which it requires to complete each of its oscillations, when suspended from that axis. This time being known, the length of a *simple* pendulum which will oscillate in the same time, may be found (art. 277.), and this length is the distance of the centre of *oscillation*, that is of the

centre of *percussion*, from the axis of suspension.* Thus, then, to determine by experiment the distance of the centre of percussion from the axis, let the axis be placed in a horizontal position, and the body suspended, freely, from it: let the body then be slightly *deflected* from the position in which it rests, and allowed to oscillate about it; observe the time, in seconds, of any one of its oscillations (they will all be equal); square this number of seconds, and multiply it by 3·2616; the product will be the distance required.

Having thus ascertained the position of the centre of percussion, and knowing that of the centre of gravity, we can readily determine from these the centre of *gyration*. We have only to take the product of the distances of these two points from the axis of suspension, and extract the square root of

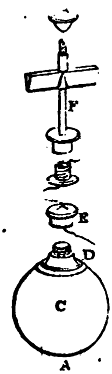
* There is, moreover, a very simple method of determining the centre of percussion or oscillation, in respect to a particular axis of suspension, from a knowledge of what is its position when suspended from another axis. Thus wishing to determine the centre of percussion of a *forge* hammer, we might suspend it from any length of cord like a pendulum, and observe the time of one of its oscillations. This fact, with a knowledge of the position of its centre of gravity, would be sufficient to enable us to determine the centre of percussion of the hammer, about the axis round which it actually works. This fact is mentioned here because there is a great practical inconvenience in determining the oscillations of so large a mass, when suspended thus from a string, rather than about the axis round which it works. The calculation to which reference is made above, would not probably be intelligible to those not versed in analytical mechanics, and those who are will need no explanation of it.

that product: the number thus obtained will determine the distance of the centre of *gyration* from the axis. The centre of percussion is the same with the centre of spontaneous rotation (art. 230.).

283. The Pendulum of Borda.

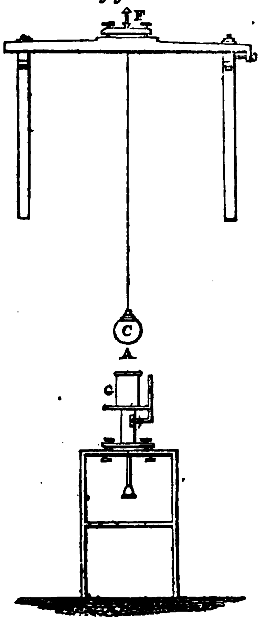
The parts of this pendulum are represented in the accompanying figure. C is a sphere of platinum, a metal, less than any other subject to alteration in its dimensions, from changes in temperature. To this ball the piece D is made to adhere, by turning its under surface of a cup-like form, and accurately of the same curvature as the surface of the sphere, and then rubbing a little grease upon it. E is a piece which screws upon D, and in the centre of which is a small hole, for the attachment of a copper wire, which is to suspend the ball. A similar contrivance attaches the opposite extremity of the wire to the suspending piece F; which is composed of a cylindrical piece of steel, passing *through*, at right angles, and fixed *in*, a triangular prism of steel, called a knife edge. When the pendulum is *used* the two ends of this knife edge are made to rest upon two agate planes, whose surfaces are accurately horizontal, and upon the same level; these are supported upon an upright iron frame, with a contrivance for adjusting their position, and the pendulum hangs suspended be-

fig. 79.



tween them. The frame, with the pendulum supported upon it, is represented in the next cut. Since, by reason

fig. 80.



of variations in temperature, the wire is subject to continual variations in its length, it becomes necessary at each observation to measure its length, with accuracy. Beneath the pendulum a contrivance is represented which is specially directed to this object. It consists of an iron pedestal solidly fixed, at the top of which is a vertical stand or column, capable of being raised by means of a screw. When the length of the pendulum is to be measured, the screw is slowly turned until, in its oscillations, the ball just grazes the top of this column. The stand is then fixed. The pendulum is removed from the agate planes which support it, and is replaced by a rod, carrying a knife-edge exactly as the pendulum does. The opposite extremity of this rod is bored hollow, and a cylindrical piece of brass fitted into this hollow end, may be made by means of a screw to advance any distance out of it so as to lengthen the rod. The rod being suspended by its knife-edge on the agate planes precisely as the pendulum was, is made to

oscillate, and the screw is turned so as to lengthen it, until at length, in its oscillations, it grazes the top of the stand G, as the pendulum did. It is then taken off and measured, and its length is the precise length, between the point of suspension of the pendulum and the bottom of the ball.

It will be observed, however, that this is a *compound* pendulum, so that the length thus measured is not the length of the *simple* pendulum which would oscillate in the same time. To find that length, we must find the centre of oscillation. In the case of this pendulum, the parts of which are of very simple geometrical forms, this is done by calculation without much difficulty. And being thus found to be at a particular point C, for any given length of the wire, so that the length of the simple pendulum is C F; it is easily shown by the formulæ, that when the wire is made to vary slightly in length, the distance of C from F will vary by very nearly the same quantity. Thus then to find the length of the simple pendulum which will oscillate in the same time, we have only to diminish the measurement, taken as above, by the constant and known distance C A.

284. Borda's Method of Coincidences for observing the Time of Oscillation of a Pendulum.

Let a pendulum clock be placed behind the pendulum whose oscillations are to be observed, and let its pendulum be made, nearly but not quite, of

the same length*, or of such a length as to oscillate nearly, but not quite, in the same time; the points of suspension of the two being immediately behind one another. If now both pendulums be set in motion at the same instant, and looked at in front, after the first oscillation they will be seen to move differently, one gaining upon the other a little, at each oscillation, and this crossing of the oscillations will continue, until one has gained upon the other a complete oscillation, when for an instant their motions will *coincide*, again to deviate, in each succeeding oscillation, until another complete oscillation is gained. Neglecting then all the *separate* oscillations, let all these *coincidences* be observed for a given time, say three hours. The hand of the clock will show how many oscillations *its* pendulum has made, and the number of coincidences will show the number of oscillations which the other pendulum has gained or lost upon it. Adding or subtracting the number of coincidences, from the number of oscillations shown by the clock, we shall get the exact number made by the pendulum we wish to observe, in the three hours. Dividing the number of seconds in the three hours by this number of oscillations, we shall have the duration of each oscillation, in seconds.

* This change of the length of its pendulum will alter the going of the clock; but that is immaterial; the hand will still register the *number* of the oscillations, which is all that is required.

285. To determine experimentally the Position of the Centre of Oscillation of a Body without knowing the Force of Gravity at the Place of Observation.

It is a remarkable property of the centre of oscillation, which was first given by theory, that if a body be suspended from any point, and the time of its small oscillations about that point be observed, and if it be then suspended from another point, this second point being that which was its centre of oscillation before; then its time of oscillation will *now* be found to be precisely the same as it was when suspended from the first point, that point having become its centre of oscillation to this new point of suspension. This property is usually described as that by which the centres of suspension and oscillation are *convertible*. What is meant by it, may perhaps be more clearly understood as follows. If A be any point in a body from which it is suspended, and B be its centre of oscillation in respect to the point of suspension A, and if the body had been suspended from B instead of A, its centre of oscillation would have been A instead of B.

Since in both cases the distance of the centre of oscillation from the point of suspension would have been the same, it is clear that the *times* of oscillation would have been the same.

Thus then to determine by experiment, the centre of oscillation B of a pendulum, about any point of SUSPENSION, we have only to find by experiment, a point B about which it will oscillate in the same

time as it does about A; that is, we must suspend it from different points, until at length we find one in respect to which this equality obtains.

286. Captain Kater's Pendulum.

A most ingenious contrivance, introduced by Captain Kater, greatly facilitates the experimental determination of the position of the centre of oscillation described in the last article. On the rod of his pendulum is placed a moveable or sliding weight. By moving this weight, the form of the oscillating body, and thus the position of its centre of oscillation, may be changed, so that when by trials as described above, two points are found about which the times of oscillation are *nearly* the same, by moving this weight, they may, without any further change in the position of the points, be made *exactly* the same. By means of the slide the pendulum *itself* is in fact altered, so as to have its centre of oscillation in the point we wish it.

In Captain Kater's pendulum, the point B being roughly determined to be the centre of oscillation to the point of suspension A, triangular pieces of steel called knife-edges are fixed through the middle of the rod at those points. The projecting extremities of the knife-edges at one of these points, say A, being made to rest, by their angles, upon agate planes, the pendulum is allowed to oscillate freely, and the time of oscillation observed. Its position is then *reversed*, and it is allowed to oscillate in the same way upon the knife-edge at B. If the time of oscillation is the same as before, then B is the centre of

oscillation, and all that is required is known. If the time of oscillation be not the same, the sliding weight is *moved* until it *becomes* the same. When this is the case the centre of oscillation is in B, and A B is the length of a simple pendulum which would oscillate in the same time. If then the time of oscillation be observed, the force of gravity may be calculated by the rule (art. 278.), in which A B is to be taken for the length of the pendulum.

Sometimes two moveable weights are used, one of which is moved by means of a micrometer screw, to effect a more delicate adjustment.* It is a remarkable fact, proved by analysis, that the result, in experiments made with this pendulum, will not be affected, if for the knife-edges *cylindrical axes* be substituted.

Mr. Lubbock has shown in the "Philosophical Transactions for 1830," that a slight deviation of the knife-edges, from a position accurately transverse or perpendicular to that in which the pendulum tends to oscillate, is of no importance if *it be* a deviation sideways or horizontally; but that a deviation of one degree, *vertically*, would be sufficient to increase the number of vibrations by 3 in 24 hours. An error in placing the agate planes truly in a horizontal position, has a yet greater effect. The sixth part of a degree of deviation will in this case cause an increase of 6 vibrations in 24 hours.

* For a detailed account of the experiments of Captain Kater, see the Philosophical Transactions for 1818.

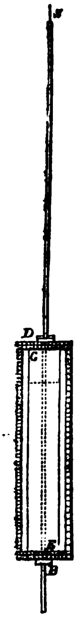
287. Compensation Pendulums.

A pendulum, of whatever material it may be formed, necessarily varies its *dimensions* with every change of *temperature*. From this cause arises a variation in the position of its *centre* of oscillation, and in the *time* of each of its oscillations. In the pendulums used for clocks, it becomes necessary to introduce some contrivance for *compensating* this variation, where great accuracy is required. The method described in the last article, of varying the position of the centre of oscillation, by means of a sliding weight, could the weight be made to slide, of itself, into a different position with each variation of temperature, might evidently answer the purpose. It is, in fact, somewhat on this principle, that the compensation pendulum is formed. The general tendency of the expansion of the material of the pendulum is evidently to *lengthen* it, and to carry its centre of oscillation lower; a compensation would be made, if there were a *part* of this pendulum whose mass, was, by its expansion, raised *higher*. The one tending to raise and the other to depress the centre of oscillation, by each additional degree of temperature, it is clear that these elements, possibly, might be so combined as to keep it exactly in the same place.

One of the simplest contrivances of this kind, at once the earliest and practically the best, is Graham's Pendulum. The rod S B of this pendulum is of steel. It carries a frame or stirrup D B, on which is supported a glass cylinder G H containing mercury. By every increase of temperature,

the steel rod *elongates*, carrying the centre of oscillation of the whole, farther from the point of suspension. But, by the same change of temperature, the mercury *rises* in the cylinder, thus carrying the centre of oscillation *upwards*, and towards the point of suspension. A right adjustment of the quantity of mercury to the length of the rod, will cause these two opposite effects to *neutralise* one another, and preserve the centre of oscillation in its original position. To determine this quantity of mercury, it is customary to assume, that the *surface* of the mercury must be made to remain always at the same distance from the point of suspension. So that, by whatever distance it may be depressed by the elongation of the rod, it may be raised the same distance, by its own expansion. The computation on this principle is easily made. We have only to know the fraction of its length, by which a rod of steel elongates for each additional degree of temperature, and the similar fraction by which a given quantity of mercury increases its bulk. The former fraction is ·00000636, and the latter ·00001066. From this last fraction, we could *readily* ascertain by how much the column of mercury in the glass cylinder, increased its height, and elevated its centre of oscillation, were it not that the same variation of temperature which causes an expansion of the mercury, causes also an expansion of the glass of the vessel which contains it, increasing

fig. 81.



the *capacity* of the vessel. Nevertheless, this disturbing cause may also be taken into the calculation, or at any rate allowance may be made for it by experiment; and thus the quantity of mercury in the glass cistern may be so adjusted as to preserve a position of the centre of oscillation, which approaches to uniformity.

There are, however, causes of variation in the position of the centre of oscillation, and in the time of oscillation, *other* than those which have been spoken of, and which appear scarcely to admit of compensation. The first is the difference of the *times* requisite to communicate the variation of each degree of temperature to the two metals, mercury and steel. From a communication recently made by Mr. Dent to the British Association of Science, it appears that the mercury of this pendulum requires nearly four times the interval to acquire a given variation of temperature that the steel rod does. During the whole of this interval the pendulum cannot then be in a state of compensation, and there must be a variation in its beats. Another cause of variation, first noticed by Mr. Dent, and uncompensated, is in the varying elasticity of the spring. This elasticity diminishes as the temperature increases, and to this cause Mr. Dent traced, in some of his experiments, an error of nearly 2 seconds in 24 hours, produced by an artificial elevation of the temperature of the spring to 95°. He has recommended the substitution of a cast iron cylinder for the reception of the mercury, instead of one of glass. A much more truly cylindrical form can be given to such a vessel, by turning, than a glass cylinder can possibly receive; it can be more easily fixed to the rod;

moreover it possesses this great practical advantage, that the mercury can be *boiled* in it, to expel the bubbles of air which, when it is first filled, or after it has been packed up and removed, are very liable to adhere to its interior surface, displacing the mercury. In his experiments to determine the qualities of a pendulum, thus constructed, Mr. Dent's attention was directed to the fact, hitherto unobserved, that the rate of the pendulum was singularly affected by radiant heat. He found that heat radiated from the fire of the room, in which his experiments were made, affected differently the pendulum having the glass vessel, and that having the iron vessel; the mercury in the former preserving a temperature always 5° higher, than that in the latter. The heat radiated from a lamp, was even sufficient to produce an inequality of 2°, and it was only got rid of, completely by screening both the fire and the lamp.

Directed by this fact, Mr. Dent recommends that the cistern of the mercurial pendulum should always be *blackened* with a composition of lamp black and spirits of wine.

288. Harrison's Compensation Pendulum.

In this pendulum, known as the *gridiron* pendulum, a system of bars, of *steel* and *brass*, are combined in such a way, as that whilst the elongation of the *steel* bars tends to depress the *bob* E of the pendulum, that of the *brass* bars tends to elevate it; and the lengths of these bars are so adjusted, that the depression *thus* produced by the former, for each degree of temperature, shall just be equalled by the elevation produced by the latter.

The shaded lines in the accompanying figure represent the *steel* bars, and the light lines the *brass* bars. The pendulum is suspended from the cross piece A B, to the extremities of which are fixed the steel bars A C and B D, carrying the cross bar C D, through a hole, in which the rod which carries the *bob* passes. On this cross piece C D, rest the two *brass* bars *c a* and *d b*, supporting the cross piece *a b*. Now, it is evident that the cross piece *a b* is *depressed* by the elongation of the rods A C and B D, carrying with them the piece C D, and that it is *elevated* by the elongation of the brass bars *c a* and *d b*. If then, the bars were of such lengths that the elongation of the one pair *should* just equal that of the other, then the bar *a b* would exactly keep its place; or if they were of such lengths that the elongation of the brass bars exceeded that of the steel bars, then the bar *a b* would be elevated, and by a proper adjustment we might thus cause it to be elevated, by any quantity we chose. Reasoning in the same manner, with regard to the next pair of the steel bars of the system, and the next pair of brass bars, it is apparent that, supposing the bar at *a b* to retain its position, we can cause the bar *e d* to retain its position, or to vary it in any way we like, by properly adjusting the lengths of the bars; and that

fig. 82.

this control over the position of the bar *e d*, is rendered yet more perfect, by that which we possess, by a similar adjustment, over the position of *a b*. Being thus able to give to the position of *e d* any elevation we like for each variation of a degree of temperature, we can cause it to raise itself by just as much as that variation of a degree of temperature, causes the single steel bar which carries the *bob* E, to elongate; so that the bob itself shall accurately remain at the same height.

The proper lengths of the bars are easily calculated, from the consideration that the elongation of each pair of bars of brass, ultimately elevates the bar *e d*, whilst that of each pair of steel bars, ultimately depresses it. Now there are three pair of bars of steel, and three of brass in the pendulum shown in the cut; moreover, each pair of steel bars is longer than its corresponding pair of brass bars. If then brass only expanded by the same quantity as steel, for each degree of temperature, then the bar *e d* would be *less* raised by each variation of a degree, than it would be depressed, or, on the whole, it would *sink* for each additional degree, instead of rising as it is required to do. Brass elongates, however, more than steel, for each additional degree of temperature, and it is for this reason that it is used: the expansion of brass is, for every degree of temperature, about $\frac{5}{3}$ds that of steel. It is, however, a mistake to suppose that an adjustment of the rods which preserves the position of the bob E, preserves a uniformity in the oscillations of the pendulum. That uniformity can only be produced, by a constant position of the *centre of oscillation* of the

whole. Now the variations of the lengths of the bars inclosed in the figure A C B D, necessarily produce a variation in the position of the centre of oscillation of that figure, and, therefore of the whole pendulum.

CHAP. VI.

THE RETARDATION OF MOTION — THE PRINCIPLE OF VIRTUAL VELOCITIES — THE MEASURE OF THE DYNAMICAL EFFECT OR THE ACTION OF AN AGENT — THE DYNAMICAL EFFECTS OF DIFFERENT AGENTS — THE MOVING AND WORKING POWERS IN A MACHINE. — THE MOVING AND WORKING POWERS IN ANY MACHINE ARE EQUAL, ABSTRACTION BEING MADE OF THE RESISTANCES WHICH OPPOSE THEMSELVES TO THE MOTIONS OF THE PARTS OF THE MACHINE UPON ONE ANOTHER — THE MOVING POWER IN A STEAM-ENGINE — THE WORKING POWER IN A STEAM-ENGINE.

289. The Retardation of a Body's Motion.

If a body, having acquired a certain velocity by the action of any *accelerating* force, be brought to rest, and then projected back again with an equal velocity, in such a way that it shall traverse in the opposite direction the same path as it did before, being acted upon at the same points of its path by exactly the same forces; but now in *opposite* directions to its motion, as before they acted in the *same* directions, so as now to have become *retarding* instead of *accelerating* forces; then will these take away the force of the body's motion at the same places, precisely by the same quantities that before they increased it; so that, in describing the same length of path, it will now *lose* as much of its velocity as before it *gained* in that length of path. Thus, then, in the same length of path in which before it gained *all* its velocity it will now lose it *all*, and will stop, of its own accord, precisely at the

same point from which before it began to move. A stone, for instance, falling from any height to the ground, and then being projected upwards with a velocity equal to that which it acquired in falling from that height, will ascend again (or, rather, *would* ascend, if the air offered no resistance to its motion) precisely to the same height from which it fell. For a like reason, if the body P (*fig.* 75. art. 272.) be allowed to descend freely on the curve from P to A, and then projected back again from A towards P, with a velocity equal to that which it acquired in its descent, it will ascend (friction and the resistance of the air not being considered) precisely to P, and there of its own accord stop. It is manifest that exactly the same result must follow, if, instead of projecting the body thus backward, up the curve A P, we place another equal and similar curve at A, similarly inclined, but turned the other way, so that the two shall form similar branches of the same curve, like those D B and D C of the curve B C D (*fig.* 78. art. 274.): the body will then *project itself* up one of these curves with the velocity which it has acquired in descending down the other, and will ascend upon the former to a height precisely equal to that from which it has descended on the latter; so that, if it fall from B, then (friction, and the resistance of the air not being considered) it will ascend to C. This reasoning, which is true of a body descending upon a curve, manifestly applies to a body suspended to a string, and oscillating like a pendulum. This suspension is, indeed, but another way of causing the body to descend on a curve.

290. The Velocity of a Body's projection up a Curve may be found by observing the Height to which it ascends upon it.

If a body be projected up a curve, and we observe the vertical height to which it ascends to *lose* all its velocity of projection, we know the height from which it must fall to *acquire* an equal velocity.

We can find, then, what the velocity of its projection was, for we can tell what would be the velocity acquired in falling down the curve from the observed height; that velocity being the same as would be acquired in falling *freely*, or without the curve, through that height. (See arts. 271. and 267.) It is thus that, in the Ballistic Pendulum (art. 215.), the velocity with which the pendulum begins to move, and hence that with which the ball first strikes upon it, is determined by observing the height to which it first oscillates. The following experiment, illustrative of the principle stated in this article, was made by Desaguliers. He took two hollow cylinders, each of them closed at one extremity, and, having filled them with gunpowder, he caused the open extremity of the one to fit into the open extremity of the other. To similar points in the sides of these cylinders were then attached strings of the same length, fastened at their other extremities to the same point in a horizontal axis; and, the whole hanging freely from these strings, the gunpowder was exploded.

The force of motion communicated to each by the explosion should, according to the principles explained in article 211., be the same. Whence,

knowing their masses, might readily be calculated the *ratio* of the velocities of the bodies immediately after the explosion, and hence the relation of the vertical heights to which they would afterwards respectively ascend. This calculation being made, and the heights being *observed*, the experiment and calculation were found accurately to coincide.*

291. THE DEPTH TO WHICH A CANNON OR MUSKET BALL ENTERS INTO A BLOCK OF WOOD, OR A MASS OF EARTH AGAINST WHICH IT IS FIRED, VARIES AS THE SQUARE OF THE VELOCITY WITH WHICH IT IMPINGES UPON IT.

The resistance of such a mass is evidently the same, or nearly so, at every point to which the ball enters: it constitutes therefore a *uniformly* retarding force. Now, if the ball be supposed to *emerge* again from the mass into which it has been fired, commencing its motion from the point to which it has before been made to sink into it; if, moreover, at every point of its motion of emergence it be imagined to be *accelerated* by a force precisely equal to that by which it was, as it entered, *retarded* at that point; the resistance being, in fact, conceived to be turned in the opposite direction, and converted into

* The object of Desaguliers, in making this experiment, was to verify that "law of mechanics which is known as that of the equality of action and re-action." The discussion of this law is advisedly omitted in this work. To every person acquainted with the elementary principles of algebra it will be apparent that in the experiment of Desaguliers, the heights to which the cylinders ascended should be inversely as the squares of their weights.

accelerating forces, then (art. 289.) it will acquire, at the point where it actually emerges, a velocity precisely equal to that with which it before entered the mass there; since, moreover, the force with which it was resisted when it entered the mass was a uniformly *retarding* force, the force with which it will be accelerated, as, on this hypothesis, it leaves it, will be a *uniformly accelerating* force, like that of gravity, and subject to the same description of law. The velocity which it acquires in thus leaving it will then be equal to the square root of some constant number multiplied by the depth (art. 267.), or it will vary as the square root of the depth. Thus, then, the velocity of *the first impact* varies as the square root of the depth, and conversely the depth varies as the square of the velocity of impact. This fact was proved experimentally in a great number of instances by Robins. (See *Robin's Mathematical Tracts*, by Wilson, vol. i. p. 152.)

292. The Principle of Virtual Velocities.

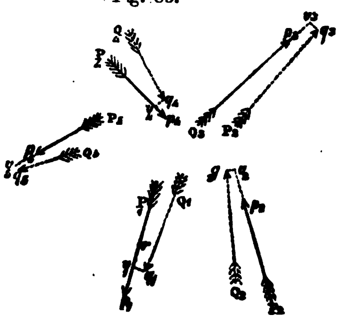
The principle known by this name arises out of that relation between forces of motion and forces of pressure, which has been pointed out in the preceding pages of this work (art. 253.). It embraces every question of equilibrium, and may be considered as including the whole science of statics*;

* The principles of the parallelogram of forces, and the equality of moments upon either of which the whole science of statics may be considered to be founded, may readily be deduced from it.

and it is especially important that it should be known to practical men under its accurate and most general form, because vague and exceedingly erroneous notions of it are prevalent amongst workmen, and conclusions false at once in practice and in theory are deduced from it. To understand what is meant by the *virtual velocity* of a force (which is the only difficulty in the matter), let a system of forces be supposed to be in equilibrium, and let the points of application of two or more of these forces be supposed to be capable of *displacement*, the displacement of any one point bringing about a displacement of the rest. Suppose, moreover, a displacement of this kind to be actually made in the system, but let it be an exceedingly *small* displacement, so that all the moveable points of application afterwards occupy positions different from those they occupied before, but exceedingly near to them, and all the forces applied to them act in directions different from those in which they acted before, but exceedingly near to those directions. From the new point of *application* of each force, drop a perpendicular upon the previous *direction* of that force; then the line intercepted between the previous point of application of that force and the foot of this perpendicular, will be what is called the VIRTUAL VELOCITY of the force. This definition will be more readily understood by a reference to the accompanying diagram, where the arrows $P_1\,p_2$, $P_2\,p_2$, $P_3\,p_3$, $P_4\,p_4$, $P_5\,p_5$, are supposed to represent forces in equilibrium applied to the points p_1, p_2, p_3, p_4, which points are supposed moreover to be

capable of *displacement* under certain limitations. A small displacement is made in one of these points of application, as, for instance, p_1, which is moved to any other point near to it, as q_1, the force upon that point now acting in the direction Q_1 q_1. This displacement of the direction and point of application of one of the forces necessarily brings about a corresponding displacement of all the rest; their new positions are supposed to be represented by Q_2 q_2, Q_3 q_3, Q_4 q_4, Q_5 q_5, and their new points of application by q_2 q_3 q_4 q_5. From these last mentioned points let perpendiculars q_1v, q_2v_2, q_3v_3, q_4v_4, q_5v_5, be supposed to be drawn upon the previous directions of the forces, or these directions produced if necessary; then the lines p_1v_1, p_2v_2, p_3v_3, p_4v_4, p_5v_5, intercepted, on the directions of the original directions of the forces, between their points of application, and the feet of the perpendiculars, are the VIRTUAL VELOCITIES of their respective forces. This being thoroughly understood, the enunciation of the principle of virtual velocities becomes easy. It is this:—

Fig. 83.



293. IF ANY NUMBER OF FORCES BE UNDER ANY CIRCUMSTANCES IN EQUILIBRIUM, AND TO

ANY OR ALL OF THEIR POINTS OF APPLICATION THERE BE COMMUNICATED INDEFINITELY SMALL MOTIONS IN ANY DIRECTIONS; THEN THESE FORCES, BEING EACH MULTIPLIED BY ITS CORRESPONDING VIRTUAL VELOCITY, AND THE SUM OF THESE PRODUCTS BEING TAKEN IN RESPECT TO THOSE FORCES, THE DISPLACEMENTS OF WHOSE POINTS OF APPLICATION ARE TOWARDS THE DIRECTIONS OF THEIR FORCES, AND THE SUM IN RESPECT TO THOSE WHOSE DISPLACEMENTS ARE FROM THE DIRECTIONS OF THEIR FORCES; THE ONE SUM SHALL EQUAL THE OTHER.

Thus, referring to the diagram, let the forces P_1, P_2, P_3, — be supposed to be respectively multiplied* by their virtual velocities, p_1v_1, p_2v_2, p_3v_3, p_4v_4, p_5v_5; then, it being observed that the displacements of the points p_2, p_3, and p_5, are *towards* the directions of the forces acting upon those points, whilst the displacements of the points p_1 and p_4 are *from* the directions of the forces acting at those points; by the principle of virtual velocities, the sum of the above-mentioned products, in respect to the first three, shall equal their sum in respect to the two others. That is the sum of the products P_2 by p_2v_2, P_3 by p_3v_3, P_5 by p_5v_5, shall equal the sum of the products, P_1 by p_1v_1, and P_4 by p_4v_4.†

It is evident that if the displacement of any point take *place* actually *in* the direction of the force

* It is here meant that the *number* of units in the force is to be multiplied by the *number* of units of length in the virtual velocity.

† This relation is expressed algebraically thus:—

$$P_2 \cdot P_2 v_2 + P_3 \cdot P_3 v_3 + P_5 \, P_5 v_5 = P_1 \, P_1 v_1 + P_4 \, P_4 v_4$$

applied at that point, then the perpendicular will vanish, and the virtual velocity will be the *actual displacement* of the point of application.

If, for instance, the point p_1 had been displaced not to q_1, but actually in the line of direction of the force P_1 or along the line P_1 p_1 to any point r, then p_1 r, the *actual displacement* of the point of application of P_1 would have been also its *virtual velocity*.

If, moreover, the system to which the forces are applied had been such that, the point of application of any one being displaced actually in the *line of direction* of that force, the points of application of all the rest should have been displaced *in the lines of direction* of their respective forces, then the *actual displacements* of all would have been their *virtual velocities*.

A particular case of the principle of virtual velocities may then be enunciated under the following form:—

"*When the relation of the parts of a system acted upon by any number of forces is such that the point of application of any one force being displaced in the line of the direction of that force, then the displacements thereby produced in all the other points of application shall be in the lines of direction of their respective forces; then each force being multiplied by its actual displacement, the sum of these products in respect to those whose displacements are from the directions of the forces shall equal the sum in respect to those whose displacements are towards those directions.*"

The circumstances here supposed obtain in respect to almost all the simple MECHANICAL POWERS, as

they are usually applied, and in respect to a great number of compounded machines, especially such as act by animal power.

Take, for instance, the various applications of the systems of LEVERS, shown in page 129. In each, when the lever is first put into operation, the points of application of the power and weight are made to move in *vertical* directions; that is, in the lines of direction in which they severally act. The virtual velocity of each is therefore its *actual displacement*, so that by the principle of virtual velocities the displacement or motion of the point of application of W, multiplied by W, is equal to the displacement or motion of the point of application of P, multiplied by P.

Now, it is evident in *fig.* 25., that since A is eight times as far from the fulcrum as W, therefore the displacement of A must equal eight times that of W. Thus, then, it follows, that W, multiplied by the displacement of W, equals P multiplied by eight times the displacement of W, and therefore, that W equals eight times P; as it was shown to be by the principle of the equality of moments.

Again, in *fig.* 26., since A is nine times as far from the axis of motion as W is, it evidently moves nine times as fast; therefore, by the principle of virtual velocities, W, multiplied by the displacement of W, equals P multiplied by nine times the displacement of W, so that W equals nine times P, as it ought.

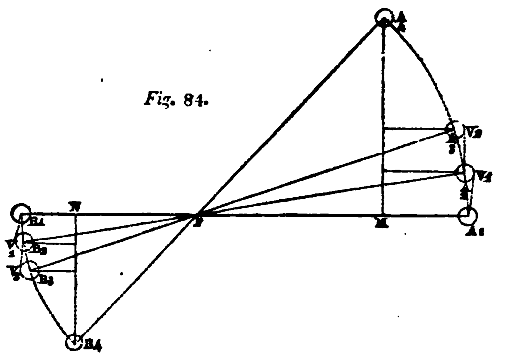
Again, in the wheel and axle (art. 135. *fig.* 31.), the displacements of the power and weight evidently take place in the lines of the directions of those forces; these displacements are therefore the

virtual velocities. So that by the principle of virtual velocities, W, multiplied by the displacement of W, is equal to P multiplied by the displacement of P; but it is evident that the displacement of W (being the length of string wound *on* the lesser cylinder) is to that of P (being the length of string wound *off* the greater cylinder) as O A to O B: hence it follows, by this principle, that W, multipled by O A, is equal to P multiplied by O B; which relation was also shown to result from the principle of the equality of moments.

If the relation of the parts of the system be such that after the first small displacement, causing all the various points of application to take up new positions near the first, the forces shall, under these altered circumstances, be still in equilibrium; then a *second* small displacement, similarly produced in each out of its second position into a *third* will, like the first, be subject to the principle of virtual velocities; and if, in these third positions, they are in equilibrium, then a *fourth* displacement will be subject to the same law, and so on. From this it follows, that if the system be such that the forces applied to it are *continually* in equilibrium, throughout all the displacements to which they are subjected, then if, after any number of such displacements, each force be multiplied by the *sum* of all the virtual velocities corresponding to these displacements, the equality spoken of before shall obtain between the sum of these products, in respect to those displacements which take place *towards* the direction of the force and the sum of those which take place *from* it.

Thus, for instance, if the balls A_1 and B_1, and the bar which connects them, be in equilibrium about

Fig. 84.



the point F, that point, supporting the centre of gravity of the whole system, and the whole be turned round into a series of new positions, differing slightly from one another, and represented by the dotted lines, then, since in each position the system will be in equilibrium, it follows, by the principle of virtual velocities, that the weight B_1, multiplied by the sum of the virtual velocities $B_1 V_1$, $B_2 V_2$, &c., shall equal the weight A_1, multiplied by the sum of the virtual velocities $A_1 V_1$, $A_2 V_2$, &c. Now the former sum is evidently equal to the vertical line B_4N, and the latter to A_4M; thus, then, it follows that the product of B_4N by B_1 equals the product of $A_4 M$ by A.

If the displacements of all the forces of the system take place actually in the lines of the operation of the forces, and the equilibrium remain after

every displacement, then the condition of an *exceedingly small* displacement disappears from the enunciation of the general principle. In this particular case, each force being multiplied by its actual displacement, however great it may be, the sum of these products, in respect to those displacements which take place towards the direction in which the force acts, shall equal the sum in respect to those which take place *from* that direction.

This is the principle known to workmen, as that by which what is gained in power is lost in velocity. Its application is limited to the particular case last described; applied beyond those limits, it leads to serious errors.

All the systems of pulleys represented in *fig*. 65. p. 216. offer illustrations of it. In the first, the single fixed pulley, it is evident that the displacement of the power is exactly equal to that of the weight; and, since the product of the former, by its displacement must equal that of the latter by its displacement, it is evident that to make up this equality the power must equal the weight.

In the second system, the string which carries the power evidently lengthens by as much as the two strings which carry the weight, together shorten; that is, by twice as much as either shortens separately; so that the displacement of the power is equal to *twice* that of the weight. By the principle of virtual velocities, then, the weight multiplied by the weight's displacement equals the power multiplied by twice the weight's displacement; so that the weight equals twice the power.

In the fourth system, the power evidently descends

by twice as much as the first moveable pulley ascends.

Again, this last ascends by twice as much as the second moveable pulley ascends; so that the power ascends by four times as much as the second moveable pulley. This second moveable pulley ascends similarly, by twice as much as the third, and by four times as much as the fourth; so that, on the whole, it is evident that the power is displaced by eight times as much as the weight. By the principle of virtual velocities, the weight, then, multiplied by the weight's displacement, equals the power multiplied by eight times the weight's displacement. So that the weight equals eight times the power. A similar method of reasoning may be very easily applied to all the other systems, except the third, which offers some difficulty.

In this system the displacement of the power is made up of the lengthening of the string to which it is attached and the descent of the pulley over which that string passes. Now, the lengthening of the string which carries the power results partly from the ascent of the weight, and is in this respect the same as in the last case of the single moveable pulley, equalling twice the ascent of the weight; and partly it results from the descent of the pulley over which it passes, in this respect equalling the descent of that pulley, and therefore equalling the ascent of the weight; so that, upon the whole, the string which carries the power lengthens by three times the ascent of the weight; again, the pulley over which this string passes descends by as much as the weight ascends,

so that altogether the power P descends by four times as much as the weight ascends. By the principle of virtual velocities, therefore, the weight equals four times the power.

294. Of Machines.

A machine is an assemblage of parts destined to receive the operation of an agent, and to transmit it to the point where it is to be applied, modifying it in the transmission, according to the circumstances under which it is to be applied. Thus, in a machine there are to be considered, 1st, the circumstances under which the operation of the moving power is received; 2dly, the circumstances by which it is modified during its transmission; 3dly, the circumstances under which it is applied at its working points. The power which operates directly from the agent we shall here call the MOVING POWER OR ACTION on the machine; the power actually applied by the machine at its working points in the performance of its work, we shall call the WORKING POWER OR ACTION on the machine. It is evident that the *moving power* produces the *working power*, and *also* the motion of all the parts if the machine, overcoming the resistances which oppose themselves to the motions of those parts; so that the *working* power is essentially less than the *moving* power in all cases, and in complex machines *greatly* less, by reason of the great number of surfaces which in those machines are made to move upon one another, and the great amount of the resistances which for that reason oppose themselves to their motion.

295. THE STATE OF THE MOTION OF A MACHINE IS, AT FIRST, A STATE OF ACCELERATED MOTION.

This is evident from the principles laid down in art. 253. Each part of the machine must have, before it can move, a FORCE OF MOTION or MOMENTUM communicated to it, and such momentum being in its nature an accumulation of pressures, requires, in every case, TIME, and a series of impulses to its accumulation.

296. THE FORCES OPERATING IN A MACHINE BEING IN EQUILIBRIUM IN EVERY RELATIVE POSITION WHICH THE PARTS OF THAT MACHINE CAN BE MADE TO ASSUME, ANY MOMENTUM OR FORCE OF MOTION THROWN INTO THE MACHINE WILL REMAIN IN IT CONTINUALLY, UMIMPAIRED AND UNALTERED.

In the statement of this principle, all consideration of the resistance of the air is omitted, and the friction of bodies in motion is supposed not to be affected by the velocity of motion (see art. 172.). The truth of it is immediately evident from the consideration, that the forces operating upon the machine — including the friction of its parts, and every other form of its resistances—being supposed in every position of its parts to be in equilibrium*, it follows that there cannot at any period of its

* The equilibrium here spoken of, and every where else in this work, is that of the state immediately bordering upon motion.

motion be any force *opposing* itself to the force of the motion of its parts; this force, then, by the principle of the permanence of the force of motion (art. 193.), being once communicated, must remain in the machine unimpaired.

If the forces operating upon a machine be not in the state of equilibrium bordering upon motion when motion is first communicated; or if this condition of equilibrium does not continue throughout the motion of the parts of the machine; then the whole quantity of motion operating in the machine will continually *vary;* if the *power* be in excess it will *increase*, if the *resistance* be in excess it will *diminish*. In the former case the excess of the power over that necessary to produce equilibrium (remaining unopposed) continually generates additional momentum; in the latter case the excess of the resistance, over that portion of it which is overcome by the power, operating in a direction opposite to the motion, continually diminishes, and eventually destroys it.

Although in the first period of the motion of a machine, the power operating in it may be greater than that which would produce an equilibrium with the resistance, yet *practically*, in every machine, that relation of these forces, which is necessary to their equilibrium (and which is accompanied by a permanence of the force of motion), grows up shortly after the motion has commenced. It is a LAW imposed in the economy of the creation around us, that no motion shall pass a certain finite limit.

A few examples will render this sufficiently evident: —

A ship, when at *rest* upon the water, and with her anchor weighed, is in a state of equilibrium bordering upon motion; the pressures upon her bows and stern are equal, and any force, however slight, acting upon her horizontally in the direction of her length, would be sufficient to move her. Her sails are unfurled, and she receives the impulse of the wind, a power which, if it continued, as at first, unopposed, would continually accumulate velocity in her, until she flew through the water as fleet at least as the wind itself. That equilibrium, however, of the forces upon her head and stern, which obtained at first, *does not remain;* the forces upon the head, constituting the resistance, increase with the motion*, and those upon the stern diminish; and in a short time the impulse of the wind upon the sails, and the pressure of the water upon the stern, come to be together precisely equalled by the increased resistance upon the bows. The state of equilibrium is now, then, *reproduced;* and as long as it is kept up, the vessel moves on with the quantity of force of motion which it had when it passed into this state of equilibrium, unimpaired.

Again, let us suppose a *pulley* suspended at any height, however great, above the earth's surface, and a string of equal length to pass over it, carrying at its extremities two unequal weights. Suppose the greater weight to be drawn up, and the whole machine then to be left to itself, the excess of the greater over the lesser weight will evidently be an *unopposed power*, and will communicate motion to the system; which motion, by the continual im-

* They increase as the square of the velocity.

pulses of this power, would be continually accelerated, with no other limit than that of the height through which the weight is allowed to descend; so that by increasing this height we could accumulate velocity and force of motion to any conceivable extent, were it not for the resistance of the air; this would effectually limit any such accumulation. It is a resistance which would be found rapidly to increase with the velocity of the descent, and which would soon become so great as entirely to baffle any further effort of the power to increase the rapidity of the motion; in short this resistance would soon pass into a state of equilibrium with the moving power, and from that period the velocity of the descent would be *uniform*, becoming what is technically called the *terminal* velocity. It is shown by theory, and has been confirmed by numerous experiments, that this *terminal* velocity of a descending body is very soon acquired, and is by no means a considerable velocity. Dr. Hutton has calculated that a leaden ball one inch in diameter, could not, by descending freely through the air (even if the air were every where of the same density as at the earth's surface) acquire a velocity of more than 260 feet per second. This velocity it would acquire in falling through 2687 feet, or about half a mile.*

* Theoretical deductions on these subjects have been more or less confirmed by numerous experiments in artillery practice. The method of the experiments was this: — Bullets fired vertically into the air, were received, on their descent, upon planks of soft wood, and the velocity of the descent was judged of from experimental data by the depths to which they sank in the wood.

We shall take as our third and last example the case of the LOCOMOTIVE CARRIAGE.

The pressure which opposes itself to the motion of a carriage upon a railroad, where the road is accurately level or horizontal, is about 8 lbs. per ton weight; so that in a train weighing, carriages and all, 10 tons, there would not be more than 180 lbs. of resistance to be counterbalanced, that the whole might be placed in a state bordering upon motion; and, as the engine of every locomotive carriage is capable of producing upon its piston a far greater pressure than this, it might be imagined that this excess of power would produce a continually accelerating motion, and that when this had attained its greatest limit, consistently with the safety of transit, the steam must be thrown off, and the pressure reduced to 180 lb., to prevent any further accumulation. In reality, however, instead of the velocity of a locomotive being thus difficult to control and keep down to limits consistent with safety, it has been found impracticable to get it up even to those limits which public expectation had fixed itself upon, and which public convenience may be supposed to demand. To a preservation of the condition of the state bordering upon motion, it is necessary that the cylinder should be continually filled and refilled with steam of the requisite pressure. Thus to a rapid motion a rapid production of steam becomes necessary; and on this the dimensions of the fire-place and boiler, and the force of the draught of air, soon place a limit. Again the resistance of the air increases with the square of the velocity with which the carriage moves; so that when it moves with any considerable

degree of velocity, the motion of the carriage comes to be opposed by this cause with a force adding itself to the resistance of its friction, and soon greatly exceeding it. The amount of this resistance on the broad surfaces of the carriages will be judged of when it is stated that it is equal to the the pressure which a wind, moving with the velocity of the carriages, would produce upon them at rest, if that wind moved exactly in the line of the road; and, moreover, that, by the experiments of Smeaton, a wind moving with the velocity of from 30 to 35 miles an hour is a very high wind, almost amounting to a gale.

297. THE DYNAMICAL EFFECT, OR THE AMOUNT OF THE ACTION OR EFFICIENCY OF ANY AGENT, IS MEASURED BY THE PRESSURE WHICH IT EXERTS MULTIPLIED BY THE SPACE THROUGH WHICH IT EXERTS IT.

For it is evident that the *pressure* exerted remaining the same, the action or effect will vary as the *space* through which it is exerted, and that the space remaining the same it will vary as the *pressure* exerted; thus, by the rules of proportion, when *both* vary, the action or effect will vary as their *product*.

Thus, for instance, a horse drawing a loaded carriage over six miles of road will exert a double action and produce a double effect when his load is doubled, and therefore his constant pressure upon it doubled; a triple effect when he draws a triple load; a quadruple effect when he draws a quadruple load over this six miles of road, and so

on; so that the space he traverses remaining the same, his effect will vary as the *pressure* which he applies. Again, his load, and therefore the pressure he applies, remaining the same, the effect he produces will vary as the *space* he traverses. Thus if he draw the same load twelve miles instead of six, his effect will be doubled; if eighteen, tripled, and so on. Since, then, his action or effect varies as the *pressure* he applies when the *space* is constant, and as the *space* when the *pressure* is constant, it follows that when *neither* is constant it varies as their *product*.

Thus the dynamical action or effect of a horse which draws a load of 6 cwt. over two miles of level road, is the same with that of a horse which draws 4 cwt. over three miles; since 6×2 is equal to 4×3.*

The dynamical effect of a weight of 4 cwt. acting to impel or to resist the motion of a machine through 10 feet, is to that of a weight of 5 cwt. acting through 12 feet as 2 to 3; since the product of 4 by 10 or 40, is to the product of 5 by 12 or 60 in that ratio.

* This equality may perhaps be understood better by some persons thus: the effect of 6 cwt. drawn over two miles is the same as that of 12 cwt. over one mile; for whether two horses draw each 3 cwt. over the same mile or draw these over two successive miles, the same dynamical effect is evidently produced. By exactly the same reasoning it is evident that the effect of 4 cwt. drawn over three miles is the same as that of 12 cwt. over one mile: both of these dynamical effects being therefore equal to 12 cwt. drawn over one mile, are equal to one another.

298. The Dynamical Efficiencies of different Agents.

There are two ways of speaking of the dynamical effect of an agent. We may speak of it as the mean effect produced in a *given period*, as for instance, one minute of the operation of that agent; or we may speak of it as the *whole* effect which that agent is capable of producing, before its operation is withdrawn, or its powers become extinct. In the former sense we speak of the mean effect which a horse drawing a load is capable of producing per minute, or of the effect which a given quantity of fuel burning in the furnace of a steam engine is capable of producing (by the intervention of the water and steam,) upon the piston per minute; in the latter sense we speak of the whole dynamical effect which a horse is capable of producing during its life; or a bushel of coals before it is burned out.

299. The Dynamical Effect of a Human Agent.

The muscular power of a man is usually made to operate either by his legs or his arms, rarely by both together. It has been estimated that by the action of his legs upon a treadwheel, he can raise his own weight, about 150 lbs., 10,000 feet per day; which gives a dynamical effect of 1,500,000 per day, or 3125 per minute, supposing the work to be continued eight hours a day.

A man who ascended a hill 10,000 feet high,

would do a good day's work; a result which corroborates the preceding.

In respect to the dynamical effect of a man working with his arms, we have the authority of Smeaton, that a good labourer can thus raise 370 lb. 10 feet high per minute; so that his dynamical effect is 3700; being somewhat greater with his arms than his legs. Desaguiliers makes the dynamical effect of a man working with his arms, 5500 per minute: this is, however, considered too high an estimate.

300. The Dynamical Effect of a Horse.

A horse drawing a weight out of a well over a pulley can, according to Desaguiliers, raise 200 lbs. for eight hours together, at the rate of $2\frac{1}{2}$ miles or 13,200 feet, per hour. This gives for the dynamical effect of a horse per minute 29,333.

The usual estimate of the dynamical effect per minute of a horse, called by engineers a HORSE'S POWER, is 33,000.

Mr. Smeaton states it to be 22,000.

301. The Power of a living Agent to produce a given Dynamical Effect.

A distinction must be made between the dynamical effect produced by a living agent, and its power of producing that effect as affected by the circmstances under which it is produced. Thus the dynamical effect of a load of 200 lbs. raised by a horse for 8 hours a day, at the rate of $2\frac{1}{2}$ miles an hour, is the same with that of 20 lbs. raised for the same

period at 25 miles an hour; but the power of producing this effect, considered as residing in the horse, is not the same; in fact, the action exerted by the horse to produce these two effects is different; he has to carry the weight of his body, lifting it a certain height at every step, much farther in the one case than the other. The distinction between the two, is that between the moving and the working power in a machine. The moving action or effect includes the motion communicated to the machinery of the horse's body, the working action or effect only that applied to the load.

An animal is best capable of exerting its muscular power against any resisting force, when it is at rest. When it is in motion, a portion of its muscular force is consumed in its motion. If the rate at which a horse is travelling per hour in miles be subtracted from 12, and the remainder squared, a number will be obtained, which will, *it is said*, represent the number of pounds of traction which the horse is capable of exerting, when it moves with this velocity.

Thus, if the horse be moving at the rate of 4 miles per hour, this number being subtracted from 12, gives 8, which squared is 64. So that the horse could, according to this rule, walking at 4 miles per hour, be able to *draw* with a force of 64 lbs. Now 4 miles per hour is 352 feet per minute. The dynamical effect per minute of a horse, thus drawing, would then be 22,528.

A waggon loaded with 86 tons, and therefore requiring a traction of $\frac{1}{12}$ of this weight, or $1\frac{2}{3}$ tons, may be drawn by 8 horses, at $2\frac{1}{2}$ miles an hour,

for 8 hours daily. This gives a dynamical effect per minute of 41,066 for each horse.

A mail coach, of 2 tons weight, and travelling at the rate of 10 miles per hour, may be worked on a turnpike road both ways, by as many horses as there are miles of road. The dynamical effect per minute may in this case be calculated as before: it will be found to be 8215, being scarcely ⅕ of the effect which the horses would have been capable of producing at the slower rate of the waggon.

302. The Dynamical Effect of One Pound of Coals.

The *power of heat*, which slumbers among the particles of a mass of coal, is best called into operation as a dynamical agent by combining it with *water* under the form of steam. According to Mr. Watt, a bushel of coals (84 lbs.) will convert into steam 10 cubic feet of water, so that 8·4 lbs. is sufficient to vaporise 1 cubic foot. Now, 1 cubic foot of water, according to Tredgold (p. 153.), will expand itself into 1711 cubic feet of steam at temperature 212°, and retaining an elasticity equal to the pressure of one atmosphere. These 1711 cubic feet of steam are therefore capable of propelling a piston of 1 foot square, under the pressure of one atmosphere, through a distance of 1711 feet. Now, the pressure of the atmosphere on a surface 1 foot square, is 2120 lbs. These 8·4 lbs. of coals, thus converting into steam a cubic foot of water, are capable therefore, through this intervention of the steam, of producing a dynamical

effect represented by the product 1711×2120, or by 3,627,320.

This effect being produced by 8·4 lbs., the effect of 1 lb. is obtained by dividing it by 8·4; by which division we find 431,824 for the dynamical effect which 1 lb. of coals is capable of producing.

303. THE DYNAMICAL EFFECT OF ANY AGENT OPERATING THROUGH A MACHINE WHICH MOVES WITH A UNIFORM MOTION, IS THE SAME WHATEVER THAT MACHINE MAY BE, PROVIDED ONLY THE RESISTANCES OPPOSED TO THE MOTIONS OF THE PARTS OF THE MACHINE BY FRICTION AND OTHER OPPOSING CAUSES BE THE SAME.

For to the state of the uniform motion of a machine there is necessary that state of the equilibrium of the pressures acting upon it which borders upon motion (see art. 294.). And this state of the equilibrium of the pressures acting upon the machine supposes, by the principle of virtual velocities, that the product of the power by the space it describes should equal the sum of the products of the resistances* by the spaces they severally describe. Now, the product of any *pressure* by the *space* through which it is made to act is its DYNAMICAL EFFECT.

* The resistance upon any point of a machine implies a force acting in a direction opposite to that in which the motion of the point takes place. The power and the resistances in the machine here spoken of, are all supposed to operate actually in the lines of direction in which the points to which they are applied move.

Including then, among these resistances, together with those upon the *working* points of the machine, those offered by the *frictions* of its various intermediate moving parts upon one another, the uncounterbalanced weights of certain of them which are raised as the motion goes on, and the resistance of the air upon the motion of all; it follows that the dynamical effect of the power is equal to the sum of the dynamical effects of the resistances; and that separating the resistances upon the working points of a machine from the rest of the resistances upon it, and supposing these last to be in every respect the same in different machines; then the same agent operating *equally* (that is, with the same dynamical effect upon the receiving organ) through these different machines, will produce the same aggregate dynamical effect upon the working points of all.

That the state of the *uniform* motion of the machine should have been attained is necessary to the application of this principle, as is expressly stated in the enunciation of it; for in that state of *accelerating* motion which precedes the *uniform* motion of the machine, the distribution of pressure and motion will vary not only with the frictions and *uncounterbalanced* weights of the parts of different machines, but with their actual weights and dimensions, and the distribution of their dimensions in respect to their axes of motion (arts. 221. and 225.).

Since neither in these respects, nor in respect to the frictions of their various surfaces of motion upon one another, or their uncounterbalanced

weights, can there be a positive equality between any two; and since in respect to machines generally there is in all these respects a great inequality, it follows that generally the dynamical effects produced upon the working points of different machines by equal operations of the same agent ARE NOT THE SAME; and, therefore, that to estimate the actually working effects of the same agent on different machines, it is necessary to know what portion of the dynamical effect, made to operate in each machine, is *consumed* in the *resistances* opposed to the machine, *elsewhere* than at its working points, and with this view to distinguish *between the moving and working powers, or the dynamical effects produced at the moving and at the working points*; between THE EFFECTS PRODUCED AT THE POINT WHICH RECEIVES THE OPERATION OF THE AGENT AND AT THE POINTS WHICH APPLY IT.

304. THE DYNAMICAL EFFECT UPON THE MOVING POINT, OR THE MOVING POWER, IN A STEAM ENGINE.

In a steam engine the operation of the agent (the steam) is received upon the piston. To estimate the dynamical effect of this agent upon the moving point, we have then to determine the pressure of the steam upon the piston and the velocity in feet with which the piston moves per minute; the product of these will give the dynamical effect upon the piston per minute. This is termed THE POWER of the engine. Compared with the dynamical effect of a horse per minute, which

we have seen to be 33,000, it determines what is called the HORSE'S POWER of the engine. There is very great difficulty, however, in determining the elasticity of the steam in the cylinder and its actual pressure upon the piston. The steam gauge determines it under all circumstances with sufficient accuracy in the *boiler;* but the elasticities of the steam in the cylinder and in the boiler are not the same: the former is influenced by the rapid state of the motion of the steam through the narrow passage of the steam pipes and its expansion into the body of the cylinder, and especially it is influenced by the greater or less opposition which the piston offers to this expansion. The determination of all these conditions is a problem of great difficulty, and as yet it is an *unsolved* problem of practical mechanics.

An instrument has indeed been contrived for measuring the elastic force of the steam in the cylinder, called the *Steam Indicator*. See *Tredgold on the Steam Engine* (art. 560.). This instrument, at best but an imperfect one, although many years ago used by Watt, has only, we believe, of late come to be employed to any extent by steam engine manufacturers for estimating the powers of their engines. It appears to admit of improvement, and will probably before long be taken for the constant guide of the practical engineer.

We are not aware of any published experiments with the Indicator of sufficient precision and authority to warrant their mention here. Whenever such experiments shall be made, valuable theoretical results cannot fail to be deducible from them.

It is customary for the engine maker to *assume* that his engine is made to work with a certain velocity of the piston and with a certain pressure upon it; and different makers have been accustomed to assign different values to these quantities. The engines of Watt were made to work with a pressure of 7 lbs. on the square inch, and the piston to travel at 220 feet per minute. Tredgold gives, as the best velocity of the piston, 120 times the square root of the length of the stroke, in feet. It is very questionable whether any of these conclusions, considered as theoretical conclusions, are founded on sufficient data.

As an example of the calculation of the dynamical effect upon the piston of a steam engine, let us take the following: —

The cylinder of an engine has a diameter of 36 inches, and its piston a stroke of 7 feet, making 16 double strokes a minute; the pressure upon the piston of this engine was shown by the steam indicator to average 10 lbs. to the square inch. From these data it may be calculated that the area of the piston was 1017·8 square inches, and the whole pressure upon it 10,178 lbs.; moreover, that it moved at the rate of 224 feet per minute; so that the dynamical effect per minute produced upon it was represented by the product of these numbers or by the number 2,279,872; which, taking the dynamical effect of a horse per minute to be 33,000, makes the *horse-power*, as it is called, of the engine or the effect produced upon its piston (not its *working power*) equal to that of 69 horses. The actual pressure of 10 lbs. per square inch upon the piston of this en-

gine was determined by Mr. Glyn with the steam indicator. The engine was probably *made* to work with 7 lbs. or 8 lbs. pressure, and would have been called by the maker an engine of 55-horse power. Had this engine worked without friction of its machinery, this moving dynamical effect or moving power of 69 horses, would have been propagated through it without diminution, and distributed among its working points, would have constituted its working or useful effect.

305. THE DYNAMICAL EFFECT UPON THE WORKING POINTS OR THE WORKING POWER OF A STEAM ENGINE.

The dynamical effect produced at the working points in a steam engine, is equal to the sum of the pressures exerted there and performing the work, each being multiplied by the space over which it is made to operate.

The following example is from the monthly reports of the working of the Cornish engines; it will sufficiently illustrate the method according to which this calculation is usually made.*

* In the year 1811, the principal mining proprietors in Cornwall determined, with a view to the encouragement of the skilful manufacture and working of engines, to ascertain from monthly reports, made by competent persons and with the requisite precautions, and to make public, the useful effect of their respective engines during that month, together with the consumption of coals and the steam pressure in the cylinder. For this purpose a mechanical contrivance, called the *counter*, was annexed to each engine, and accurately registered its number of strokes; and this registration, with the measured dimensions of its pump and stroke, are sufficient data for determining its useful effect, as shown in the above example.

The engine at the mine called the Wheal Hope, works three pumps, and the length of the stroke of each is 8 feet: their pistons support and lift, at every stroke, columns of water, whose joint weights are 27,766 lbs., and in the month of December 1826, they are stated to have made 261,890 strokes. Hence it may be calculated that the velocity of the pistons was 46·9 feet per minute, and 27,766 lbs. of water being moved with this velocity, that the working effect per minute, was the product of these two numbers, or 13,022,254.

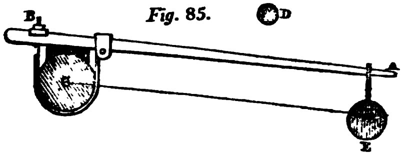
If the whole distance travelled by the pistons in the month had been multiplied by the mean pressure upon them, so as to obtain the *whole* working effect *in the month*, and this product had been divided by the number of bushels of coals consumed in the month, which was 1242, the quotient would have been the working effect of each bushel of coals in that engine, and it would have been found to be 46,838,246. This number is called the DUTY of the engine. It includes in its amount not only the qualities of the engine, but of the fuel, and the economy of the stokers in the use of it; and especially, it would seem to depend upon the greater or less escape of the heat, by radiation from the surface of the boiler.

306. Practical Method of determining the Dynamical Effect at any working Point in a Machine, or the working Power operating at that Point.

Let the work be thrown off from the shaft which

conveys the power to that working point*, whose dynamical effect is to be estimated. Let then a *friction-strap* or *break-wheel*, such as that shown in the accompanying figure, to which is connected

Fig. 85.



the rod or bar AB, be placed upon the shaft, and its revolution with the shaft being prevented by the stop D, let the strap be tightened upon the shaft by means of the screw B, until the motion of the machine is again brought back by the friction of the strap, exactly to what it was before the work was thrown off, a fact which will be indicated by the shaft making now precisely as many revolutions per minute as it did then. This being accomplished, it is certain that the *friction* of the strap is precisely equal to the *resistance* of the work; and that the power before expended in performing the *work*, is precisely equal to the power now expended in overcoming *the friction of the strap*. It only remains, therefore, to determine this last. For this purpose let a weight be suspended from the extremity of the rod, and gradually increased, until

* The power will, in the majority of cases, be found to be conveyed to each working point of the machine by such a shaft, which may be considered as the channel along which it flows. In any case where it is not, a shaft may be introduced and made the medium of communication, for the express purpose of this admeasurement.

the rod at length descends from the stop D, (against which it has hitherto been pressed, and by the resistance of which the friction of the strap has hitherto been overcome,) and assumes the horizontal position shown in the figure. An equilibrium then manifestly exists between the weight E, acting on the arm of the lever C F, and the friction, acting on the circumference of the shaft. From this relation, the friction upon the shaft may at once be calculated; and this friction in pounds, multiplied by the distance in feet, traversed by the circumference of the shaft per minute, gives the dynamical effect of the friction at the shaft, and therefore the *power* upon the working point, which was to be determined.

307. The Theory of the Steam Engine.

Could we determine from a knowledge of the dimensions, and the combination of the parts of a steam engine — its cranks, axles, levers, pistons, &c. — and the frictions of their surfaces of contact, the conditions of the *equilibrium* of the pressures acting in the machine, when in its state bordering upon motion; could we, in fact, determine accurately, under the form of an analytical expression, that precise relation which exists between a power operating upon the piston of a steam engine, and the resistances opposed to the motion of the machine at its working points, when motion is about to ensue by the power overcoming the resistances at those points — friction being of course rigidly, included in the computation; and did this analytical formula or computation apply itself to all the various positions of

the piston, and therefore of the beam, crank, levers, &c.; then we should know accurately under what steam pressure upon the piston the engine would perform any given work, and one of the most important elements of the theory of the steam engine would be determined.

The next step in the investigation would be to find, if it were possible, from given dimensions of the furnace and boiler, the quantity of steam which the engine would produce, and throw per minute into the cylinder, of such a density as that its elasticity should be sufficient to produce the required pressure per square inch upon the piston. Every time the cylinder was filled with steam of this density, the piston would be driven along it; and the number of times per minute that it would be so filled, would be known by a comparison of its capacity with the quantity of steam of the same density, generated per minute in the boiler. The pressure upon the piston being thus known, and its velocity, the whole moving and working effect of the engine, would seem to be known, and its theory completely determined.

Three important elements in the computation have, however, been here omitted:—

1st. The *temperature* under which the steam fills the cylinder influences greatly its elasticity, and therefore its pressure upon the piston.

2dly. The *velocity* under which the steam passes through the steam-pipe, from the boiler to the cylinder, controlling as it does the supply of steam to the piston, and depending for its amount upon the relative densities of the steam in the boiler and

cylinder, of necessity influences the result; and must be supposed to do so *appreciably*, until the contrary is proved, or at least rendered probable.

3dly. The elasticity of the steam in the cylinder is undoubtedly, in some degree, and *probably* to a great extent, affected by the state of *motion* produced in it by the influent jet of steam from the steam-pipe; and, like the last, this disturbing cause must be supposed to have an appreciable amount, until the contrary is proved.

These conditions, thrown into the problem, greatly add to its difficulties, and appear to place it far beyond the limits of any solution which has yet been offered.

Of the various discussions of the theory of the steam-engine which have been propounded for the guidance of practical men, there are two which may here be noticed;—those of Mr. Tredgold and M. de Pambour.

The theory of Mr. Tredgold *appears* to assume, that the steam pressure upon the piston is wholly controlled and governed by the pressure in the boiler, and entirely independent of the resistance upon the piston. It is scarcely possible to extract any other meaning from the calculation given by that author of the working or useful pressure on the piston of a non-condensing engine*, (see Tredgold, art. 367.) unless, indeed, the whole effect

* The following is the calculation given by him of the effective or working pressure upon the piston (that is, the pressure upon the piston, deducting the friction of the parts of the engine and the resistances opposed to its motion by all other causes acting to transmit it).

of the resistance on the piston, upon the steam pressure, be supposed to be included in his determination of the first small element, ·0069 of the calculation.

According to this calculation, the *actual* pressure of the steam upon the piston, neglecting the effect

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The effective pressure upon the piston is less than that in the boiler, considered as unity.

By the force producing motion of the steam into the cylinder - - -	·0069
By the cooling in the cylinder and pipes -	·0160
By the friction of the piston and waste -	·2000
By the force required to expel the steam into the atmosphere - - -	·0069
By the force expended in opening valves, and friction of the parts of the engine -	·0622
By the steam being cut off before the termination of the stroke - -	·1000
	·3920

To the expression in the text of the opinion he has formed of the principles on which this calculation of the power of a steam-engine and others of the same class are founded, the author begs here to add, that it is by no means his wish to be considered as extending this opinion to the general character of Mr. Tredgold's work. That work contains a vast mass of practical information, which will be sought for in vain elsewhere; and the many admirable plates and valuable papers which have been added to the last edition of it, published by Mr. Weale, will no doubt obtain for it a place in the library of every man interested in the progress of practical science. Nevertheless, in justice to the real interests of science, the author is compelled to express an opinion that every single question connected with the theory of the steam-engine ought, in the existing state of our knowledge, to be received with distrust and caution.

of cutting off the steam before the termination of the stroke, is only less than that in the boiler by the small fraction ·0229.

Now the pressure in the boiler can be *measured*, and thence that upon the piston *calculated*, allowing the loss of this small fraction of its amount in passing from the boiler to the cylinder, so as to determine the *moving* dynamical effect or moving power of the engine according to Mr. Tredgold's rule; and it would be found, by comparing it with the *working* dynamical effect, or *working* power, to amount only to from one third to two thirds of it. To reconcile the two, then, enormous allowance must be made by those who adopt this rule for friction and other causes opposed to the motion of the engine.

Mr. Tredgold accordingly assigns to the piston alone a friction amounting to no less than $\frac{1}{5}$th of the whole pressure upon it, and to the friction of the machinery by which the motion of the piston is transmitted $\frac{6}{100}$ths. Whence it may be calculated, that if an engine had a working power of 100 horses, 40 would be necessary to draw its piston alone, and 12 to move the remaining portion of its machinery. (See De Pambour's Theory of the Steam-Engine, p. 7.)

It is due to the interests of science to state that these calculations appear to be grounded in no sound or recognised principles: they are deduced from formulæ which are to be considered as scarcely more than empirical, and which do not appear to be borne out by the practice of the steam-engine.

The theory of M. de Pambour makes the elas-

ticity of the steam in the cylinder to depend entirely upon the resistance which the piston opposes to it, and the motion of the piston to be governed entirely by the quantity of steam generated by the engine per minute, at a given temperature, which he calls its vaporising power. The elasticity of the steam in the cylinder is, however, dependent upon its temperature as well as its density; and to compare it, and therefore the pressure upon the piston, with the vaporising power of the engine, it is necessary to establish a relation between the two.

M. de Pambour states, from numerous experiments made simultaneously with the thermometer and manometer, applied both to the boiler of a steam-engine and also to the tube, through which the steam, after having terminated its effect, escaped into the atmosphere, that during all its action in the engine the steam remains in the state technically denoted by the name of *saturated* steam; that is, it remains at the maximum density *for its temperature*. This fact, on the discussion of which it is impossible here to enter, establishes the *required* relation of density and temperature, and leads to a solution of the problem under the conditions supposed.

If confirmed by subsequent observations, it cannot but be considered a very valuable addition to the theory of the steam-engine.

APPENDIX.

APPENDIX.

TABLE I.

COMPRESSIONS PRODUCED IN DIFFERENT SUBSTANCES BY EACH ADDITIONAL PRESSURE OF ONE ATMOSPHERE, MEASURED IN MILLIONTHS OF THE WHOLE VOLUME OR BULK.

Œrsted.		Colladon and Sturm.	
Substances experimented on.	Millionths.	Substances experimented on	Millionths.
Mercury - -	1	Mercury - -	5·3
Alcohol - -	20	Sulphuric acid -	32·0
Sulphuret of carbon	30	Nitric acid -	32·2
Water - -	46·1	Ammonia - -	34·7
Sulphuric ether -	60	Acetic acid -	42·2
		Water containing air	49·5
		Water freed from air	51·3
		Nritric ether -	71·5
		Essence of terebinthum - -	73·0
		Acetic ether - -	71·5
		Hydrochloric ether under the 1st atmosphere -	85·9
		Ditto, under the 9th atmosphere -	82·5
		Alcohol under the 1st atmosphere -	96·5
		Ditto, under the 9th atmosphere -	93·
		Sulphuric ether under the 1st atmosphere, at temp. 0° cent. -	133·0
		Ditto, ditto, at temp. 11° cent. -	141·0
		Ditto, under 24th atmosphere, at temp. 0° cent. -	150·0

TABLE II.

Liquefaction of the Gases.

Names of the Gases liquefied.	Temperature in Degrees of the Centig. Ther.	Pressure at which Liquefaction is produced in Atmospheres.
Sulphurous acid - -	7	2
Cyanogen - -	7	3·6
Chlorine - - -	15·5	4
Ammonia - -	0	5
The same - - -	10	6·5
Muriatic acid - -	−16	20
The same - -	− 4	25
The same - -	10	40
Carbonic acid - -	−11	20
The same - -	0	36
Nitrous oxide - -	0	44
The same - -	7	51

TABLE III.

EXTENSIBILITY.

Experiments on the direct Extensibility of Wood and Iron.

Substance extended.	Load per Square Inch in Tons.	Extension in Millionths.	Name of Experimenter.
Bars of oak - -	1	1176	Minard and Desormes.
Iron wire, No. 18 (in cables)	1	91	Vicat.
— 17 —	1	85	—
Bar iron - - -	1	82	Engineers of the Pont des Invalides.
— - - -	15	2500	Minard and Desormes.
— - - -	18	10000	—
— - - -	20	20000	—
— - - -	23	50000	—
— - - -	25	rupture	—

TABLE IV.

EXPERIMENTS BY MR. BARLOW ON THE DIRECT EXTENSIBILITY OF WROUGHT IRON.*

Extending Weight in Tons.	Parts of the Bar extended by each additional Ton, in MILLIONTHS of the whole Length. BARS ONE INCH SQUARE. Bar No. I.	Bar No. II.	Bar No. III.	Bar No. IV.
1	0	0	0	0
2	20	0	160	150
3	62	73	150	130
4	93	80	130	140
5	109	90	120	140
6	110	110	110	130
7	—	90	120	100
8	93	80	120	80
9	—	100	120	elasticity destroyed.

	BARS TWO INCHES SQUARE. Bar No. V.	Bar No. VI.	Bar No. VII.
8	180	150	125
10	140	120	110
12	110	100	50
14	110	80	50
16	110	85	50
18	110	80	105
20	100	75	100
22	100	70	95
24	100	75	95
26	100	80	95
28	95	80	95
30	90	95	95
32	95	95	90
34	85	110	85
36	75	full elasticity.	90
38	95		95
40	145 elasticity exceeded.		95 elasticity perfect.

* Compiled from a Report addressed to the Directors of the London and Birmingham Railway. Fellowes, 1835.

TABLE V.—*continued.*

Substances experimented on.	Tonacity in Tons per Square Inch.	Name of Experimentor.	Crushing Force in Tons per Square In.	Name of Experimentor.
Lead wire - - -	1 1	Guyton		
Stone, slate (Welsh) -	5·7			
Marble (white) -	4	- -	1·4	Rennie
Givry - - -	1			
Portland - - -	$\frac{1}{8}$	- -	1·6	—
Craigleith freestone -	- -	- -	2·4	—
Bramley fall sandstone	- -	- -	2·7	—
Cornish granite -	- -	- -	2·8	—
Peterhead ditto -	- -	- -	3·7	—
Limestone (compact blk.)	- -	- -	4	—
Purbeck - - -	- -	- -	4	—
Aberdeen granite -	- -	- -	5	—
Brick, pale red - -	·13	- -	·56	—
red - - - - -	- -	- -	·8	—
Hammersmith (pavior's)	- -	- -	1	—
ditto (burnt)	- -	- -	1·4	—
Chalk - - - - -	- -	- -	·22	—
Plaster of Paris - -	·03			
Glass, plate - - -	4			
Bone (ox) - - -	2·2			
Hemp fibres glued together	41			
Strips of paper glued together	13			
Wood, Box, spec. gravity ·862	9	Barlow		
Ash - - - ·6	8	—		
Teak - - - ·9	7	—		
Beech - - ·7	5	—		
Oak - - - ·92	5	—	17	—
Ditto - - ·77	4	—		
Fir - - - ·6	5	—		
Pear - - ·646	$4\frac{1}{4}$	—		
Mahogany - ·637	$3\frac{1}{8}$	—		
Elm - - - -	6	- -	·57	—
Pine, American -	6	- -	·73	—
Deal, white - -	6	- -	·86	—

Torsion.

M. Savart has shown, in a series of experiments, detailed in the *Annales de Chimie*, August, 1829, on the torsion of bars of different sections and dimensions —

1st. That the ANGLES of torsion are in every case proportional to the FORCES of torsion, so long as the torsion of the bar remains within the elastic limits.

2dly. That in bars of the same section, subjected to the same forces of torsion, the angles of torsion are directly proportional to the LENGTHS of the bars.

TABLE VI.

EXPERIMENTS BY M. DULEAU UPON THE ANGLE OF TORSION IN BARS OF IRON.

Nature of the Specimen.	Length of the Part twisted.	Side of Square, or Diameter of Cylinder.	Angle of Torsion produced by a Pressure of 22 lbs. act- at a Leverage of 1·22 Feet.
	Feet.	Inches.	Degrees.
Round iron, English, marked Dowlais -	7·9	·78	4
Round iron, Perigord	9·5	·91	3
Square iron, English, marked C 2 -	13·5	·79	6½
Square iron, Perigord	8·3	·8	3·8
Flat iron, English -	9·6	1·32 x ·337	11·4

TABLE VII.

EXPERIMENTS BY MR. G. RENNIE ON THE RUPTURE OF SQUARE BARS OF DIFFERENT METALS BY TORSION: THE FORCE BEING MADE TO ACT AT THE EXTREMITY OF A LEVER TWO FEET IN LENGTH.

Description of Material.	Length of Piece.	Side of Square Section.	Mean Weight producing Rupture.	
	Inches.	Inches.	Lbs.	Oz.
Iron cast, horizontally	0	¼	9	15
vertically -	0	¼	10	10
horizontally	½	¼	7	3
- -	¾	¼	8	1
- -	1	¼	8	8
vertically -	½	¼	10	1
- -	¾	¼	8	9
- -	1	¼	8	5
- -	6	¼	9	12
horizontally	0	½	93	12
- -	0	½	74	0
- -	10	½	52	0
Steel -	0	¼	17	1
Wrought iron, English	0	¼	10	2
Swedish	0	¼	9	8
Gun metal, hard -	0	¼	5	0
Yellow brass, fine -	0	¼	4	11
Copper, cast - -	0	¼	4	5
Tin - - -	0	¼	1	0

TABLE VIII.

Experiments by Mr. Bramah on the Rupture by Torsion of Square Bars by Weights acting at a Leverage of Three Feet.

Description of Material.	Length of Piece.	Side of Square Section.	Weight producing Rupture.
	Inches.	Inches.	Lbs.
Cast iron, alloyed with $\frac{1}{16}$th of copper - -	12	$1\frac{1}{16}$	215
- -	24	$1\frac{1}{16}$	213
Mixture of equal parts of old Adelphi and Alfreton - - -	12	$1\frac{1}{16}$	330
- -	12	$1\frac{1}{16}$	310
- -	24	$1\frac{1}{16}$	280
Cast iron - -	12	1	238
- -	24	1	218

TABLE IX.

Experiments by Mr. Dunlop on the Rupture by Torsion of Cylindrical Bars of Cast Iron, with Weights acting at a Leverage of Fourteen Feet Two Inches.

Length of the Bar.	Diameter.	Weights producing Rupture.
Inches.	Inches.	Lbs.
$2\frac{3}{4}$	2	250
$3\frac{1}{4}$	$2\frac{1}{4}$	384
3	$2\frac{1}{2}$	408
3	$2\frac{3}{4}$	700
4	$3\frac{1}{4}$	1170
5	$3\frac{1}{2}$	1240
5	$3\frac{3}{4}$	1662
5	4	1938
6	$4\frac{1}{4}$	2158

Mr. Hodgkinson's Experiments on the Mechanical Properties of Cast Iron.

The experiments of Mr. Hodgkinson and Mr. Fairbairn have been published, in the Seventh Report of the British Association of Science, since our chapter on the strength of materials went to press. Their great practical importance will sufficiently account for their introduction here, as an appendix to that chapter. They have reference—

1st. To the resistance of cast iron to rupture by extension.
2d. To the resistance of cast iron to rupture by compression.
3d. To the resistance of cast iron to rupture by transverse strain.
4th. To the destruction of the elastic properties of the material as the body advances to rupture.
5th. To the influence of time upon the conditions of rupture.
6th. To certain relations of the internal structure of metals to their conditions of rupture.
7th. To the relative properties in all these respects of HOT AND COLD BLAST IRON.

The experiments on tension and compression were made by means of a lever constructed for the purpose by Mr. Fairbairn, and admirably adapted to its use. A table given at the end of this paper contains their principal results.

From this table it appears that the resistance of cast iron to rupture by *extension* varies from 6 to 9 tons upon the square inch; and that to rupture by *compression* from 36 to 65 tons.

A series of experiments was directed to the verification of the commonly assumed principle, that the forces resisting rupture by extension, are — the material being the same — as the areas of the sections of rupture; and they appear fully to have

established this principle, not only in respect to iron but to wood.

The experiments of Mr. Hodgkinson on *transverse* strain present less of novelty and importance; they fully, however, confirm the views previously taken on this subject by him, and detailed in articles 66. 68, &c. A series of them, directed to the verification of the commonly assumed principle, "that the strengths of rectangular beams of the same width, to resist rupture by transverse strain, are as the squares of their depths," fully established that law.

With regard to the destruction of the elastic properties of the material, as it approaches to rupture, the experiments of Mr. Hodgkinson possess great interest and importance.

It has been asserted by Mr. Tredgold, and commonly assumed, that this destruction of elastic power, or displacement beyond the elastic limit, does not manifest itself until the load exceeds *one third* the *breaking weight.*

Mr. Hodgkinson found that, in some instances, this effect was produced, and manifested in a permanent *set* of the material, when the load did not exceed *one sixteenth* of the breaking weight. Thus, a bar one inch square, supported between props $4\frac{1}{2}$ feet apart, which broke when loaded with 496 lb., showed a *permanent deflection*, or *set*, when loaded with 16 lb. In other cases, permanent sets were given by loads of 7 lb. and 14 lb., the breaking weights being respectively 364 lb. and 1120 lb. These sets were therefore given by $\frac{1}{52}$d and $\frac{1}{80}$th the breaking weights respectively. Thus, then, there would seem to be no such limits, *in respect to transverse strain*, as those known by the name of elastic limits; and it follows from these experiments that the principle of loading a beam within the elastic limit has no foundation in practice.

It was ascertained by a very ingenious experiment, that a bar, subjected, under precisely the same circumstances, to extension and compression by transverse strain, gave, for *equal loads*, *equal deflections*, in the two cases.

The most remarkable results on the subject of transverse strain were, however, those of Mr. Fairbairn, having reference

to the influence of TIME upon the deflection produced by a given load.

A bar one inch square, supported between props $4\frac{1}{2}$ feet apart, and loaded with 280 lbs., being about $\frac{5}{8}$ths its breaking weight, had its deflection accurately measured, from month to month, for fifteen months, and it was found that, throughout that period, the deflection was CONTINUALLY INCREASING; the whole increase in that period amounting to the fraction ·043 of an inch. A bar of the same dimensions, similarly supported, and loaded with 336 lbs., being about $\frac{3}{4}$ths of its breaking weight, increased its deflection similarly, and in the same period, by the fraction ·077 of an inch. Another similar bar, loaded with about $\frac{7}{8}$ths the breaking weight, similarly increased its deflection by the ·088th of an inch. The deflection of these bars still daily advances under the same loads, and, a sufficient period having elapsed, will no doubt proceed to *rupture*. A *fourth* bar of the same size was loaded with 448 lbs., being very nearly its breaking load. It bore it for thirty-seven days, increasing its deflection during the first few days by the fraction ·282 of an inch; thence retaining the *same deflection* until it broke.

The fact thus established, that a beam loaded beyond a certain limit continually *yields* to the load, but with an exceedingly slow progression, unless the load very nearly approach the breaking load, is one of vast practical importance; *it opens an entirely new field of speculation and inquiry.* The questions, what are the *limits* of loading (if any) beyond which this continual progression to rupture *begins?* what are the various *rates* of progression corresponding to *different* loads beyond that limit? and what are the effects of *temperature* on these circumstances? remain, as yet, almost unanswered.

Another interesting feature of Mr. Hodgkinson's experiments has, however, reference to certain relations of the internal structure of cast iron to the conditions of its rupture.

In the compression of short columns of different heights, and of the same diameter, he found that where the height of the column exceeded a certain limit, the crushing force be-

came *constant*, not varying as the height of the column was increased, until it reached another limit; at which second limit the column began to yield, not strictly by the crushing, but by the *bending* of its material.

The *first* limit was a height of little less than three times the radius of the column; the *second* limit was about six times the radius of the column. For columns of different heights, between these limits, and having equal diameters, the force producing rupture by compression, was nearly the *same*. When the column was *less* than the lower limit, the crushing force became *greater*; and when it was *greater* than the higher limit, the crushing force became *less*.

These facts were at once explained by an examination of the fragments of the ruptured columns. In all cases where the height of the column exceeded a certain limit, the section of rupture was found to be a plane inclined at nearly the same angle to the axis of the column. The mean value of this angle was 55°, and in no case did the inclination of the section vary from that angle more than 3°.

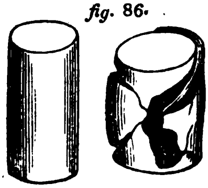
Now the limiting height of the column at which this oblique section first began to be distinctly and completely made, was precisely that (equal to three times the radius) at which the force producing rupture became independent of height. One of these facts, indeed, completely explains the other. For every height of the column above that limit, the section of rupture being a plane inclined at the same angle to the axis of the column, was a plane of the same size; so that in each case the cohesion of the same number of particles was to be overcome, that the rupture might be produced; and the cohesion of the same number of particles being to be overcome under the same circumstances for each different height, the same force would be required to overcome that cohesion; until at length that height (six times the radius) was attained at which the column began to *bend*. This height once reached, a pressure continually less as the column was longer became, of course, sufficient to break it.

This property is not, however, limited to cast iron; similar experiments were made by Mr. Hodgkinson, and by Rondelet,

with columns of wrought iron, wood, bone, marble, and other stones, and with the same result.

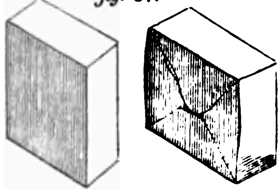
Although the angle with the axis of the direction of rupture was always the same, yet the particular position round the axis in which the section was made was not the same. There may evidently be an *infinite number* of such planes round a given point in the axis of the column, all inclined at the same angle of 55° to its axis; and there is no reason, in the nature of the material itself, provided it be homogeneous, why it should affect one of these planes of section rather than another. In the majority of cases *one* of these planes of section will, however, be determined in preference to the rest, by some *want* of homogeniety in the material, or by some *inequality* in the *distribution* of the *compressing force* upon the top of the column. Still such a particular determination of the section of rupture may *not* possibly present itself. In that case, the rupture, having *no* tendency to take place in one direction rather than another, will take place in all directions at once; and thus the surface of rupture will assume the form of the surface of a double cone, of which the two component cones have a common apex, and from which the sides of the column will break away. In the accompanying figures are represented the fragments of a column which broke under these circumstances in the experiments of Mr. Hodgkinson.

fig. 86.



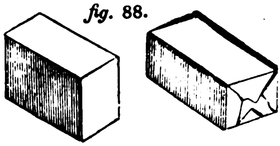
In the case of rectangular columns, the section of rupture will manifestly be the *narrowest* section which can be made at the given inclination, or that sloping towards the narrower face of the rectangle; because a section inclined at the same angle, but sloping towards the wider face, would oppose to rupture the cohesion of a much greater number of particles than the other or narrower section. In the majority of cases, this section will be made from one end of the top of the column rather than the other; but it may take place from both ends at once. This case occurred, too, in the experiments of Mr. Hodgkinson, and is represented in the cut,

fig. 87.



where the two planes of rupture from the opposite ends of the top of the column are seen crossing one another at the centre, and dividing the column into four wedge-like masses. The last cut represents fragments obtained in a similar experiment with a shorter rectangular column, where the height was not sufficient to allow of the one part *sliding* on the other along its plane of fracture. Thus it became apparent that the material had its first and easiest direction of fracture at a given angle of inclination to the direction of the pressure; so that its first and easiest fracture would take place, if allowed to do so, by the sliding of one portion of it on the surface of another, at a given angle of inclination to the axis of the pressure. And thus was completely explained the great increase of the strength of the column when it was so short (less than three times the radius) that one portion could not thus slide upon the other; the height of the column being less than the perpendicular height of the true plane of section, the upper portion was, in this case, manifestly prevented from sliding upon the lower, by the resting of its base upon the mass which supported the column.

fig. 88.



Now not the least interesting feature of these experiments is, that their results had long ago been anticipated by theory.

It is evident that when a column sustains a pressure in the direction of its length, the tendency of the column to yield, by the sliding of one portion upon the other along an oblique section, will be influenced by two causes: *first*, it will be *greater* as the inclination of the section to the direction of the pressure is less; on the principle, that the tendency of a heavy mass to slide upon an inclined plane is greater, as the inclination of the plane to the vertical is less: *secondly*, it will be *less* as the inclination of the section to the direction of the

pressure is less; inasmuch as the number of the cohering particles in such a section, and therefore the actual coherence of the whole section, is greater as the section is more oblique. Thus, then, by the operation of one of these causes, as the section is more oblique, the tendency to slide along it is *greater*, and by the operation of the other it is *less*. There must then be a particular obliquity of section for which these causes most effectually neutralise one another, and the tendency to rupture is the least. The position of this section was discussed by Coulomb, as early as the year 1773, (Mémoires des savans Etrangers, 1773,) and was found, neglecting the weight of the material of the column, and the *friction* of the surfaces which slip upon one another, and considering only the *coherence* of these surfaces, to be inclined at 45° to the axis of the column. Allowing for the effect of friction, and supposing, as is very probable, that under these circumstances of intimate contact it gives a limiting angle of resistance of 20°, this theoretical result of Coulomb is brought precisely to the practical result of Mr. Hodgkinson, giving 55° for the obliquity of the section of fracture.

A yet further confirmation of this fact is found in some experiments of Professor Daniel, detailed in the first volume of the Journal of the Royal Institution. Having immersed some rectangular bars of hammered lead for a considerable time in mercury, the solid metal became saturated with the fluid. Any friction which the two surfaces of any section, slipping upon one another, might have had, was thus taken away by the intervention of the mercury, and the cohesion of the particles of the bar was so destroyed, that it could not sustain its own weight. Under these circumstances the theory of Coulomb evidently points to an angle of 45°, as that at which the surfaces should slip. This is precisely the angle at which they were found to slip.

From these facts it is apparent, that if columns be taken of different diameters, and of heights so great as not to allow of their bending, but yet sufficient to allow of a perfect separation of the plane of fracture; that is, if they be taken of heights lying between three times and six times the radius of each; then their strengths being as the numbers of particles in their

planes of fracture respectively, will be as the areas of those planes; moreover, the planes of fracture being inclined at equal angles to the axes of the cylinders, their areas will be as the transverse sections of the cylinders; so that, in fact, the *strengths* of the columns will be as the areas of their transverse sections. This law Mr. Hodgkinson verified. Thus, for instance, the mean of three experiments upon a column $\frac{1}{2}$ of an inch in diameter, gave for the crushing force 6,426 lbs., whilst the mean of four on a column $\frac{3}{4}$ of an inch in diameter, gave 14,542 lbs. The diameters of these columns were as 2 to 3; these sections were, therefore, as 4 to 9; and this is near the ratio of the crushing weights.

A series of experiments was directed by Mr. Hodgkinson to the verification of this law, usually assumed in respect to the transverse strength of rectangular beams, that, when their lengths and breadths are the same, their strengths are as the squares of their depths.

His experiments fully established this law. Thus he placed between props, 4 feet 6 inches apart, castings of Carron iron No. 2., which were all 1 inch broad, and respectively 1, 3, and 5 inches deep; these broke respectively with weights of 452 lbs., 3,843 lbs., and 100,50 lbs.; which are very nearly as the numbers 1, 9, 25; that is, as the squares of the depths.

The following table contains a general summary of the results obtained by Mr. Hodgkinson, in respect to the direct strengths of hot and cold blast iron to resist compression and extension.

TABLE X.

Description of Metal.		Compressive Force per Square Inch.	Tensile Force per Square Inch.	Ratio.
Devon Iron, No. 3.	Hot blast	145,435	21,907	6·638:1
Buffery Iron, No. 1.	Hot blast	86,397	13,434	6·431:1
Ditto, No. 1.	Cold blast	93,385	17,466	5·346:1
Coed-Talon Iron, No. 2.	Hot blast	82,734	16,676	4·961:1
Ditto	Cold blast	81,770	18,855	4·337:1
Carron Iron, No. 2.	Hot blast	108,540	13,505	8·037:1
Ditto	Cold blast	106,375	16,683	6·376:1
Ditto, No. 3.	Hot blast	133,440	17,755	7·515:1
Ditto	Cold blast	115,442	14,200	8·129:1

TABLE XI.

General Summary of Results, as derived from the Experiments on Transverse Strength of Hot and Cold Blast Irons.

Description of Metal.		Ratio of the strengths, that of the cold blast being represented by 1000.	Ratio of the powers to sustain impact (cold blast being 1000).
Carron iron, No. 2.	-	1000 : 990·9	1000 : 1005·1
Devon, No. 3.	-	1000 : 1416·9	1000 : 2785·6
Buffery, No. 1.	-	1000 : 930·7	1000 : 962·1
Coed-Talon, No. 2.	-	1000 : 1007	1000 : 1234
Ditto, No. 3.	-	1000 : 927	1000 : 925
Elsicar and Milton	-	1000 : 818	1000 : 875
Carron, No. 3.	-	1000 : 1181	1000 : 1201
Muirkirk, No. 1.	-	1000 : 927	1000 : 823
Mean	-	1000 : 1024·8	1000 : 1226·3

On the whole, then, it appears that the strength of hot blast iron to resist transverse *strain* is greater than that of cold-blast iron, in the ratio of 1024·8 : 1000; and that its strength to resist *impact* is greater, in the proportion of 1226·3 : 1000.

On the Chemical Composition of Hot and Cold Blast Irons, as analysed by Dr. Thompson.*

The following differences of the two descriptions of metal resulted from the investigations of Dr. Thompson: —

1. The specific gravity of hot blast iron is greater than that of cold blast iron, by about the 22d part.

2. It was found that manganese, silicon, and aluminum were united with carbon in the composition of *all* cast iron; but that, of these foreign ingredients, carbon, silicon, and aluminum entered into the *composition* of the hot blast iron in a much less proportion than into the cold blast iron;

* Report of Brit. Ass. Sci. vol. vi.

in short, that the hot blast was greatly *purer* than the cold blast iron.

The mean result of five analyses of different irons gave for the *hot* blast iron No. 1. the proportion of 6½ atoms of iron* to 1 of carbon, silicon, aluminum; and for the *cold* blast iron No. 1. the proportion 3½ : 1.

The proportions in which carbon, silicon, and aluminum entered into the *cold* blast iron were 4, 1, 1; and those in which they entered into the hot blast iron, 12, 5, 2.

No trace of the ingredients silicon and aluminum was found by Dr. Thompson in the best *steel;* but only the iron, manganese, and carbon; and he gives it as his opinion, that the union of these two ingredients, silicon and aluminum, in all English iron, is the reason why good steel can never be made from it.

Dr. Thompson gives the following explanation of the *economy* of the hot blast:—

"The whole of the oxygen of the air of the *hot* blast combines with the fuel *as soon as it enters* into the furnace; whilst the oxygen of the air of the *cold* blast is *not* all consumed immediately, but makes its way upwards, and is *gradually* consumed in its ascent, producing a *scattered* heat, which is of no use in smelting the iron, but serves only to consume the fuel. When the hot blast is used the combustion is *concentrated* towards the bottom of the furnace; with the cold blast it is much more diffused. Hence the reason of the saving of the coals in the former case, which constitutes the great advantage attending the new method. This greater concentration of the combustion must subject the iron to a greater heat than when the combustion is more scattered. Hence the greater rapidity of the process, and consequently the additional quantity of the cast iron obtained from the furnace in a given time."

* With the iron is here included the small fractional proportion of manganese.

TABLE XII.

Of the Specific Gravities of various Materials of Construction, together with the Weight of a Cubic Foot of each Material.

Materials.	Specific Gravity.	Weight of a cubic foot in lbs. avoirdupois.	Remarks.
Acacia, English	0·710	44·3	
Alabaster	2·730	170·6	
Ash (middle)	0·727	45·4	
(outside)	0·702	43·8	
Beech	0·696	43·5	
Birch, common, (middle)	0·792	49·5	
(outside)	0·630	39·3	
Box	0·990	61·8	
Brass (cast)	8·370	523·0	369 cubic inches weigh 1 cwt.
Brick, common, (pale red)	1·557 to 2·000	97·3 to 125	
stock, (red)	1·841 to 2·168	115·0 to 135·5	
Dutch clinker	1·482	92·62	
Welsh fire	2·408	150·5	
Brickwork	-	95·0 to 117·0	A rod of new brickwork weighs 16 tons.

TABLE XII. (*continued.*)

Materials.	Specific Gravity.	Weight of a cubic foot in lbs. avoirdupois.	Remarks.
Cane	0·400	25·0	
Cedar (S. America)	0·457	28·5	Loses none of its specific gravity in seasoning.
Chalk	2·315	144·7	13 cubic feet weigh 1 ton.
Chestnut (sweet)	0·610	38·1	
(horse)	0·570	35·6	
Clay	2·000	125·0	
(with gravel)	2·560	160·0	
Coal (Newcastle)	1·269	79·3	
Coke	0·744	46·5	
Copper (sheet)	8·750	549·0	352·4 cubic inches weigh 1 cwt.
(cast)	8·607	537·9	
Deal, Christiana, (middle)	0·698	43·6	
(outside)	0·680	42·5	
Memel, (middle)	0·590	36·8	
(outside)	0·590	36·8	
English	0·470	29·3	

Scotch, (white)	10·498		31·1	
(yellow)	0·472		29·3	
Earth, common	1·52 to 2·00		95 to 125	
Elm, green	0·693 to 0·940		44·4 to 58·7	
seasoned, (middle)	0·554		34·6	64·4 cubic feet weigh 1 ton. Its specific gravity is diminished by ·352 in seasoning.
seasoned, (outside)	0·532		33·2	
Fir, New England	0·553		34.5	
Riga	0·753		47·0	47·1 cubic feet weigh 1 ton. The specific gravity is not diminished in seasoning.
Mar Forest	0·696		43·5	51·65 cubic feet weigh 1 ton.
Spruce	0·551		34·4	
Flint	2·58 to 2·63		161 to 164	
Flooring	- -		- -	The weight of a superficial foot of flooring is, according to Tredgold, 40lbs., including ceiling, counter-floor, and iron girders. When covered with people, 120lbs. per foot is added.
Gold (cast)	19·238		1202·3	
Granite, Aberdeen	2·625		164·0	

TABLE XII. (*continued.*)

Materials.	Specific Gravity.	Weight of a cubic foot in lbs. avoirdupois.	Remarks.
Granite, Cornish	2·662	166·3	
red Egyptian	2·654	165·8	13·05 cubic feet weigh 1 ton.
Gravel	-	120	
Gun metal (cast)	8·153	509.5	Copper 8 parts, tin 1.
Gypsum (opaque)	2·168	135·5	
Hawthorn	0·910	56·8	
Holly	0·760	47·5	
Iron, cast, (mean quality)	7·207	450	430·25 cubic inches weigh 1 cwt.
Buffery (Birmingham) No. 1. hot blast	6·998	437·3	These specific gravities were determined by Messrs. Hodgkinson and Fairbairn, and are taken from their paper on the strength and properties of cast iron, published in the seventh report of the British Association of Science.
Do. cold blast	7·079	442·4	
Milton (Yorkshire) No. 1. hot blast	6·976	436·0	
Elsicar (Yorkshire) No. 1. cold blast	7·030	439·3	
Coed-Talon (N. Wales) No. 2. hot blast	6·968	435·5	
Do. cold blast	6·955	434.7	
Do. No. 3. hot blast	6·970	435·6	

Do. cold blast	7·194	449·6	These specific gravities were determined by Messrs. Hodgkinson and Fairbairn, and are taken from their paper on the strength and properties of cast iron, published in the seventh report of the British Association of Science.
Carron (Scotch) No 2. hot blast	7·046	440·1	
Do. do. cold blast	7·066	441·6	
Do. No. 3. hot blast	7·056	441·0	
Do. do. cold blast	7·094	443·6	
Muirkirk (Scotch) No. 1. hot blast	6·953	434·5	
Do. do. cold blast	7·113	444·5	
Devon (Scotch) No. 3. hot blast	7·251	453·1	
Do. do. cold blast	7·295	455·9	
wrought	7·6 to 7·8	475 to 487·5	397·6 cubic inches weigh 1 cwt., at specific gravity 7 6
Do. (hammered)	8·217	513·6	
Larch, seasoned, (red)	·496 to ·640	31 to 40	
Do. (white)	·364	22·75	
Scotch, (very dry)	0·470	29·3	
Lead (cast)	11·352	709·5	272·8 cubic inches weigh 1 cwt.
(milled sheet)	11·407	712·9	
Lignum vitæ	1·333	83·3	
Mahogany (Honduras)	0·56	35·0	64 cubic feet weigh 1 ton.
(Spanish)	0·816 to 0·852	51 to 53·3	
Maple (Norway)	0·793	49·5	

TABLE XII. (*continued.*)

Materials.	Specific Gravity.	Weight of a cubic foot in lbs. avoirdupois.	Remarks.
Marl - - -	1·60 to 2·87	100 to 179·3	
Marble, white Italian -	2·726	170·3	13 cubic feet weigh 1 ton.
black Brabant -	2·697	168·5	
Millstone - - -	2·484	155·2	
Mulberry - - -	0·660	41·2	
Mortar (hair) dry - -	1·384	86·5	
(various) dry - -	1·414 to 1·893	88·3 to 118·3	
Oak, English - -	·969	60·5	36 cubic feet weigh 1 ton.
Canadian - -	·872	54·5	
Dantzic - - -	·756	47·25	
Adriatic - -	·993	62·0	47 cubic feet weigh 1 ton.
fast grown - -	·937	58·5	
slowly grown - -	·845	52·8	
English (butt) - -	1·113	69·5	These pieces were from the same tree. The first lost specific gravity ·370 in seasoning, and the second ·294.
Do. (top) - -	1·071	66·9	

bog oak	1·60	65·3	Lost no portion of its specific gravity in seasoning.
the finest quality, two years in store	·748	46·7	This piece was of remarkable strength.
Pebble	2·644	166·5	
Pewter	7·248	453·0	
Pine, pitch, Virginia, butt of tree	·628	36·7	Lost specific gravity ·031 in seasoning.
Do. top of same tree	·540	33·7	·011
yellow Canada, butt	·683	42·6	·222
Do. top of same tree	·495	30·9	·075
red Canada, butt	·672	42·0	·103
Do. top of same tree	·570	35·6	·067
spruce, Halifax, butt	·587	36·6	·007
Do. top of same tree	·541	33·8	Gained specific gravity ·031.
Platinum	20·336	1271·0	
Plaster, cast	1·286	80·37	
Puzzolano	2·570 to 2·850	160·6 to 178·1	
Poon (East Indies) butt	·651	40·7	Lost specific gravity ·075.
Do. top of same tree	·771	48·2	Lost specific gravity ·076.
Poplar	·360	22·5	
Porphyry (red)	2·871	179·0	

TABLE XII. (*continued.*)

Materials.	Specific Gravity.	Weight of a cubic foot of lbs. avoirdupois.	Remarks.
Rope (hempen) - -	- -	- -	A common rope*, 1 foot long, and 1 inch in circumference, weighs from ·044 to ·046 lbs. In a cable it weighs ·027 lbs.
Roofs - -	- -	- -	The weight of a square foot of Welsh slating is 11¼ lbs.; that of a square foot of plain tiling is 16¼ lbs. The greatest force of the wind on a roof may be estimated at 40 lbs. per foot. (Tredgold.)
Rubble-work - -	- .	140·0	
Sand, quartz - -	2·750	171·8	
Do. common -	1·454 to 1·886	90·87 to 117·87	
Shingle - - -	1·424	89	
Silver - - -	11·091	693·0	
Slate (Welsh) - -	2·752	172·6	
Steel (soft) - -	7·780	486·2	
(razor tempered) -	7·840	490·0	

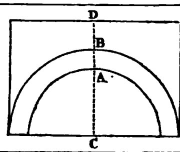
* To find the weight in lbs. which a rope will bear, square its girt in inches, and multiply by 200 for common ropes, and by 120 for cables.—*Tredgold.*

Stone, Portland - -	2·113	132·0	
Bath - - -	1·975	123·4	
Craigleith - -	2·362	147·6	
Dundee - - -	2·621	163·8	
Paving - -	2·416	151·0	15 cubic feet weigh 1 ton.
Limestone - -	3·179	198·6	11¼ cubic feet weigh 1 ton.
Stone-work, (hewn) - -	- -	160·0	
Teak - - -	0·745	46·5	
Java, (seasoned) -	0·697	43·5	
Tile, (common) - -	1·815 to 1·858	113·4 to 116·1	
Tin, (cast) Banca - -	7·217	451·0	
English block - -	7·295	455·8	
Malacca - - -	6·126	382·8	
Water, river - -	1·000	62·5	
sea - - -	1·0271	64·2	
Walnut - - -	0·671	41·9	
Whalebone - -	1·3	81·0	
Willow, (green) - -	0·619	38·68	
(dry) - - -	0·404 to ·568	25¼ to 35½	
Yew, (Spanish) - -	0·807	50·4	
Zinc, (cast) Goslar - -	7·215	450·9	

TABLE XIII.

The Horizontal Thrust of a Semicircular Arch whose Extrados is a Horizontal Straight Line.

Horizontal Thrust.



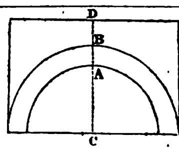
Values of $\frac{AB}{AC}$	$\frac{BD}{AC}=0$	$\frac{BD}{AC}=0·1$	$\frac{BD}{AC}=0·2$	$\frac{BD}{AC}=0·3$	$\frac{BD}{AC}=0·4$	$\frac{BD}{AC}=0·5$	$\frac{BD}{AC}=1·0$
0·05	0·08174	0·14797	0·21762	0·28877	0·36060	0·43277	0·79541
0·10	0·10279	0·16370	0·22588	0·28862	0·35164	0·41481	0·73161
0·15	0·11894	0·17480	0·23111	0·28764	0·34429	0·40100	0·68504
0·20	0·13073	0·18191	0·23322	0·28460	0·33603	0·38747	0·64488
0·25	0·13871	0·18558	0·23237	0·27922	0·32607	0·37293	0·60727
0·30	0·14333	0·18604	0·22874	0·27145	0·31416	0·35687	0·57041
0·35	0·14054	0·18379	0·22253	0·26140	0·30023	0·33907	0·53335
0·40	0·14422	0·17913	0·21415	0·24924	0·28437	0·31953	0·49560
0·45	0·14124	0·17240	0·20374	0·23520	0·26674	0·29835	0·45693
0·50	0·13649	0·16396	0·19168	0·21957	0·24760	0·27573	0·41728

Note.—This and the following table are extracted from the work of M. Garidel, entitled Tables de la Poussée des Voutes. Paris, 1837.

TABLE XIV.

The Angle of Rupture in a Semicircular Arch, the Extrados being a Horizontal Straight Line.

Angles of Rupture.



Values of $\frac{AB}{AC}$	$\frac{BD}{AC}=0$	$\frac{BD}{AC}=0·1$	$\frac{BD}{AC}=0·2$	$\frac{BD}{AC}=0·3$	$\frac{BD}{AC}=0·4$	$\frac{BD}{AC}=0·5$	$\frac{BD}{AC}=1$
0·05	68·0°	59·19°	54·04°	51·15°	49·35°	48·20°	45·74°
0·10	65·4	60·48	57·70	56·01	54·93	54·17	52·34
0·15	64·0	61·3	59·7	58·69	58·0	57·49	56·21
0·20	63·1	61·7	60·88	60·30	59·90	59·60	58·80
0·25	62·24	61·76	61·44	61·22	61·05	60·94	60·59
0·30	61·3	61·42	61·54	61·60	61·66	61·67	61·81
0·35	60·17	60·80	61·21	61·54	61·78	61·98	62·56
0·40	58·8	59·8	60·52	61·05	61·48	61·67	62·9
0·45	57·32	58·53	59·45	60·19	60·80	61·28	62·85
0·50	55·63	56·97	58·09	58·98	59·72	60·34	62·40

TABLE λ′.

THE LENGTHS OF PENDULUMS WHICH BEAT SECONDS IN DIFFERENT LATITUDES.

Stations.	Latitudes.	Observed length in millimeters of the pendulum which beats seconds at the station. N. B. The length is reduced by calculation to what it would be if the station were on the level of the sea.	Length of the pendulum in millimeters at the station, as calculated by the formula $l=0{\cdot}99102557+0{\cdot}00507188 \sin^2\lambda$, where λ represents the latitude.	Daily gain or loss which would be observed in a pendulum which beat seconds at Paris, if it were transferred to each station successively.	Name of Observer.
		Millimeters.		Seconds.	
Spitzbergen	79° 49′ 58″ N	996·0356	995·93941	+ 94·2	Sabine.
Greenland	74 32 19 N	995·7478	995·73699	+ 81·7	Sabine.
Hammerfest	70 40 05 N	995·5405	995·54163	+ 72·7	Sabine.
Drontheim	63 25 54 N	995·0200	995·08284	+ 50·1	Sabine.
Unst	60 45 28 N	994·9395	994·88712	+ 46·6	Kater.
Portsoy	57 40 59 N	994·6906	994·64791	+ 35·8	Kater.
Leith	55 58 41 N	994·5354	994·50969	+ 29·1	Kater.
Arbury Hill	52 12 55 N	994·2228	994·19348	+ 15·5	Kater.
London	51 31 08 N	994·1232	994·13360	+ 11·1	Kater.
Isles Malouines	51 31 44 S	994·1295	994·13446	+ 11·4	Duperrey.
Shanklin	50 37 24 N	994·0468	994·05610	+ 7·8	Kater.
Paris	48 50 14 N	993·8673	993·90017	+ 0·00	Duperrey.
Clermont	45 46 48 N	993·5822	993·63055	− 12·4	Biot & Mathieu.
Toulon	43 07 09 N	993·3858	993·39514	− 20·9	Duperrey.
New York	40 42 43 N	993·1682	993·18334	− 30·4	Sabine.
Formentera	38 39 56 N	992·9760	993·00533	− 38·7	Biot & Arago.

TABLE XV. (*continued.*)

Stations.	Latitudes.	Observed lengths in millimeters of the pendulum which beats seconds at the station. N. B. The length is reduced by calculation to what it would be if the station were on the level of the sea.	Length of the pendulum in millimeters at the station as calculated by the formula $l=0{\cdot}99102557+0{\cdot}00507188 \sin^2 \lambda$ where λ represents the latitude.	Daily gain or loss which would be observed in a pendulum which beat seconds at Paris, if it were transferred to each station successively.	Name of Observer.
		Millimeters.		Seconds.	
Port Jackson -	33° 51′ 39″ S	992·5879	992·60012	− 55·6	Duperrey.
Rio de Janeiro -	22 55 13 S	991·6930	991·79483	− 94·6	Freycinet.
Island Mowi -	20 52 07 N	991·7850	991·66918	− 90·5	Freycinet.
Isle of France -	20 09 56 S	991·7987	991·62832	− 89·9	Freycinet.
Jamaica -	17 56 07 N	991·4739	991·50653	−104·1	Sabine.
Island of Guam -	13 27 51 N	991·4520	991·30053	−105·0	Freycinet.
Bahia -	12 59 21 S	991·2064	991·28180	−115·7	Sabine.
Trinidad -	10 38 56 N	991·0609	991·19876	−122·1	Sabine.
Sierra Leone -	8 29 8 N	991·0953	991·13615	−120·7	Sabine.
Ascension -	7 55 9 S	991·1824	991·12185	−116·8	Duperrey.
Marauham -	2 31 43 S	990·8932	991·03544	−129·4	Sabine.
Saint Thomas -	0 24 41 N	991·1094	991·02583	−119·9	Sabine.
Island Rawak -	0 01 34 S	990·9577	991·02557	−126·5	Freycinet.

N.B. To reduce millimeters to inches, multiply by the decimal 0·03937.

Experiments on Friction, made at Metz in the Years 1831, 1832, 1833. M. Morin.

These experiments, into the mechanical details of which more precautions were introduced, and in which greater mechanical accuracy was probably attained, than in any which have preceded them; and in the measurement of the *results* of which, and the separation of the friction of the moving body from the various other elements which complicated those results, admirable theoretical skill and ingenuity were exhibited *,—have placed the question of friction entirely in a new, and a far more satisfactory position than it has before occupied. They were made at the expense of the French government, under the most favourable circumstances, by methods which have been fully and clearly detailed; and however opposed they may be in their results to all former experiments, and especially to those of Coulomb, it is impossible not to yield to them the greatest confidence.

The principal conclusions drawn from these experiments may be stated as follows:—

They show the friction of two surfaces which have been for *a considerable time in contact* to be not only different in its *amount*, but in its *nature*, from the friction of surfaces in *continuous motion*, especially in this, that this friction of quiescence is subject to causes of variation and uncertainty, from

* The contrivance, first suggested by M. Poucelet, by which the motion of the moving surface was made to record *itself* through all the variations of its velocity, as the weight which communicated motion to it accelerated or retarded its descent, is one of the most remarkable and the most valuable contributions which theory has ever made to practical mechanics: for the details of it the reader is referred to the work of M. Morin, entitled "Nouvelles Expériences sur le Frottement." Paris, 1833. Bachelier. This instrument admits of being applied under a modified form to determine the *action* or working *dynamical effect* of any part of a machine in motion; its determinations may be extended to every period and circumstance of the motion. Applied by a very simple contrivance to the cylinder of a steam engine, it would serve admirably the purpose of a steam indicator, recording with precision every varying effort of the moving power, and indicating the exact period of the motion when each such effort was made. Results thus obtained from an extensive series of experiments would constitute a body of facts *invaluable* as facts of reference to the civil engineer.

which the friction of motion is exempt. This variation does not appear to depend upon the *extent* of the surfaces of contact; for, with different pressures, the ratio of the friction to the pressure, or the co-efficient of friction, as it is called, varied greatly, although the surfaces of contact were the same.* The uncertainty which would have been introduced into every question of practical mechanics, and especially of construction, by this consideration, is, however, removed by a second very important fact developed accidentally in the course of the experiments. It is this, that by the slightest *jar* or *shock*, the most imperceptible *movement* of the surfaces of contact, their friction is made to pass from this state accompanying *quiescence* into that entirely different state of friction which accompanies *motion*; and as every machine or structure of whatever kind may be considered to be subject to such shocks or imperceptible motions of its surfaces of contact, it is evident that the state of friction to be made the basis on which all questions of statics are to be determined, should be that last mentioned, which accompanies continuous motion. Now the LAWS of this friction, thus accompanying motion, are shown by the experiments of M. Morin to be of remarkable *uniformity* and *precision*, and that, under an extensive range of variation, as well in the *pressures* by which the surfaces are held in contact, as in the *dimensions* of those surfaces. They are these,—

1. The friction accompanying the motion of two surfaces between which no unguent is interposed, bears the same proportion to the force by which those surfaces are pressed together, whatever may be the amount of that force.

2. This friction is independent of the *extent* of the surfaces of contact.

3. Where unguents are interposed, a distinction is to be made between the case in which the surfaces are simply *unctuous*, and *in intimate contact* with one another, and the case in which the surfaces are wholly *separated* from one another

* Thus, for instance, in the case of oak upon oak with parallel fibres, the co-efficient of friction of quiescence varied under different pressures, but upon the same surface, from ·55 to ·76.

by an *interposed stratum of the unguent.* If the pressure upon a surface of contact of given dimensions be increased beyond a certain limit, the latter of these cases passes into the first; the stratum of unguent being *pressed out,* and the unctuous surfaces which it separated from one another being brought into intimate contact. As long as either of these two states remains, the laws of its friction are not affected by the presence of the unguent; but in the transition from the one state to the other, an exception is made to the independence of the friction upon the extent of the surface of contact; for supposing the extent of two surfaces of contact, between which a stratum of unguent is interposed, and which sustain a given pressure, to be continually *diminished,* it is evident that the portions of this pressure which take effect upon each element of the surfaces of contact will be continually increased, and that they may thus be so increased as to press out the interposed stratum of unguent, and cause the state of the surfaces to pass into that which we have designated as *unctuous,* thereby changing the co-efficient of friction. That law of friction, then, which is known as the law of "the independence of the surface," is to be received, in the case where a stratum of unguents is interposed, only within certain limits.

It will be understood, from what has above been said, that there are three states, in respect to friction, into which the surfaces of bodies in contact may be made successively to pass: *one,* a state in which no unguent is present; the *second,* a state in which the surfaces are unctuous, but intimately in contact; the *third,* a state in which the surfaces are separated by an entire stratum of the interposed unguent. Throughout each of these states the co-efficient of friction is the same; but it is essentially different in the different states, as will be seen from the following tables.

4. It is a law common to the friction of all the states of contact of two surfaces, that their friction, when in motion, is altogether independent of the *velocity* of the motion. M. Morin has verified this law, as well in various states of contact without interposed fluids, as in cases where water, oils, grease,

glutinous liquids, syrups, pitch, &c., were interposed in a continuous stratum.

The variety of the circumstances under which these laws obtain in respect to the friction of motion, and the accuracy with which the phenomena of motion accord with them, may be judged of from one example taken from the first set of experiments of M. Morin upon the friction of surfaces of oak whose fibres were parallel to the direction of their motion upon one another. He caused the surfaces of contact to vary their dimensions in the ratio of 1 to 84, from less than 5 square inches to nearly 3 square feet; the forces which pressed them together, he varied from 88 lbs. to 2205 lbs., and the velocities of their motion, from the slowest perceptible to 9·8 feet per second—causing them to be at one period accelerated motions, at another uniform, at a third retarded; yet throughout all this wide range of variation, he in no instance found the co-efficient of friction to deviate from the same fraction of 0·478 by more than $\frac{1}{24}$th of the amount of that fraction.

TABLE XVI.

Experiments on Friction, without Unguents, by M. Morin.

The surfaces of friction were varied from ·03336 to 2·7987 square feet, the pressures from 88 lbs. to 2205 lbs., and the velocities from a scarcely perceptible motion to 9·84 feet per second.

SURFACES OF CONTACT.	*Friction of Motion.* N.B. The Friction in this case varies but very slightly from the mean.		*Friction of Quiescence.* N.B. The Friction in this case varies considerably from the mean. In all the experiments the surface had been 15 minutes in contact.		REMARKS. The surfaces of wood were planed, and those of metal filed and polished with the greatest care, and carefully wiped after every experiment. The presence of unguents was especially guarded against.
	Co-efficient of Friction.	Limiting Angle of Resistance.	Co-efficient of Friction.	Limiting Angle of Resistance.	
Oak upon oak, the direction of the fibres being parallel to the motion	0·478	25° 33′	0·625	32° 1′	
Oak upon oak, the directions of the fibres of the moving surface being perpendicular to those of the quiescent surface and to the direction of the motion	0·324	17 58	0·540	28 23	The dimensions of the surfaces of contact were in this experiment ·947 square feet, and the results were nearly uniform. When the dimensions were diminished to ·043, a tearing of the fibre became apparent in the case of motion, and there were symptoms of the combustion of the wood; from these circumstances there resulted an irregularity in the friction, indicative of excessive pressure.

TABLE XVI. (*continued.*)

SURFACES OF CONTACT.	*Friction of Motion.* N.B. The Friction in this case varies but very slightly from the mean.		*Friction of Quiescence.* N.B. The Friction in this case varies considerably from the mean. In all the experiments the surface had been 15 minutes in contact.		REMARKS. The surfaces of wood were planed, and those of metal filed and polished with the greatest care, and carefully wiped after every experiment. The presence of unguents was especially guarded against.
	Co-efficient of Friction.	Limiting Angle of Resistance.	Co-efficient of Friction.	Limiting Angle of Resistance.	
Oak upon oak, the fibres of both surfaces being perpendicular to the direction of the motion - - -	0·336	18° 35′			
Oak upon oak, the fibres of the moving surface being perpendicular to the surface of contact, and those of the surface at rest parallel to the direction of the motion - - -	0·192	10 52	0·271	15° 10′	
Oak upon oak, the fibres of both surfaces being perpendicular to the surface of contact, or the pieces end to end - - - - -	- -	- -	0·43	23 17	
Elm upon oak, the direction of the fibres being parallel to the motion - -	0·432	23 22	0·694	34 46	It is worthy of remark that the friction of oak upon elm is but 5·9ths of that of elm upon oak.
Oak upon elm, ditto - - -	0·246	13 50	0·376	20 37	
Elm upon oak, the fibres of the moving surface (the elm) being perpendicular to those of the quiescent surface (the oak) and to the direction of the motion - - - -	0·450	24 16	0·570	29 41	

Ash upon oak, the fibres of both surfaces being parallel to the direction of the motion - - - -	0·400	21 49	0·570	29 41	
Fir upon oak, the fibres of both surfaces being parallel to the direction of the motion - - - -	0·355	19 33	0·520	27 29	
Beech upon oak, ditto - - -	0·360	19 48	0·53	27 56	
Wild pear-tree upon oak, ditto - -	0·370	20 19	0·440	23 45	
Service-tree upon oak, ditto - -	0·400	21 49	0·570	29 41	
Wrought iron upon oak, ditto - -	0·619	31 47	0·619	31 47	In the experiments in which one of the surfaces was of metal, small particles of the metal began, after a time, to be apparent upon the wood, giving it a polished metallic appearance; these were at every experiment wiped off; they indicated a wearing of the metal. The friction of motion and that of quiescence, in these experiments, coincided. The results were remarkably uniform.
Ditto, the surfaces being greased and well wetted - - -	0·256	14 22	0·649	33 0	
Wrought iron upon elm - -	0·252	14 9	- -	- -	
Wrought iron upon cast iron, the fibres of the iron being parallel to the motion	0·194	10 59	0·194	10 59	
Wrought iron upon wrought iron, the fibres of both surfaces being parallel to the motion - - -	0·138	7 52	0·137	7 49	
Cast iron upon oak, ditto - - -	0·490	26 7			
Ditto, the surfaces being greased and wetted - - - -	- -	- -	0·646	32 52	
Cast iron upon elm - - -	0·195	11 3			
Cast iron upon cast iron - -	0·152	8 39	0·162	9 13	
Ditto, water being interposed between the surfaces - - -	0·314	17 26			
Cast iron upon brass - - -	0·147	8 22			

TABLE XVI. (*continued.*)

SURFACES OF CONTACT.	*Friction of Motion.* N.B. The Friction in this case varies but very slightly from the mean.		*Friction of Quiescence.* N.B. The Friction in this case varies considerably from the mean. In all the experiments the surface had been 15 minutes in contact.		REMARKS. The surfaces of wood were planed, and those of metal filed and polished with the greatest care, and carefully wiped after every experiment. The presence of unguents was especially guarded against.
	Co-efficient of Friction.	Limiting Angle of Resistance.	Co-efficient of Friction.	Limiting Angle of Resistance.	
Oak upon cast iron, the fibres of the wood being perpendicular to the direction of the motion -	0·372	20° 25′			
Hornbeam upon cast iron — fibres parallel to motion - - -	0·394	21 31			
Wild pear-tree upon cast iron ditto -	0·436	23 34			
Steel upon cast iron - - -	0·202	11 26			
Steel upon brass - - -	0·152	8 39			
Yellow copper upon cast iron - -	0·189	10 49			
Ditto oak - -	0·617	31 41	0·617	31° 41′	
Brass upon cast iron - - -	0·217	12 15			
Brass upon wrought iron, the fibres of the iron being parallel to the motion	0·161	9 9			
Wrought iron upon brass - -	0·172	9 46			
Brass upon brass - - -	0·201	11 22			
Black leather (curried) upon oak, ditto	0·265	14 51	0·74	36 31	The friction of motion was very nearly the same whether the surface of contact was the inside or the outside of the skin.
Ox hide (such as that used for soles and for the stuffing of pistons) upon oak, ditto - - - -	rough 0·52 smooth 0·335	27 29 18 31	rough 0·605 smooth 0·43	31 11 23 17	The *constancy* of the co-efficient of the friction of motion was equally apparent in the rough and the smooth skins.
Leather as above, polished and hardened by hammering	0·296	16 30	- -	- -	

Hempen girth, or pulley band (sangle de chanvre), upon oak, the fibres of the wood and the direction of the cord being *parallel* to the motion -	0·52	27 29	0·64	32 38	
Hempen matting, woven with small cords, ditto - - -	0·32	17 45	0·50	26 34	All the above experiments, except that with curried black leather, presented the phenomenon of a change in the polish of the surfaces of the friction—a state of their surfaces necessary to, and dependent upon, the state of their motion upon one another.
Old cordage 1½ inch in diameter, ditto -	0·52	27 29	0·79	38 19	
Calcareous oolitic stone, used in building, of a moderately hard quality, called stone of Jaumont—upon the same stone - - -	0·64	32 38	0·74	36 31	
Hard calcareous stone of Brouck, of a light grey colour, susceptible of taking a fine polish (the muschelkalk), moving upon the same stone -	0·38	20 49	0·70	35 0	
The soft stone mentioned above, upon the hard - - -	0·65	33 2	0·75	36 53	
The hard stone mentioned above, upon the soft - - -	0·67	33 50	0·75	36 53	
Common brick upon the stone of Jaumont - - - -	0·65	33 2	0·65	33 2	
Oak upon ditto, the fibres of the wood being perpendicular to the surface of the stone - - -	0·38	20 49	0·63	32 13	
Wrought iron upon ditto, ditto - -	0·69	34 37	0·49	26 7	
Common brick upon the stone of Brouck	0·60	30 58	0·67	33 50	
Oak as before (endwise) upon ditto -	0·38	20 49	0·64	32 38	
Iron, ditto ditto -	0·24	13 30	0·42	22 47	

TABLE XVII.

Experiments on the Friction of Unctuous Surfaces by M. Morin.

In these experiments the surfaces, after having been smeared with an unguent, were wiped, so that no interposing layer of the unguent prevented their intimate contact.

SURFACES OF CONTACT.	Friction of Motion. Co-efficient of Friction.	Friction of Motion. Limiting Angle of Resistance.	Friction of Quiescence. Co-efficient of Friction.	Friction of Quiescence. Limiting Angle of Resistance.
Oak upon oak, the fibres being parallel to the motion	0·108	6° 10′	0·390	21° 19′
Ditto, the fibres of the moving body being perpendicular to the motion	0·143	8 9	0·314	17 26
Oak upon elm, fibres parallel	0·136	7 45		
Elm upon oak, ditto	0·119	6 48	0·420	22 47
Beech upon oak, ditto	0·330	18 16		
Elm upon elm, ditto	0·140	7 59		
Wrought iron upon elm, ditto	0·138	7 52		
Ditto upon wrought iron, ditto	0·177	10 3		
Ditto upon cast iron, ditto	- -	- -	0·118	6 44
Cast iron upon wrought iron, ditto	0·143	8 9		
Wrought iron upon brass, ditto	0·160	9 6		
Brass upon wrought iron	0·166	9 26		
Cast iron upon oak, ditto	0·107	6 7	0·100	5 43
Ditto upon elm, ditto, the unguent being tallow	0·125	7 8		
Ditto, ditto, the unguent being hog's lard and black lead	0·137	7 49		
Elm upon cast iron, fibres parallel	0·135	7 42	0·098	5 36
Cast iron upon cast iron	0·144	8 12		
Ditto upon brass	0·132	7 32		
Brass upon cast iron	0·107	6 7		
Ditto upon brass	0·134	7 38	0·164	9 19
Copper upon oak	0·100	5 43		
Yellow copper upon cast iron	0·115	6 34		
Leather (ox hide) well tanned upon cast iron, wetted	0·229	12 54	0·267	14 57
Ditto upon brass, wetted	0·244	13 43		

The distinction between the friction of surfaces to which no unguent is present, those which are merely unctuous, and those

between which a uniform stratum of the unguent is interposed, appears first to have been remarked by M. Morin; it has suggested to him what appears to be the true explanation of the difference between his results and those of Coulomb. He conceives, that in the experiments of this celebrated engineer the requisite precautions had not been taken to exclude unguents from the surfaces of contact. The slightest unctuosity, such as might present itself accidentally, unless expressly guarded against—such, for instance, as might have been left by the hands of the workman who had given the last polish to the surfaces of contact—is sufficient materially to affect the co-efficient of friction.

Thus, for instance, surfaces of oak having been rubbed with hard dry soap, and then thoroughly wiped, so as to show no traces whatever of the unguent, were found by its presence to have lost ⅔ds of their friction, the co-efficient having passed from 0·478 to 0·164.

This effect of the unguent upon the friction of the surfaces may be traced to the fact, that their motion upon one another without unguents was always found to be attended by a wearing of both the surfaces; small particles of a dark colour continually separated from them, which it was found from time to time necessary to remove, and which manifestly influenced the friction: now with the presence of an unguent the formation of these particles, and the consequent wear of the surfaces, completely ceased. Instead of a new surface of contact being continually presented by the wear, the same surface remained, receiving by the motion continually a more perfect polish.

TABLE XVIII.

Experiments on Friction with Unguents interposed, by M. Morin.

The extent of the surfaces in these experiments bore such a relation to the pressure, as to cause them to be separated from one another throughout by an interposed stratum of the unguent.

SURFACES OF CONTACT.	Friction of Motion. Co-efficient of Friction.	Friction of Quiescence. Co-efficient of Friction.	UNGUENTS.
Oak upon oak, fibres parallel	0·164	0·440	Dry soap.
Ditto ditto - -	0·075	0·164	Tallow.
Ditto ditto - -	0·067	- -	Hogs' lard.
Ditto, fibres perpendicular	0·083	0·254	Tallow.
Ditto ditto - -	0·072	- -	Hogs' lard.
Ditto ditto - -	0·250	- -	Water.
Ditto upon elm, fibres parallel	0·136	- -	Dry soap.
Ditto ditto - -	0·073	0·178	Tallow.
Ditto ditto - -	0·066	- -	Hogs' lard.
Ditto upon cast iron, ditto -	0·080	- -	Tallow.
Ditto upon wrought iron, ditto	0·098	- -	Tallow.
Beech upon oak, ditto -	0·055	- -	Tallow.
Elm upon oak, ditto -	0·137	0·411	Dry soap.
Ditto ditto -	0·070	0·142	Tallow.
Ditto ditto -	0·060	- -	Hogs' lard.
Ditto upon elm, ditto -	0·139	0·217	Dry soap.
Ditto upon cast iron, ditto -	0·066	- -	Tallow.
Wrought iron upon oak, ditto	0·256	0·649	Greased and saturated with water.
Ditto ditto ditto -	0·214	- -	Dry soap.
Ditto ditto ditto -	0·085	0·108	Tallow.
Ditto upon elm, ditto -	0·078	- -	Tallow.
Ditto ditto ditto -	0·076	- -	Hogs' lard.
Ditto ditto ditto -	0·055	- -	Olive oil.
Ditto upon cast iron, ditto	0·103	- -	Tallow.
Ditto ditto ditto -	0·076	- -	Hogs' lard.
Ditto ditto ditto -	0·066	0·100	Olive oil.
Ditto upon wrought iron, ditto	0·082	- -	Tallow.
Ditto ditto ditto -	0·081	- -	Hogs' lard.
Ditto ditto ditto -	0·070	0·115	Olive oil.

TABLE XVIII. (*continued.*)

SURFACES OF CONTACT.	FRICTION OF MOTION. Co-efficient of Friction.	FRICTION OF QUIESCENCE. Co-efficient of Friction.	UNGUENTS.
Wrought iron upon brass, fibres parallel -	0·103	- -	Tallow.
Ditto ditto ditto -	0·075	- -	Hogs' lard.
Ditto ditto ditto -	0·078	- -	Olive oil.
Cast iron upon oak, ditto -	0·189	- -	Dry soap.
Ditto ditto ditto -	0·218	0·646	Greased, and saturated with water.
Ditto ditto ditto -	0·078	0·100	Tallow.
Ditto ditto ditto -	0·075	- -	Hogs' lard.
Ditto ditto ditto -	0·075	0·100	Olive oil.
Ditto upon elm, ditto -	0·077	- -	Tallow.
Ditto ditto ditto -	0·061	- -	Olive oil.
Ditto ditto ditto -	0·091	- -	Hogs' lard and plumbago.
Ditto, ditto upon wrought iron - - -	- -	0·100	Tallow.
Ditto upon cast iron -	0·314	- -	Water.
Ditto ditto - -	0·197	- -	Soap.
Ditto ditto - -	0·100	0·100	Tallow.
Ditto ditto - -	0·070	0·100	Hogs' lard.
Ditto ditto - -	0·064	- -	Olive oil.
Ditto ditto - -	0·055	- -	Lard and plumbago.
Ditto upon brass - -	0·103	- -	Tallow.
Ditto ditto - -	0·075	- -	Hogs' lard.
Ditto ditto - -	0·078	- -	Olive oil.
Copper upon oak, fibres parallel - -	0·069	0·100	Tallow.
Yellow copper upon cast iron	0·072	0·103	Tallow.
Ditto ditto - -	0·068	- -	Hogs' lard.
Ditto ditto - -	0·066	- -	Olive oil.
Brass upon cast iron -	0·086	0·106	Tallow.
Ditto ditto - -	0·077	- -	Olive oil.
Ditto upon wrought iron -	0·081	- -	Tallow.
Ditto ditto - -	0·089	- -	Lard and plumbago.
Ditto ditto - -	0·072	- -	Olive oil.
Ditto upon brass - -	0·058	- -	Olive oil.
Steel upon cast iron -	0·105	0·108	Tallow.
Ditto ditto - -	0·081	- -	Hogs' lard.
Ditto ditto - -	0·079	- -	Olive oil.
Ditto upon wrought iron -	0·093	- -	Tallow.
Ditto ditto - -	0·076	- -	Hogs' lard.
Ditto upon brass - -	0·056	- -	Tallow.
Ditto ditto - -	0·053	- -	Olive oil.
Ditto ditto - -	0·067	- -	Lard and plumbago.
Tanned ox hide upon cast iron - - -	0·365	- -	Greased, and saturated with water.

TABLE XVIII. (*continued.*)

SURFACES OF CONTACT.	Friction of Motion. Co-efficient of Friction.	Friction of Quiescence. Co-efficient of Friction.	UNGUENTS.
Tanned ox hide upon cast iron - - -	0·159	- -	Tallow.
Ditto ditto - -	0·133	0·122	Olive oil.
Ditto upon brass - -	0·241	- -	Tallow.
Ditto ditto - -	0·191	- -	Olive oil.
Ditto upon oak - -	0·29	0·79	Water.
Hempen fibres not twisted, moving upon oak, the fibres of the hemp being placed in a direction perpendicular to the direction of the motion, and those of the oak parallel to it -	0·332	0·869	Greased, and saturated with water.
The same as above, moving upon cast iron - -	0·194	- -	Tallow.
Ditto ditto - -	0·153	- -	Olive oil.
Soft calcareous stone of Jaumont upon the same, with a layer of mortar, of sand, and lime, interposed after from 10 to 15 minutes' contact - - -	- -	0·74	

A comparison of the results enumerated in the above table leads to the following remarkable conclusion, easily fixing itself in the memory, *that with the unguents hogs' lard and olive oil interposed in a continuous stratum between them, surfaces of wood on metal, wood on wood, metal on wood, and metal on metal, when in motion, have all of them very nearly the same co-efficient of friction, the value of that co-efficient being in all cases included between* 0·07 *and* 0·08, *and the limiting angle of resistance therefore between* 4° *and* 4° 35′.

For the unguent tallow the co-efficient is the same as the above in every case, except in that of metals upon metals; this unguent seems less suited to metallic surfaces than the others, and gives for the mean value of its co-efficient 0·10, *and for its limiting angle of resistance* 5° 43′.

The experiments of which the above are results were all made under considerable pressures, such as those under which

the parts of the larger machines are accustomed to move upon one another: under such pressures the adhesion of the unguent to the surfaces of contact, and the opposition presented to their motion by its viscidity, are causes whose influence may be altogether neglected as compared with the friction. In the case of lighter machinery, as, for instance, that of clocks and watches, these considerations rise, however, into importance.

TABLE XIX.

Comparison of French and English Measures.

MEASURES OF LENGTH.	
French.	English.
Millimètre - -	0·03937 inch.
Centimètre - -	0·393708 inch.
Decimètre - -	3·937079 inches.
Mètre - -	39·37079 inches. 3·2808992 feet. 1·093633 yard.
Myriamètre - -	6·2138 miles.
MEASURES OF SUPERFICIES.	
Mètre Carré - -	1·196033 square yard.
Are - -	0·098845 rood.
Hectare - -	2·471143 acres.
MEASURES OF CAPACITY.	
Litre - -	1·760773 pint. 0·2200967 gallon.
Decalitre - -	2·2009668 gallons.
Hectolitre - -	22·009668 gallons.

TABLE XIX. (*continued.*)

MEASURES OF WEIGHT.	
French.	English.
Gramme - - -	15·438 grains, troy. 0·643 pennyweights, troy. 0·03216 ounces, troy.
Kilogramme - -	2·68027 pounds, troy. 2·20548 pounds, avoirdupois.

THE END.

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